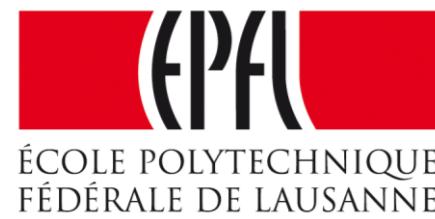


Week 3 – part 1 : Reduction of the Hodgkin-Huxley Model



Biological Modeling of Neural Networks

**Week 3 – Reducing detail:
Two-dimensional neuron models**

Wulfram Gerstner

EPFL, Lausanne, Switzerland

3.1 From Hodgkin-Huxley to 2D

- Overview: From 4 to 2 dimensions
- MathDetour 1: Exploiting similarities
- MathDetour 2: Separation of time scales

3.2 Phase Plane Analysis

- Role of nullclines

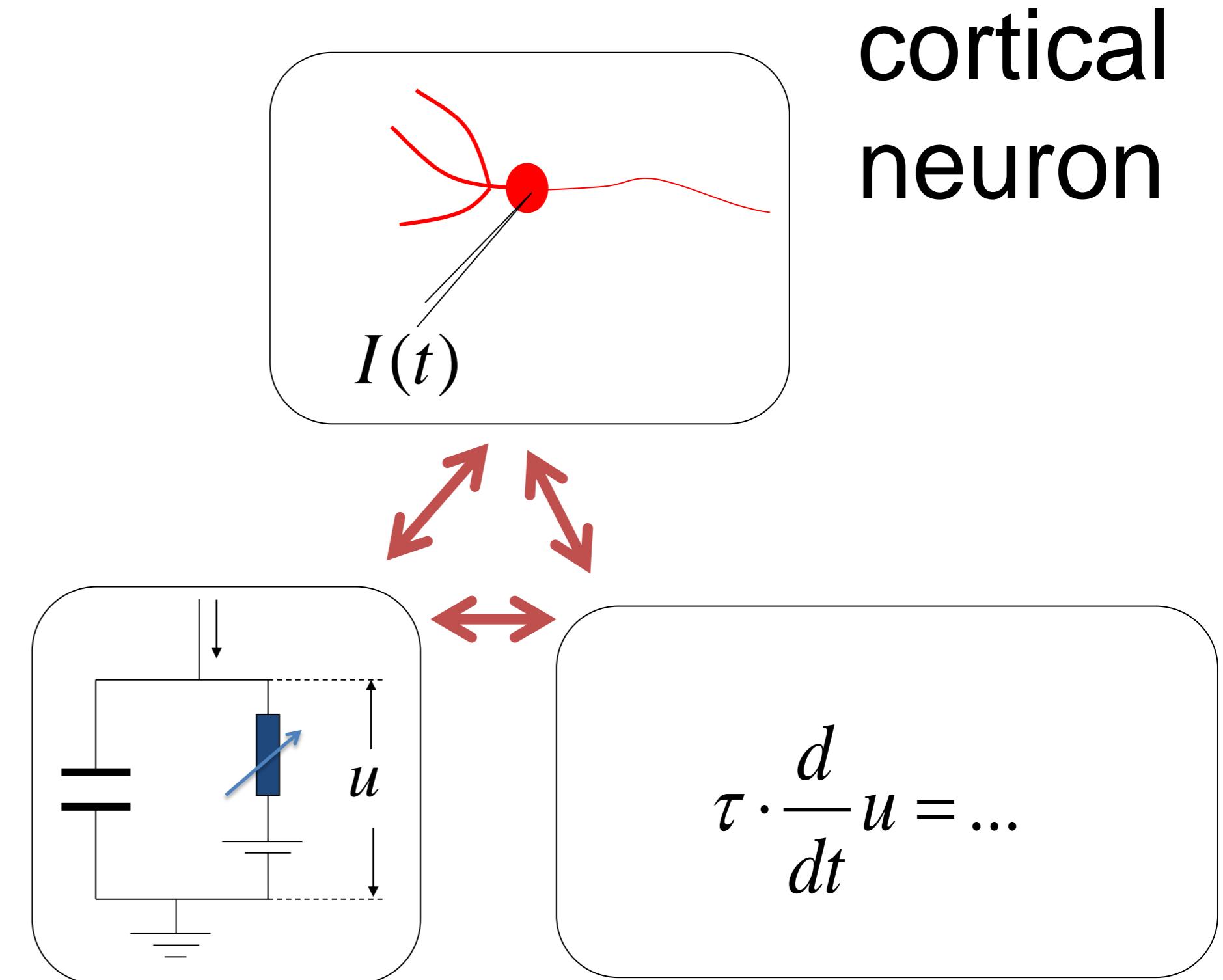
3.3 Analysis of a 2D Neuron Model

- constant input vs pulse input
- MathDetour 3: Stability of fixed points

3.4 Type I and II Neuron Models

next week!

Neuronal Dynamics – 3.1. Review :Hodgkin-Huxley Model



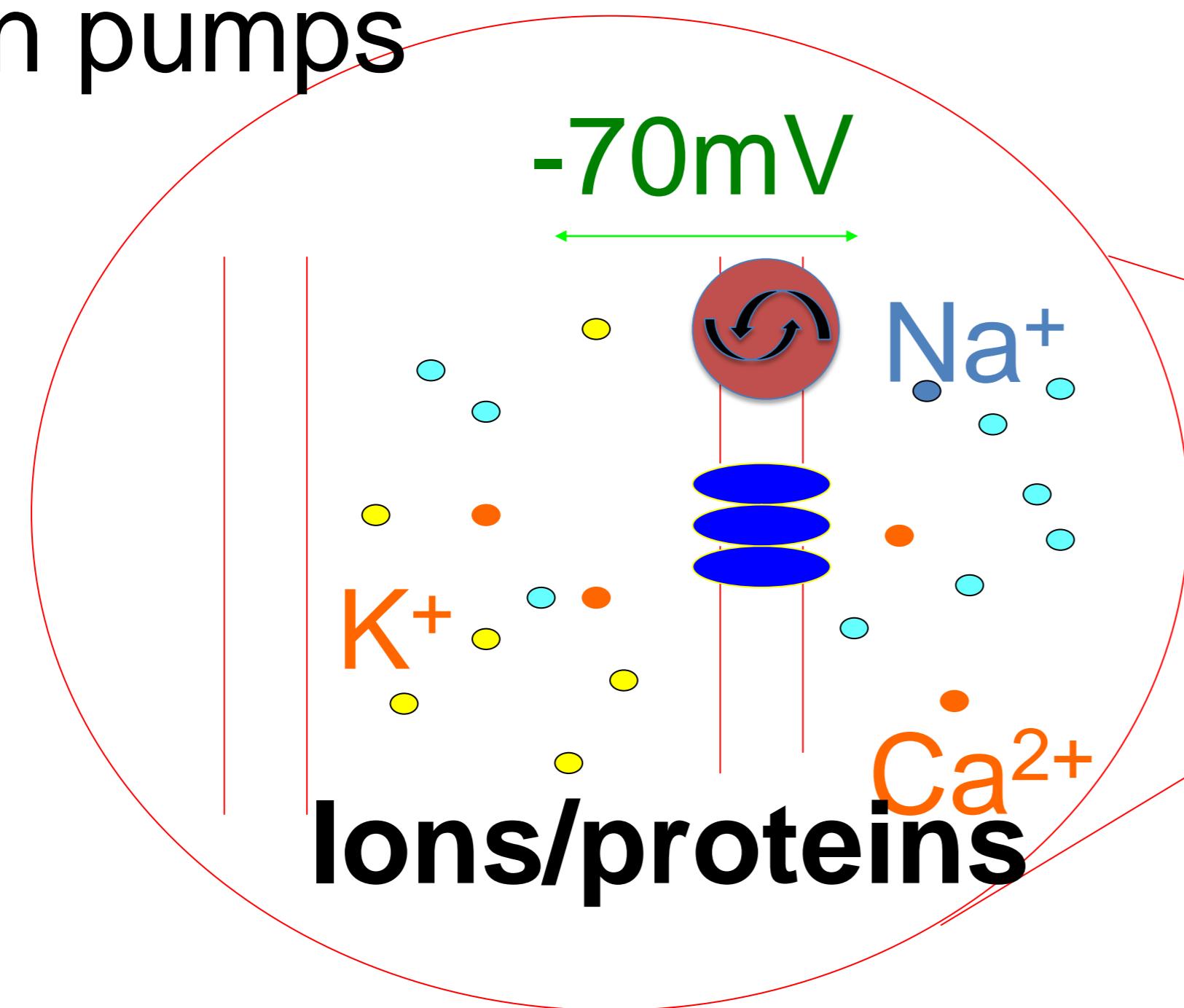
→Hodgkin-Huxley model
→Compartmental models

Neuronal Dynamics – 3.1 Review :Hodgkin-Huxley Model

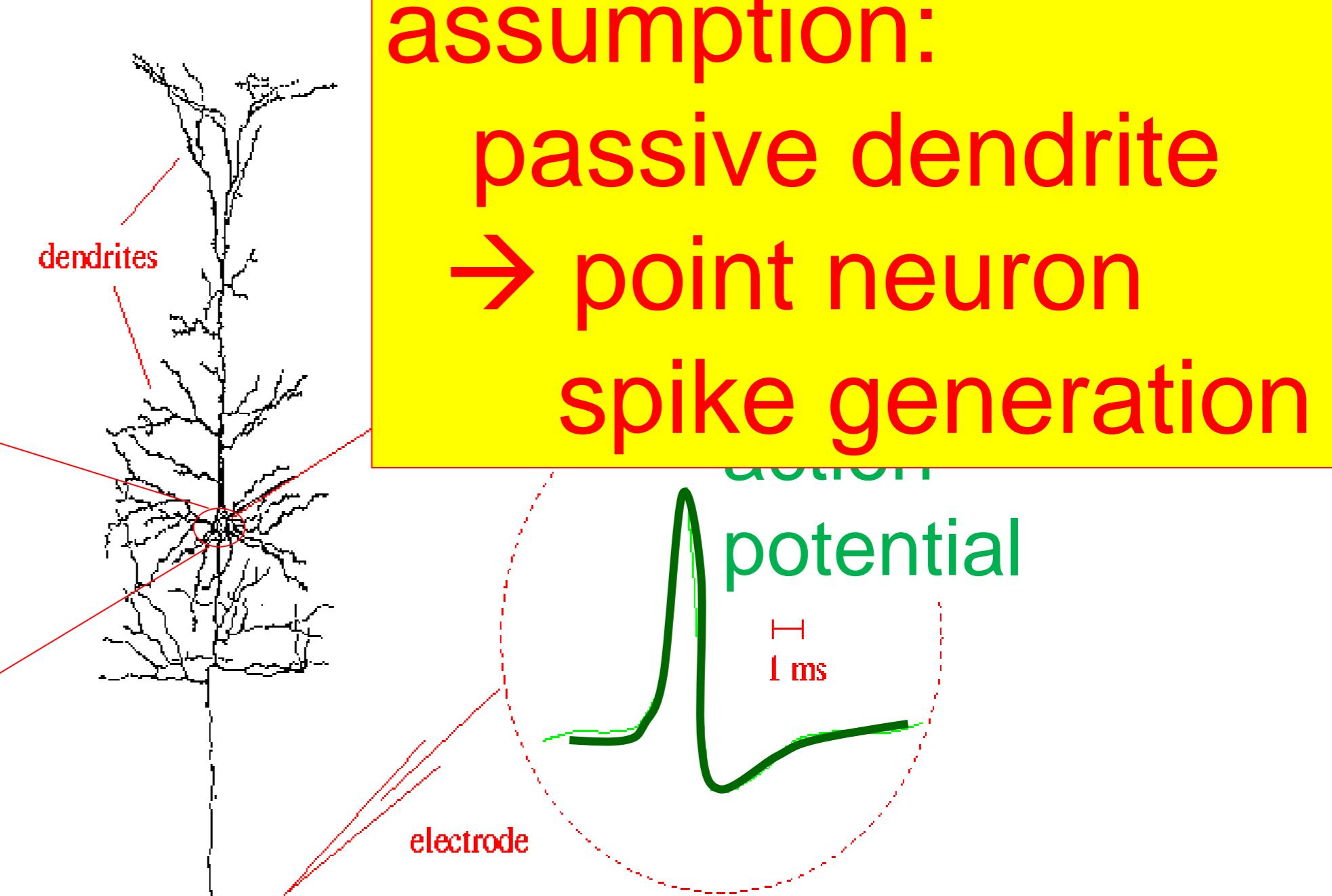
Week 2:

Cell membrane contains

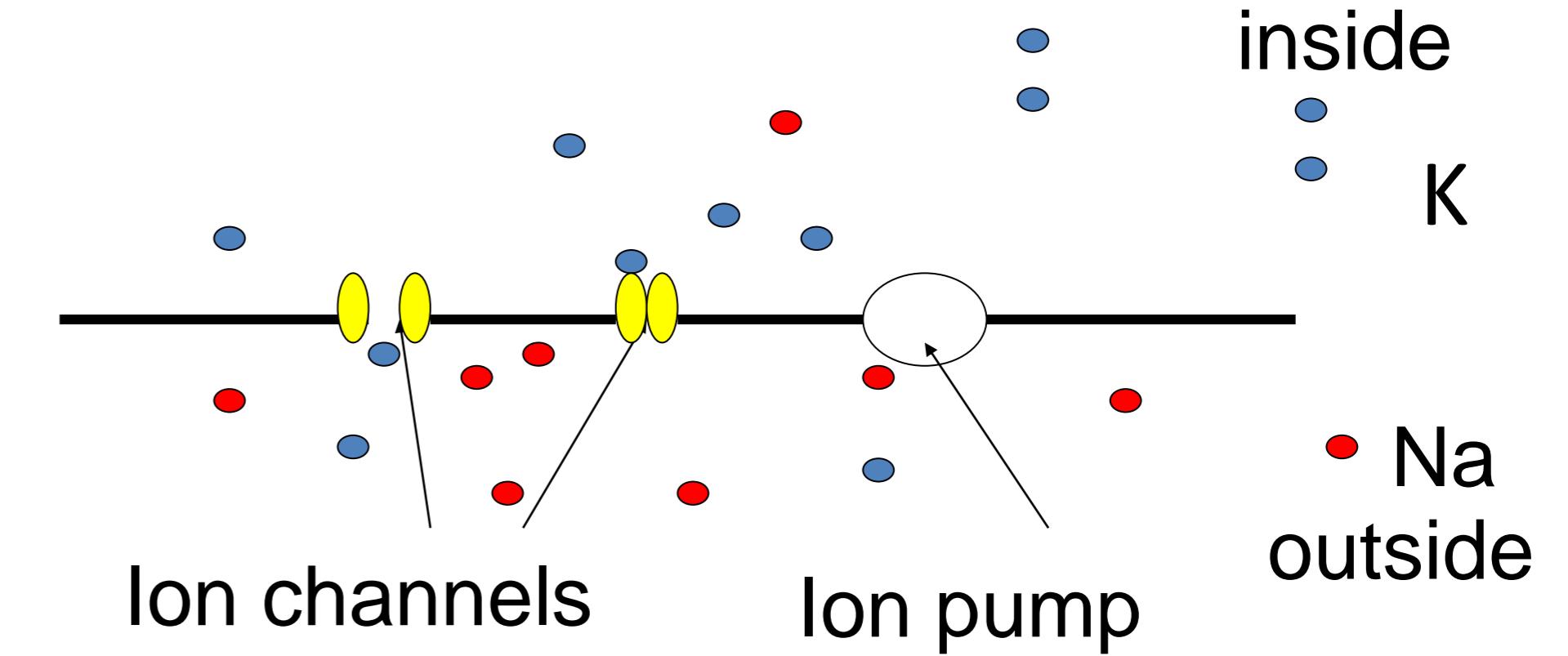
- ion channels
- ion pumps



Dendrites (week 4):
Active processes?



Neuronal Dynamics – 3.1. Review :Hodgkin-Huxley Model



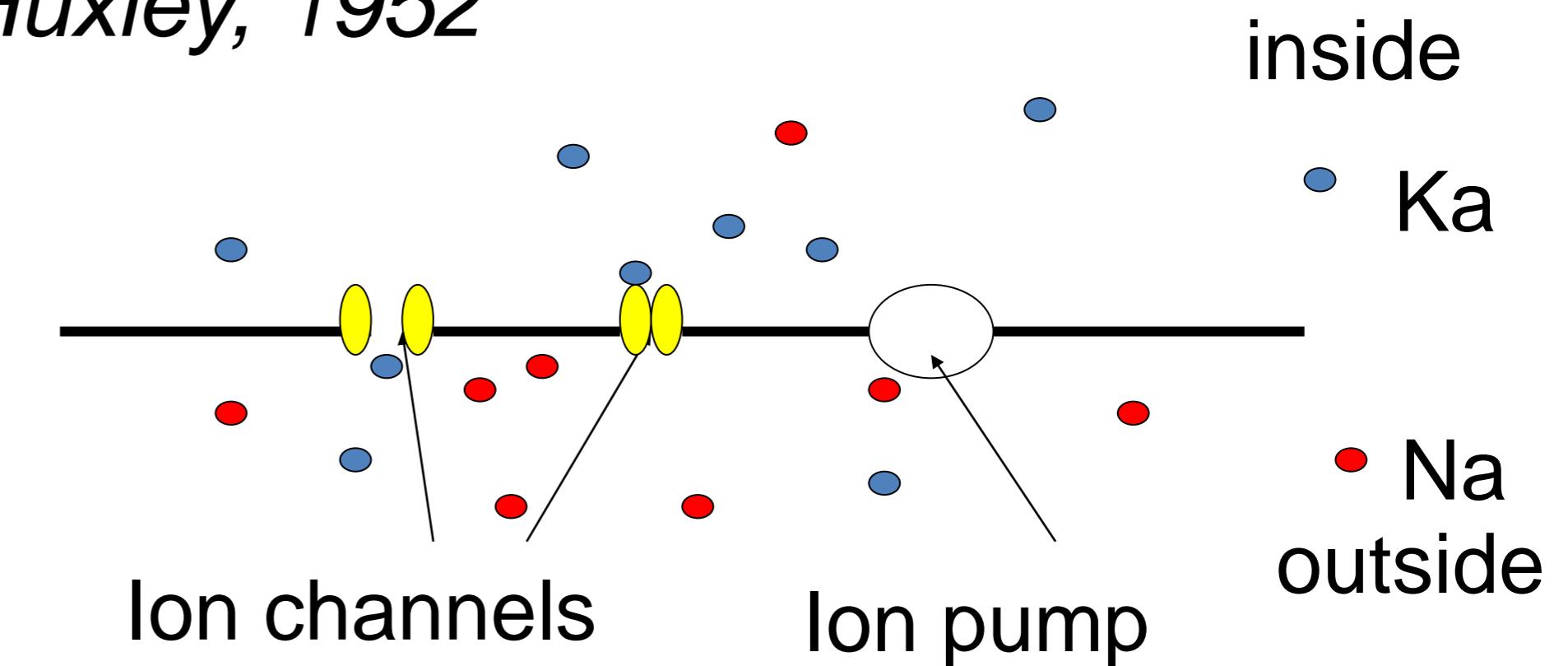
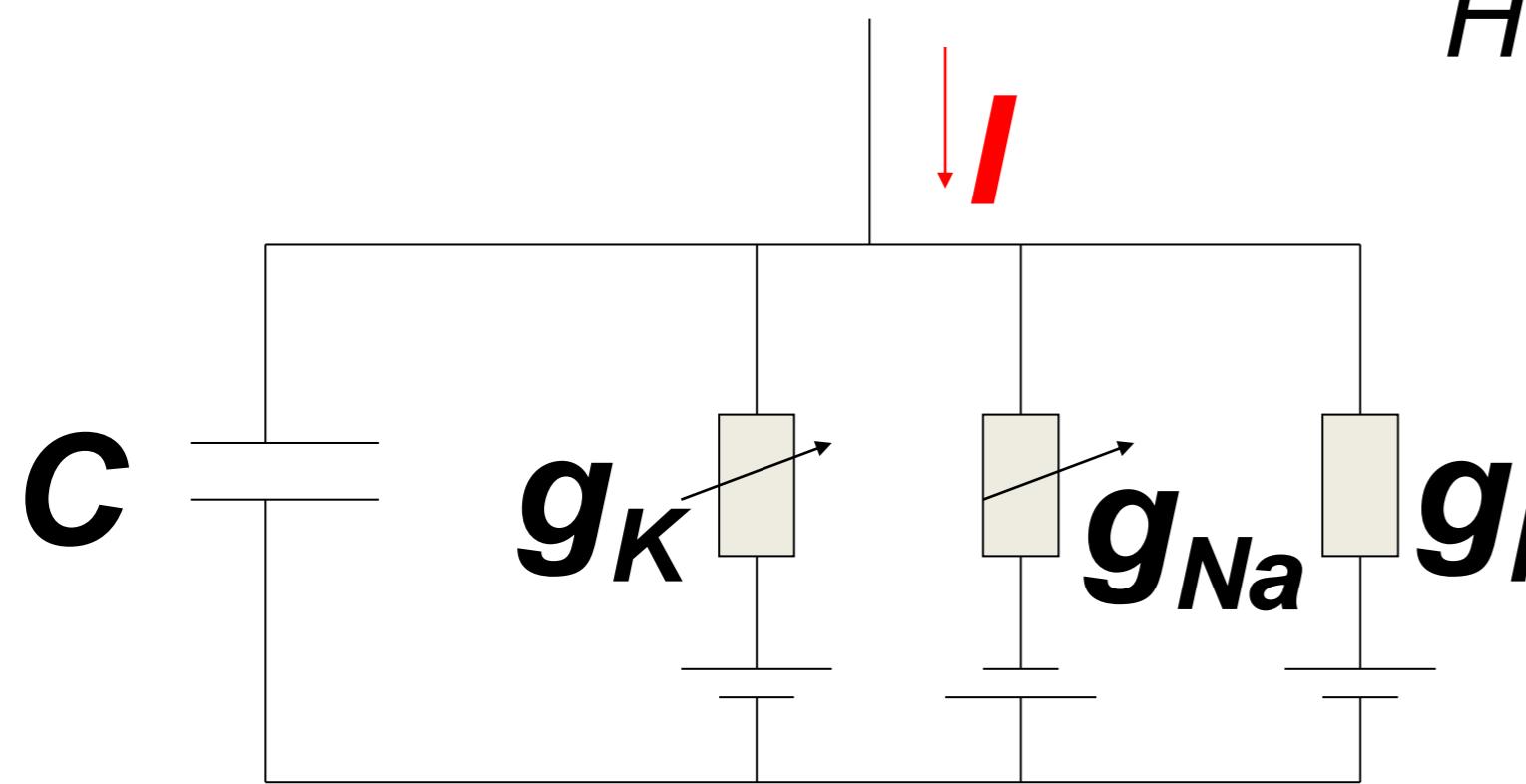
$$\Delta u = u_1 - u_2 = \frac{-kT}{q} \ln \frac{n(u_1)}{n(u_2)}$$

Reversal potential

ion pumps → concentration difference ⇔ voltage difference

Neuronal Dynamics – 3.1. Review: Hodgkin-Huxley Model

Hodgkin and Huxley, 1952

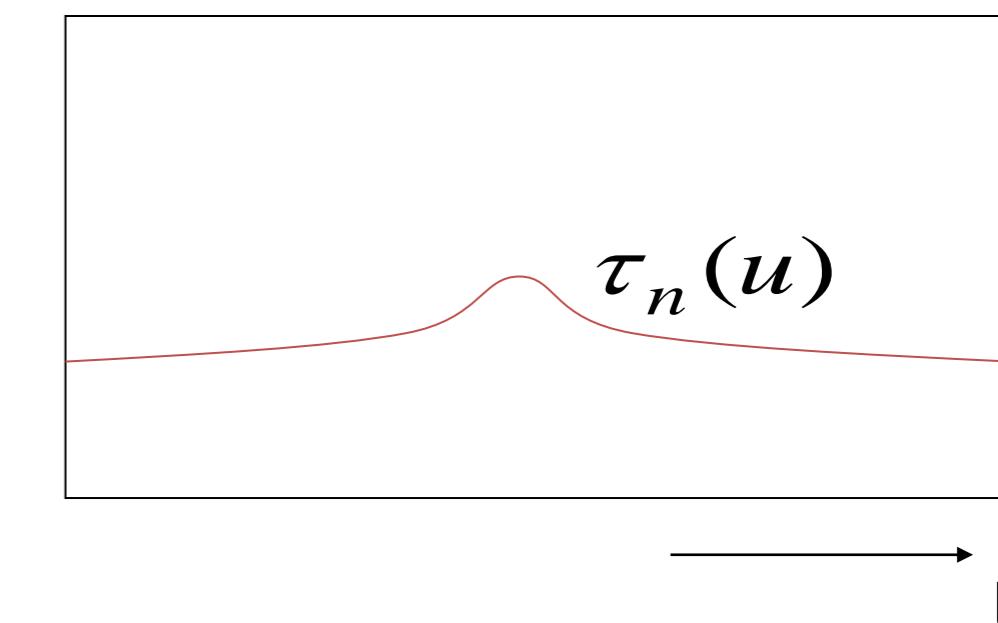
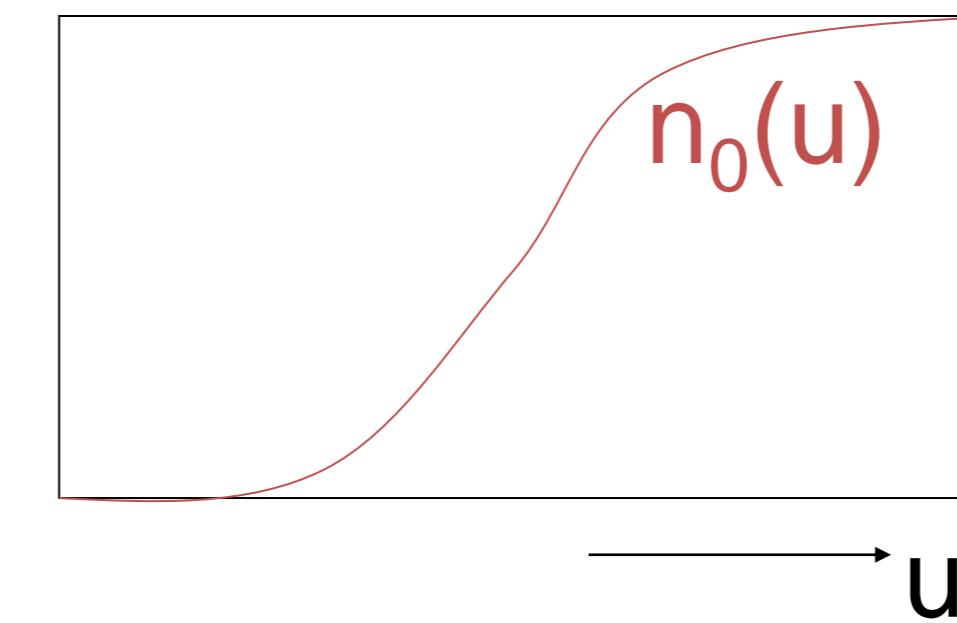


$$C \frac{du}{dt} = -g_{Na} m^3 h (u - E_{Na}) - g_K n^4 (u - E_K) - g_l (u - E_l) + I(t)$$

Annotations above the equation show I_{Na} , I_K , and I_{leak} with curly braces, and a red "stimulus" arrow pointing down to the $I(t)$ term.

**4 equations
= 4D system**

$$\frac{dm}{dt} = \frac{h m - h m \bar{m}(u) u}{\tau_m \tau_h(u) u}$$



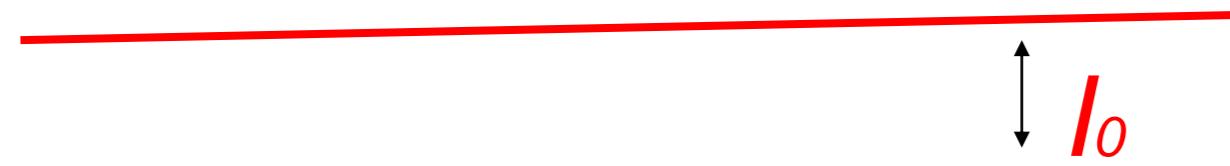
Neuronal Dynamics – 3.1. Overview and aims

Can we understand the dynamics of the HH model?

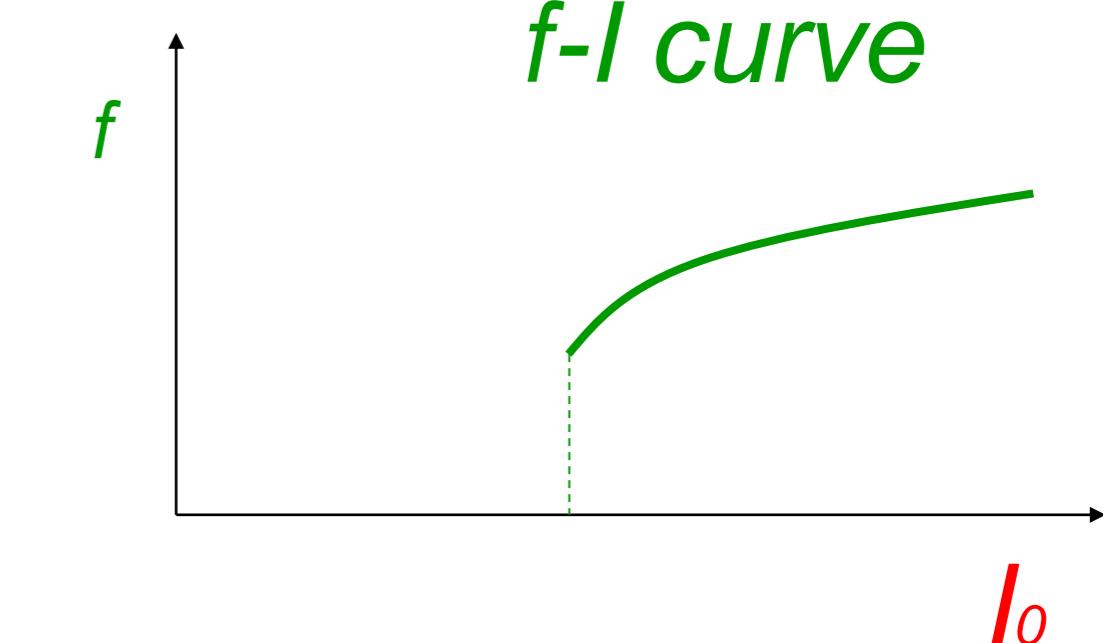
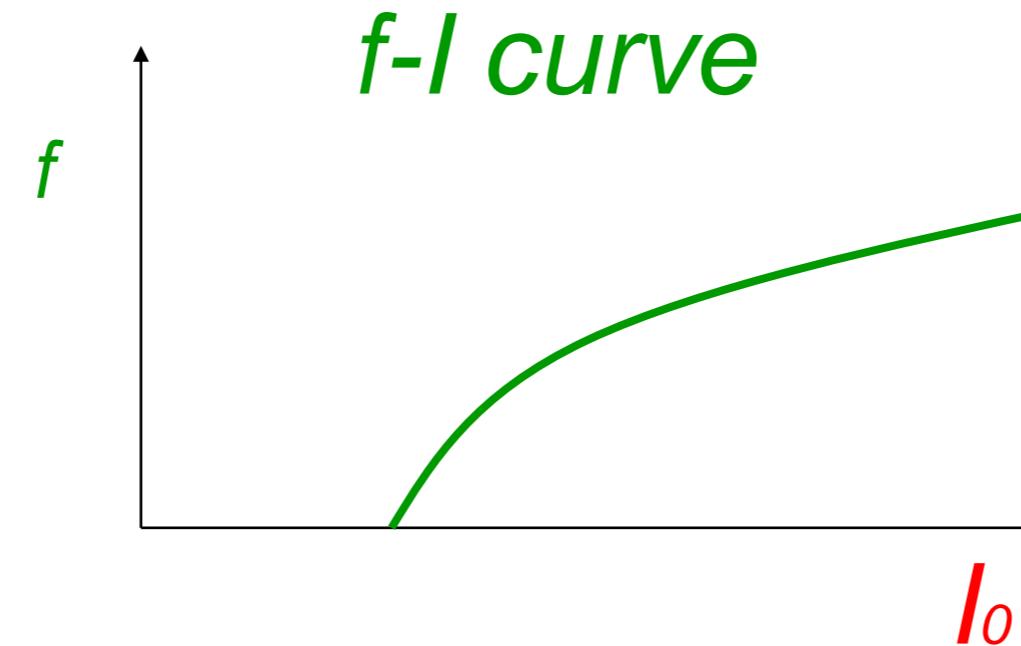
- mathematical principle of Action Potential generation?
- constant input current vs pulse input?
- Types of neuron model (type I and II)? (next week)
- threshold behavior? (next week)

→ Reduce from 4 to 2 equations

ramp input/
constant input



Type I and type II models



Neuronal Dynamics – 3.1. Overview and aims

Can we understand the dynamics of the HH model?

→ Reduce from 4 to 2 equations

Neuronal Dynamics – Quiz 3.1.

A - A biophysical point neuron model

with 3 ion channels,

each with activation and inactivation,

has a total number of equations equal to

- 3 or
- 4 or
- 6 or
- 7 or
- 8 or more

Neuronal Dynamics – 3.1. Overview and aims

Toward a
two-dimensional neuron model

- Reduction of Hodgkin-Huxley to 2 dimension
 - step 1: separation of time scales
 - step 2: exploit similarities/correlations

Neuronal Dynamics – 3.1. Reduction of Hodgkin-Huxley model

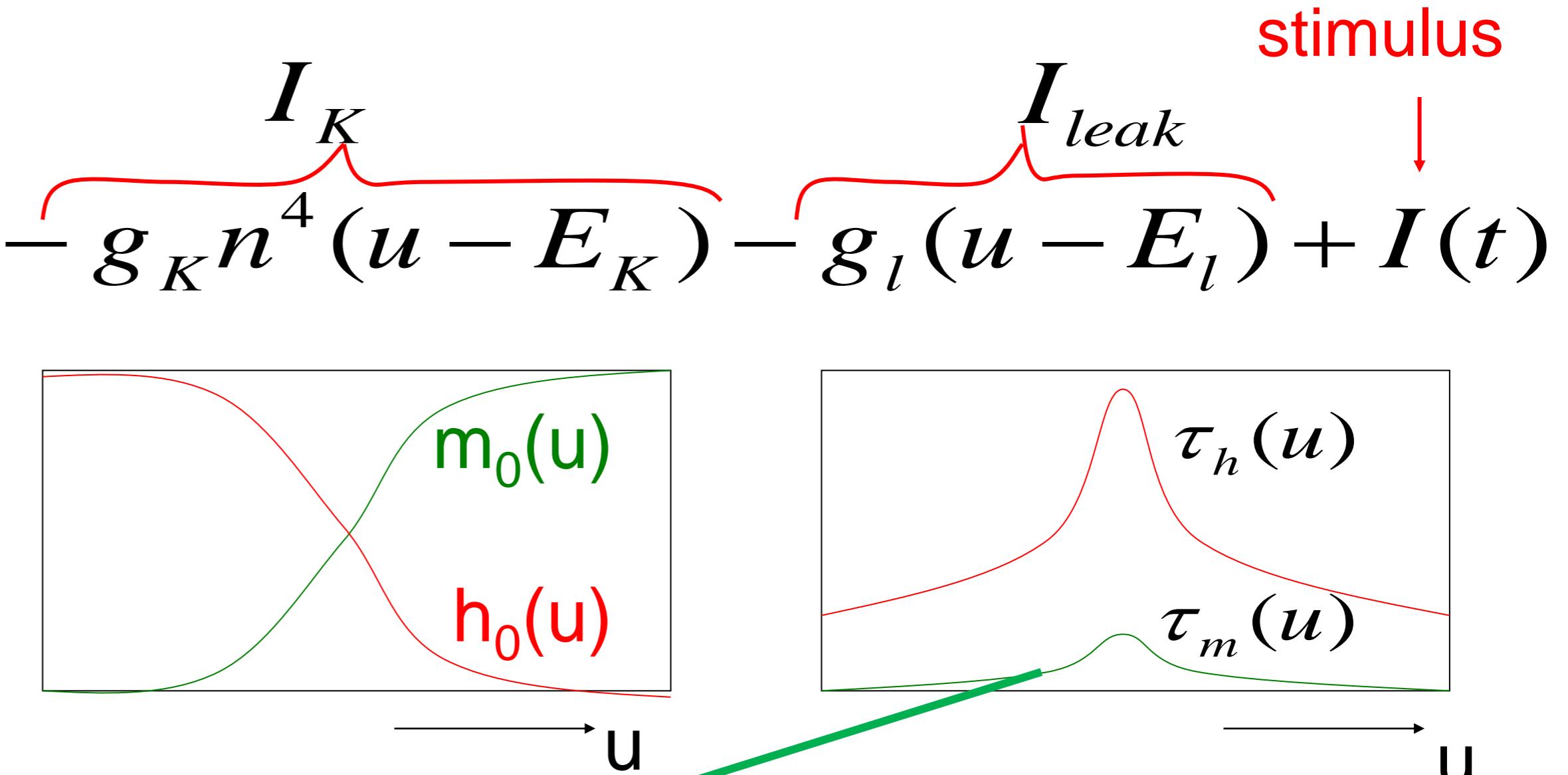
$$C \frac{du}{dt} = -g_{Na} m^3 h (u - E_{Na}) - g_K n^4 (u - E_K) - g_l (u - E_l) + I(t)$$

$$\frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n - n_0(u)}{\tau_n(u)}$$

1) dynamics of m are fast



MathDetour

$$m(t) = m_0(u(t))$$

Neuronal Dynamics – MathDetour 3.1: Separation of time scales

$$\tau_1 \frac{dx}{dt} = -x + c(t)$$

*Exercise 1 (week 3)
(later !)*

Two coupled differential equations

$$\tau_1 \frac{dx}{dt} = -x + h(y)$$

$$\tau_2 \frac{dy}{dt} = f(y) + g(x)$$

Separation of time scales

$$\tau_1 \ll \tau_2 \rightarrow x = h(y)$$

Reduced 1-dimensional system

$$\tau_2 \frac{dy}{dt} = f(y) + g(h(y))$$

Neuronal Dynamics – 3.1. Reduction of Hodgkin-Huxley model

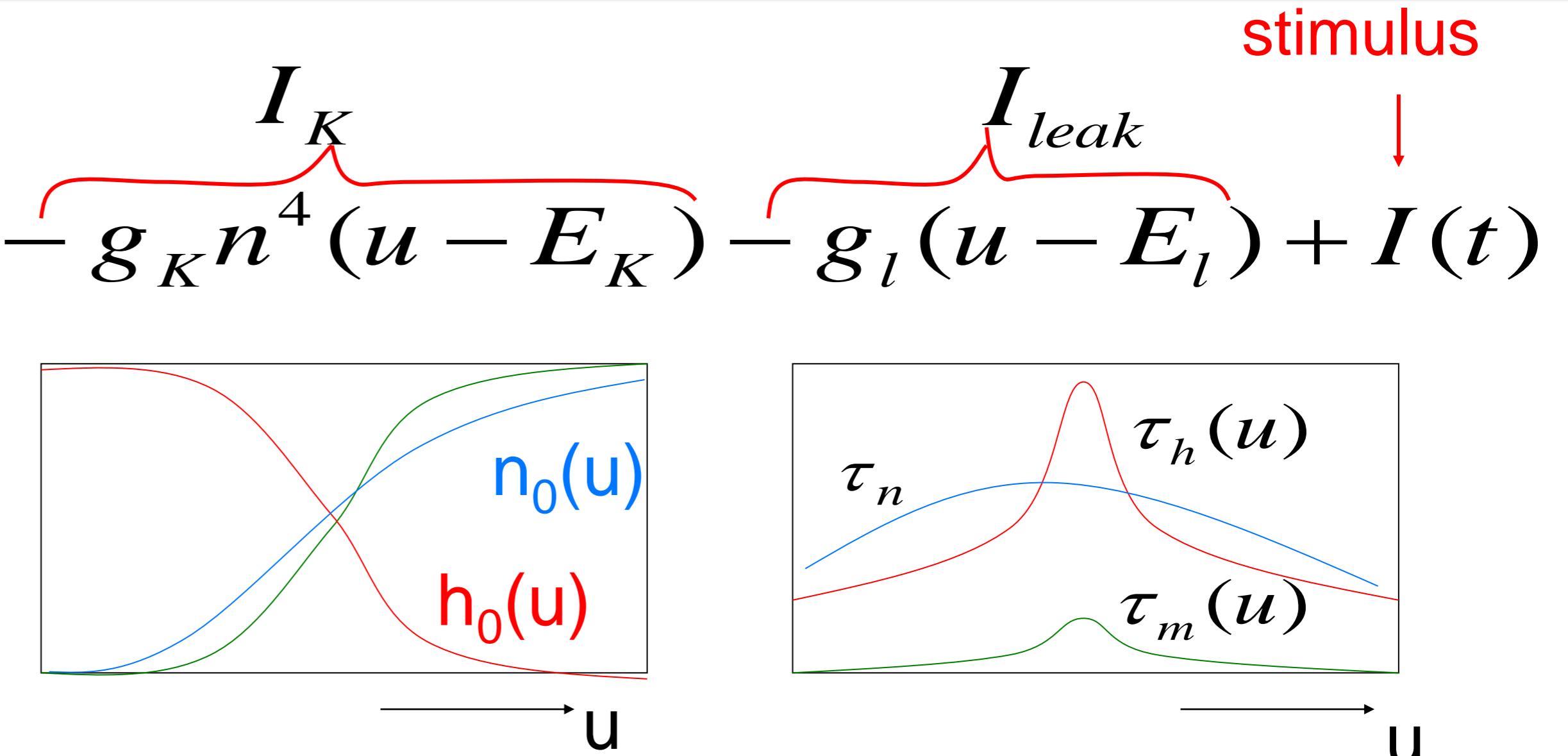
$$C \frac{du}{dt} = -g_{Na} m^3 h (u - E_{Na}) - g_K n^4 (u - E_K) - g_l (u - E_l) + I(t)$$

$$\frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n - n_0(u)}{\tau_n(u)}$$

- 1) dynamics of m are fast
- 2) dynamics of h and n are similar



$$\longrightarrow m(t) = m_0(u(t))$$

Neuronal Dynamics – 3.1. Reduction of Hodgkin-Huxley model

Reduction of Hodgkin-Huxley Model to 2 Dimension

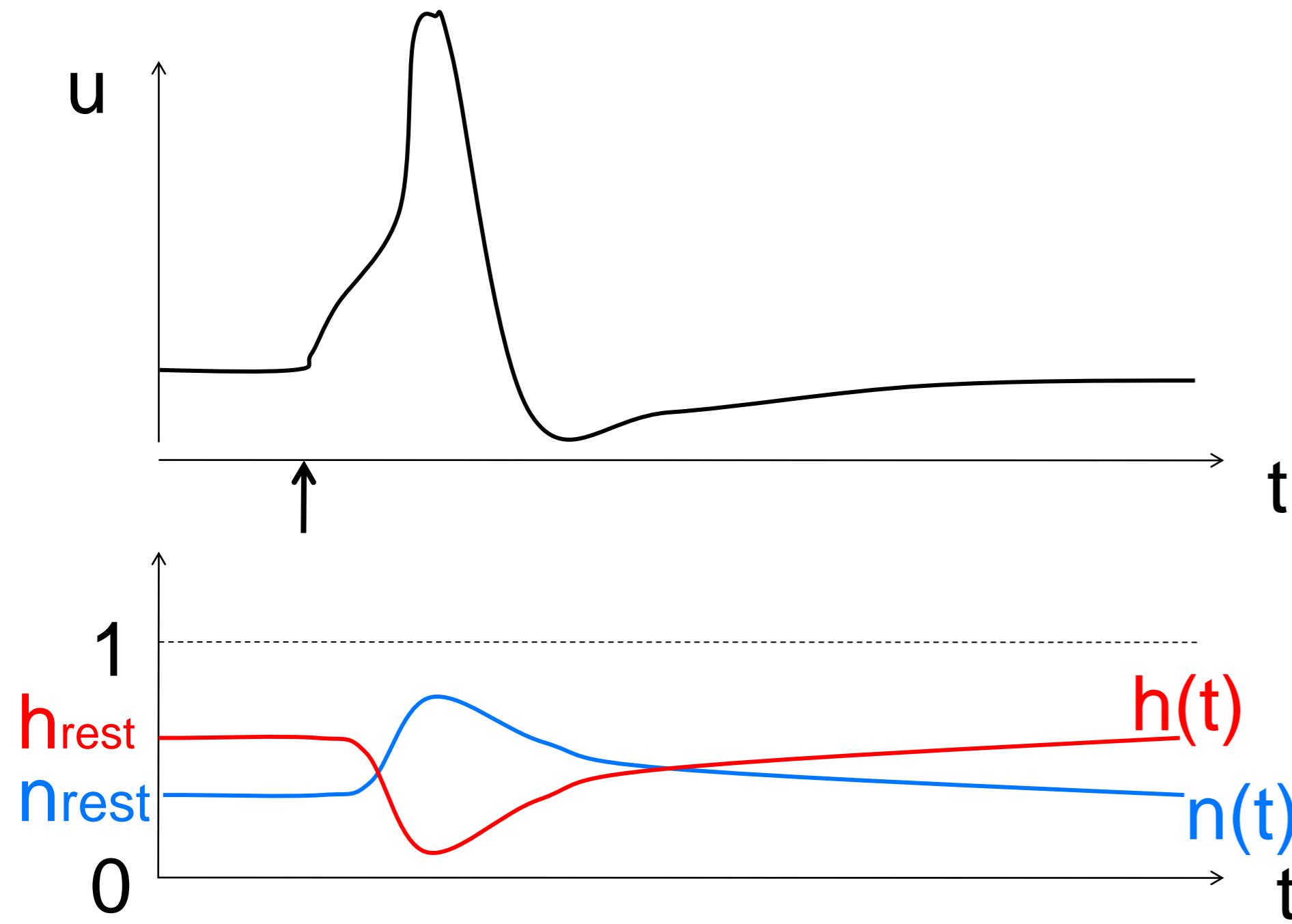
- step 1:
separation of time scales
- step 2:
exploit similarities/correlations

Now !

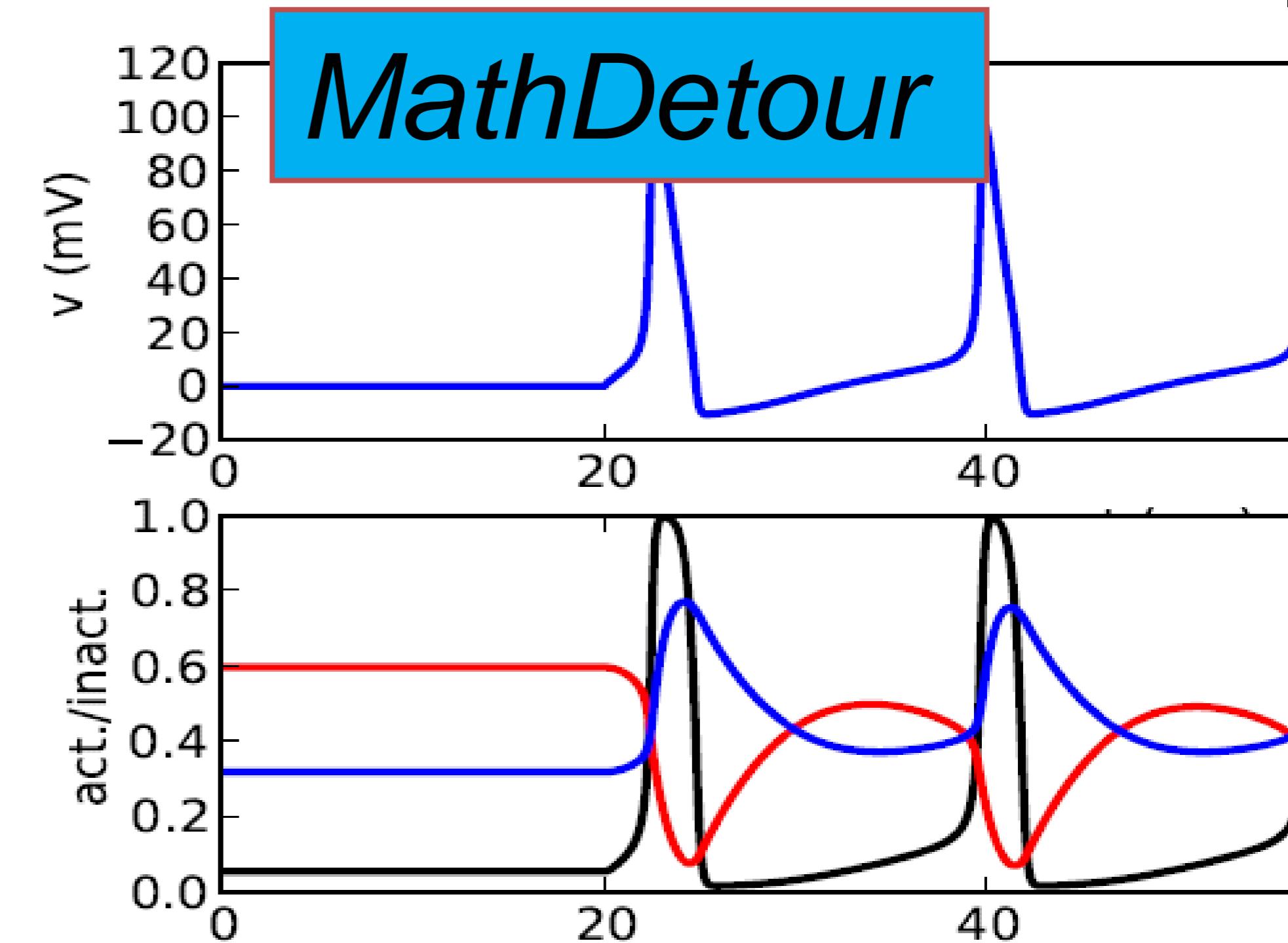
Neuronal Dynamics – 3.1. Reduction of Hodgkin-Huxley model

$$C \frac{du}{dt} = -g_{Na}m^3h(u - E_{Na}) - g_Kn^4(u - E_K) - g_l(u - E_l) + I(t)$$

2) dynamics of h and n are similar

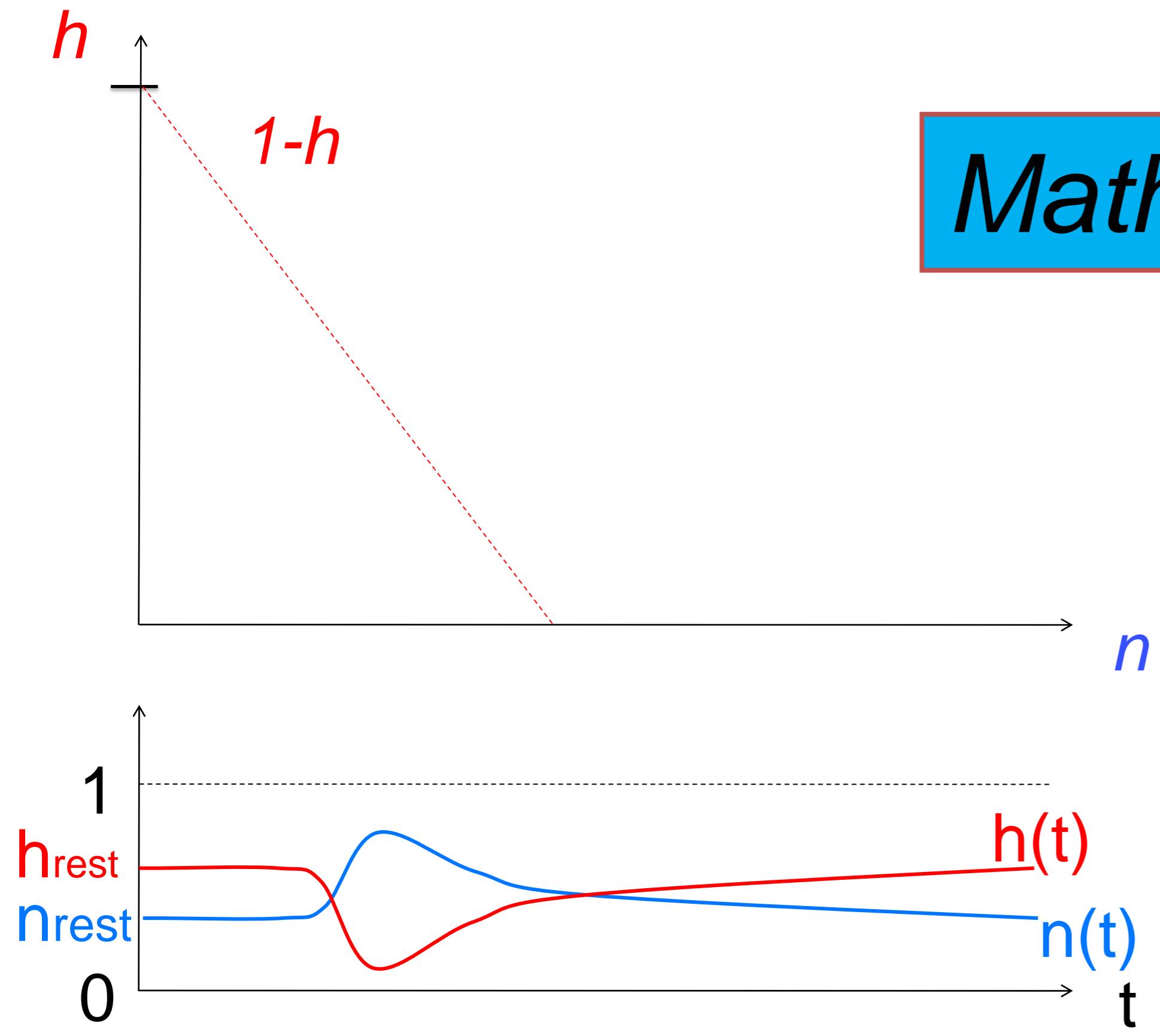


$$1 - h(t) = a n(t)$$



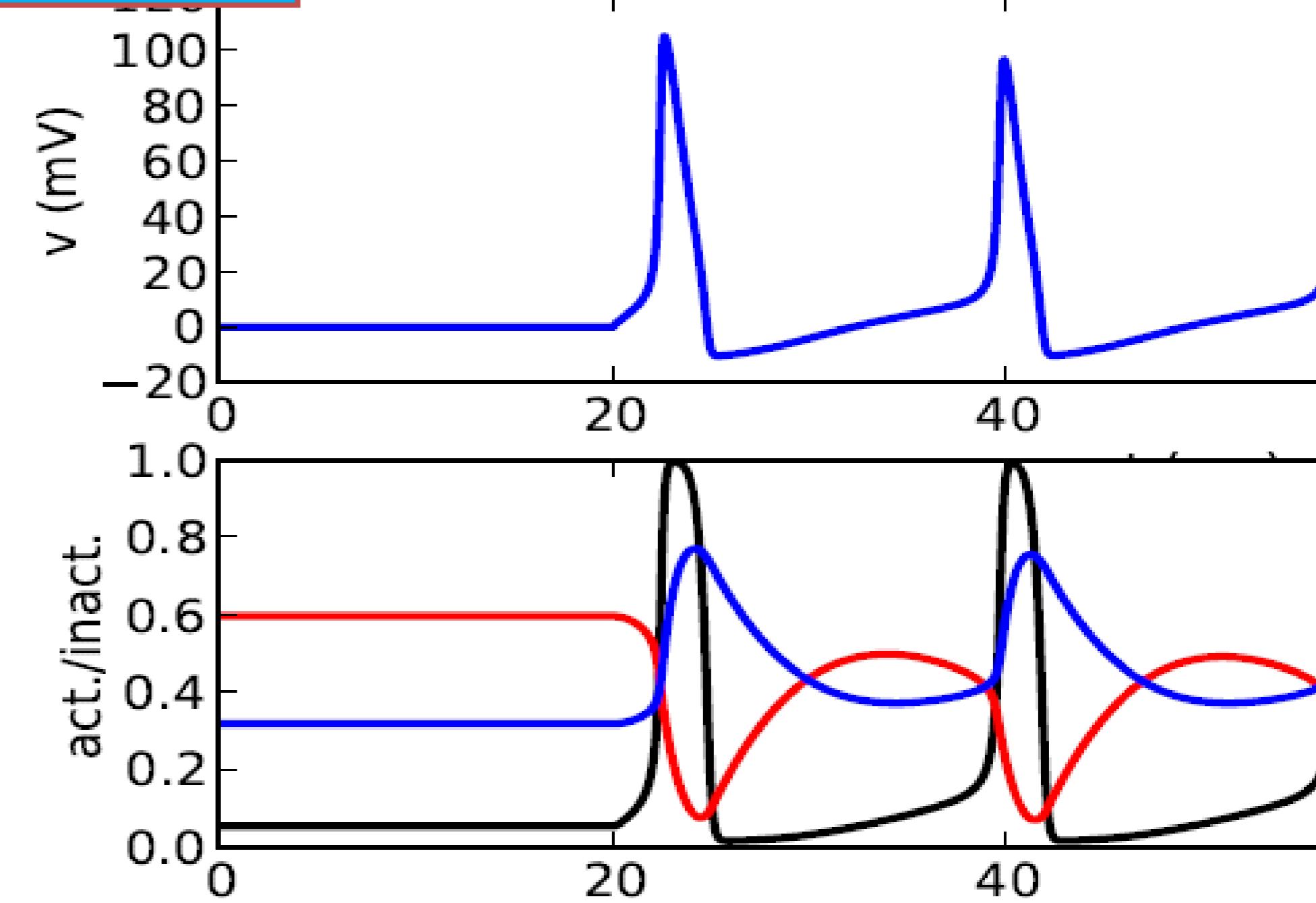
Neuronal Dynamics – Detour 3.1. Exploit similarities/correlations

dynamics of h and n are similar

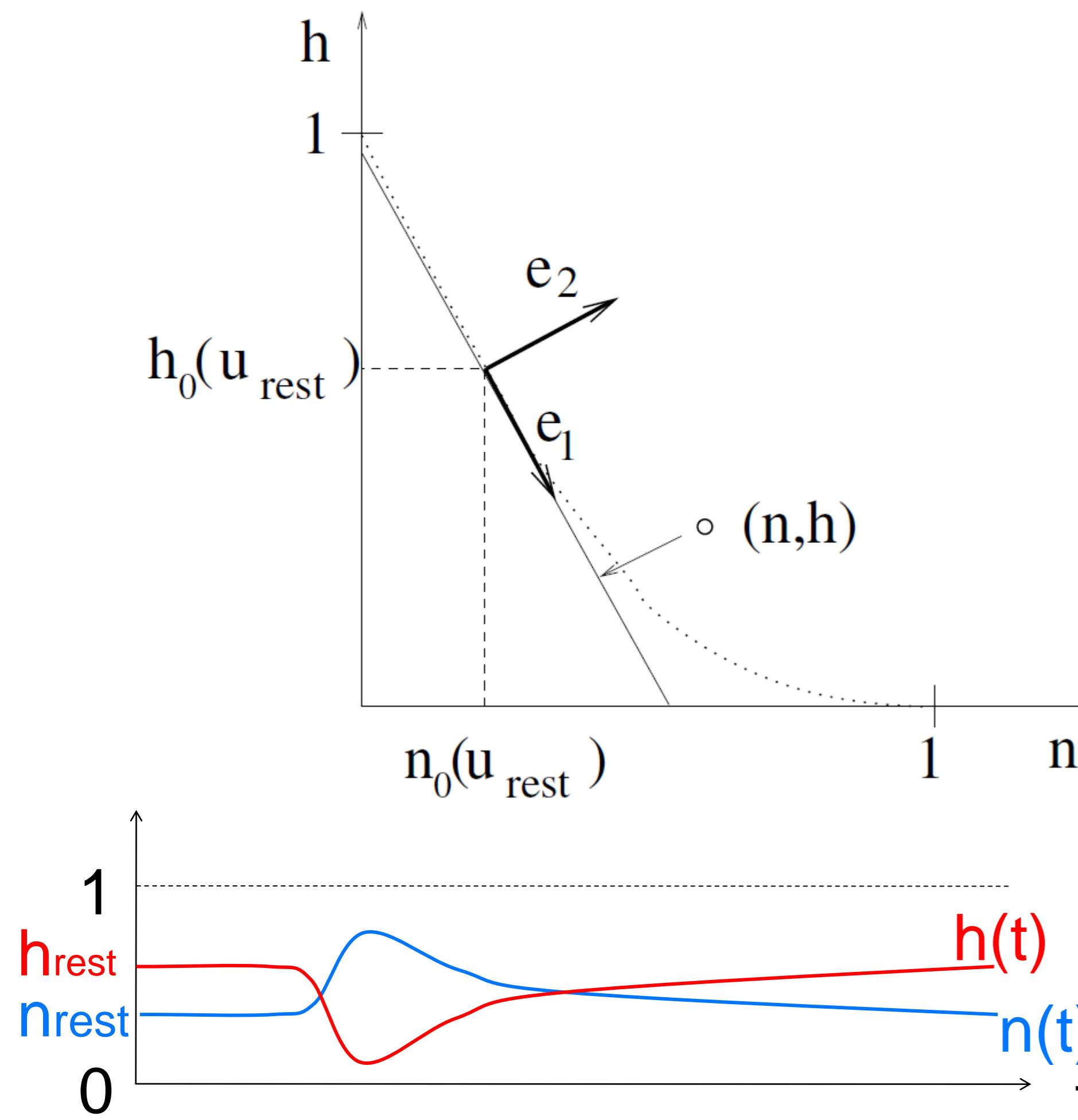


Math. argument

$$1 - h(t) = a n(t)$$



Neuronal Dynamics – Detour 3.1. Exploit similarities/correlations



dynamics of h and n are similar

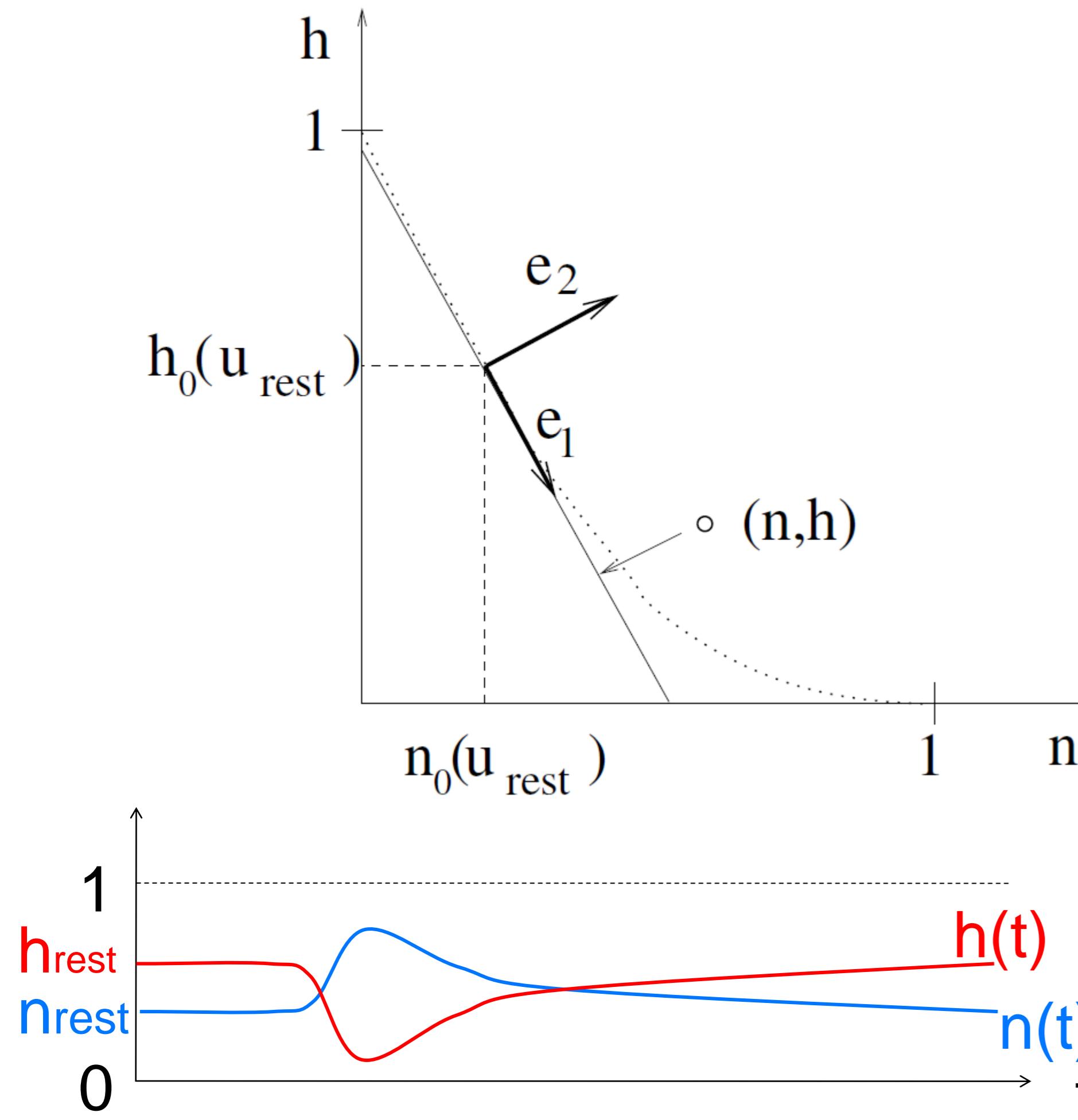
$$1 - h(t) = \alpha n(t)$$

at rest

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n - n_0(u)}{\tau_n(u)}$$

Neuronal Dynamics – Detour 3.1. Exploit similarities/correlations



dynamics of h and n are similar

- (i) Rotate coordinate system
- (ii) Suppress one coordinate
- (iii) Express dynamics in new variable

$$1 - h(t) = \alpha n(t) = w(t)$$

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n - n_0(u)}{\tau_n(u)}$$

$$\frac{dw}{dt} = -\frac{w - w_0(u)}{\tau_{\text{eff}}(u)}$$

Neuronal Dynamics – 3.1. Reduction of Hodgkin-Huxley model

$$C \frac{du}{dt} = -\underbrace{g_{Na}[m(t)]^3 h(t)(u(t) - E_{Na})}_{I_{Na}} - \underbrace{g_K[n(t)]^4 (u(t) - E_K)}_{I_K} - \underbrace{g_l(u(t) - E_l)}_{I_{leak}} + I(t)$$

$$C \frac{du}{dt} = -g_{Na} m_0(u)^3 (1-w)(u - E_{Na}) - g_K \left[\frac{w}{a} \right]^4 (u - E_K) - g_l(u - E_l) + I(t)$$

- 1) dynamics of m are fast $\longrightarrow m(t) = m_0(u(t))$
 2) dynamics of h and n are similar $\longrightarrow 1-h(t) = \underbrace{a n(t)}_{w(t)}$

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)} \longrightarrow \frac{dw}{dt} = -\frac{w - w_0(u)}{\tau_{eff}(u)}$$

$$\frac{dn}{dt} = -\frac{n - n_0(u)}{\tau_n(u)}$$

Neuronal Dynamics – 3.1. Reduction of Hodgkin-Huxley model

$$C \frac{du}{dt} = -g_{Na} m_0^3 (1-w) (u - E_{Na}) - g_K \left(\frac{w}{a}\right)^4 (u - E_K) - g_l (u - E_l) + I(t)$$

$$\frac{dw}{dt} = -\frac{w - w_0(u)}{\tau_{eff}(u)}$$

$$\tau \frac{du}{dt} = F(u(t), w(t)) + R I(t)$$

$$\tau_w \frac{dw}{dt} = G(u(t), w(t))$$

NOW Exercise 1.1-1.4: separation of time scales

$$C \frac{du}{dt} = -g_{Na} m^3 h(u - E_{Na}) - g_K n^4 (u - E_K) - g_l (u - E_l) + I(t)$$

$$\frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)}$$

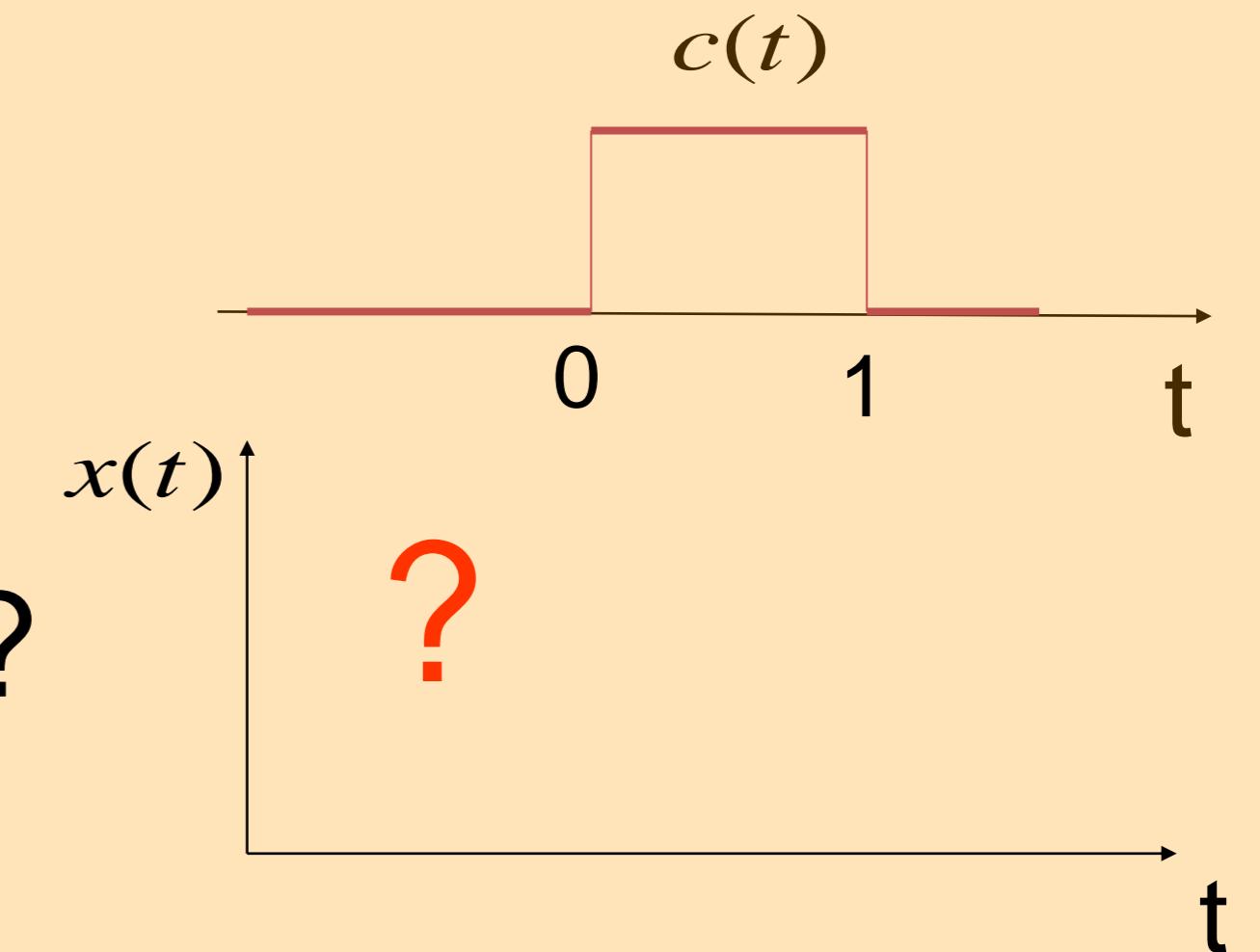
Exercises:
1.1-1.4 **now!**
1.5 **homework**

Exerc. 9h50-10h00

Next lecture:
10h15

$$A: \frac{dx}{dt} = -\frac{x - c(t)}{\tau}$$

- calculate $x(t)$!
- what if τ is small?

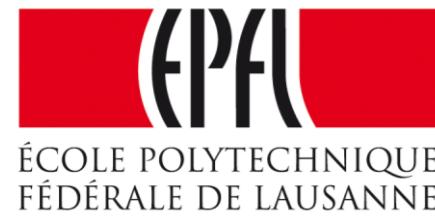


$$\frac{dm}{dt} = -\frac{m - c(u)}{\tau_m}$$

$$\frac{du}{dt} = f(u) - m$$

- B: -calculate $m(t)$
if τ is small!
- reduce to 1 eq.

Week 3 – part 1 : Reduction of the Hodgkin-Huxley Model



Biological Modeling of Neural Networks

**Week 3 – Reducing detail:
Two-dimensional neuron models**

Wulfram Gerstner

EPFL, Lausanne, Switzerland

3.1 From Hodgkin-Huxley to 2D

- Overview: From 4 to 2 dimensions
- MathDetour 1: Exploiting similarities
- MathDetour 2: Separation of time scales

3.2 Phase Plane Analysis

- Role of nullclines

3.3 Analysis of a 2D Neuron Model

- constant input vs pulse input
- MathDetour 3: Stability of fixed points

3.4 Type I and II Neuron Models

next week!

Neuronal Dynamics – MathDetour 3.1: Separation of time scales

Ex. 1-A $\tau_1 \frac{dx}{dt} = -x + c(t)$

Exercise 1 (week 3)

Two coupled differential equations

$$\tau_1 \frac{dx}{dt} = -x + c(t)$$

$$\tau_2 \frac{dy}{dt} = f(y) + g(x)$$

Separation of time scales

$$\tau_1 \ll \tau_2$$

Reduced 1-dimensional system

$$\tau_2 \frac{dy}{dt} = f(y) + g(c(t))$$

Neuronal Dynamics – MathDetour 3.2: Separation of time scales

Linear differential equation $\tau_1 \frac{dx}{dt} = -x + c(t)$



step

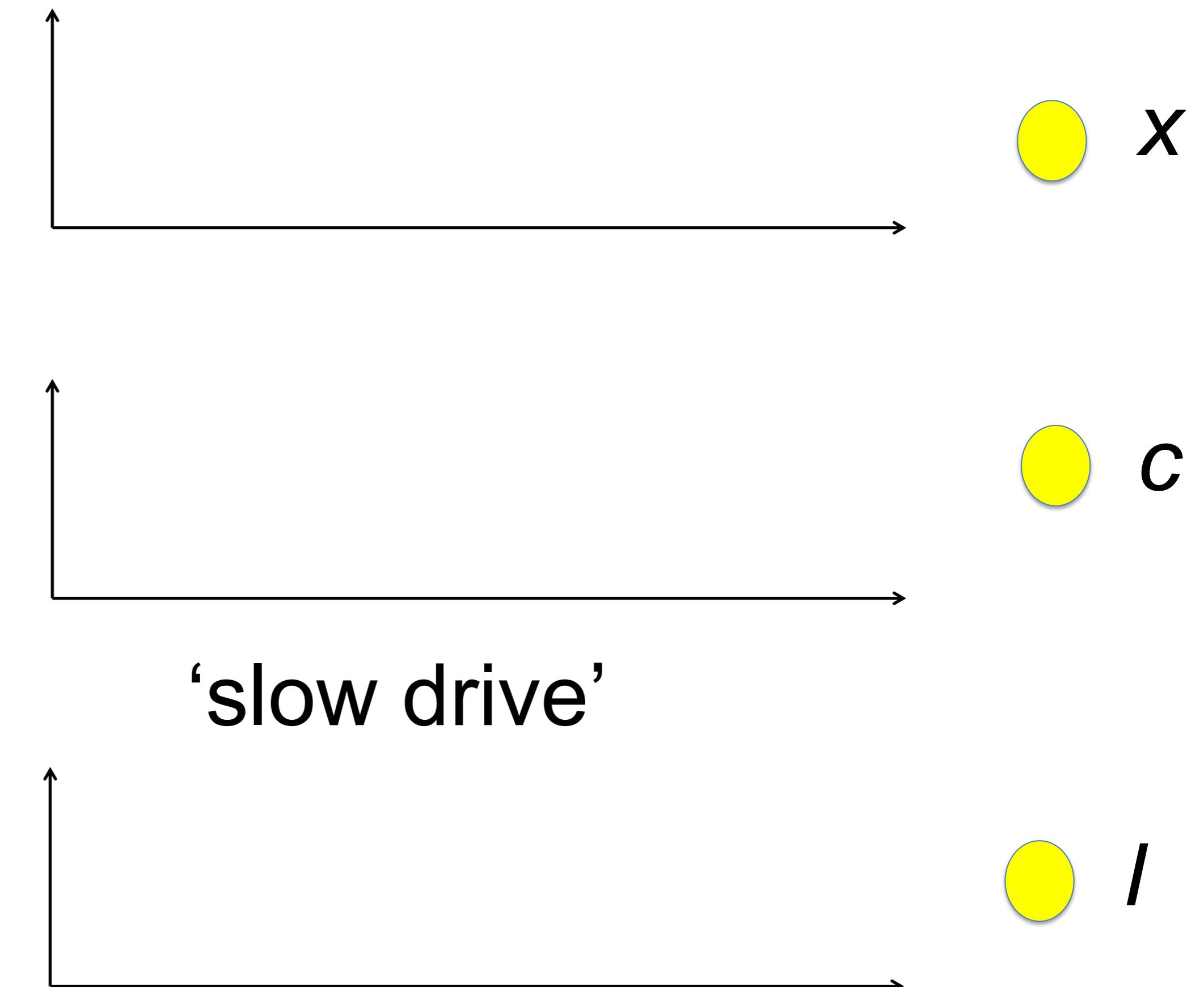
Neuronal Dynamics – MathDetour 3.2 Separation of time scales

Two coupled differential equations

$$\tau_1 \frac{dx}{dt} = -x + c(t)$$

$$\tau_2 \frac{dc}{dt} = -c + f(x) + I(t)$$

$$\tau_1 \ll \tau_2$$



Neuronal Dynamics – Reduction of Hodgkin-Huxley model

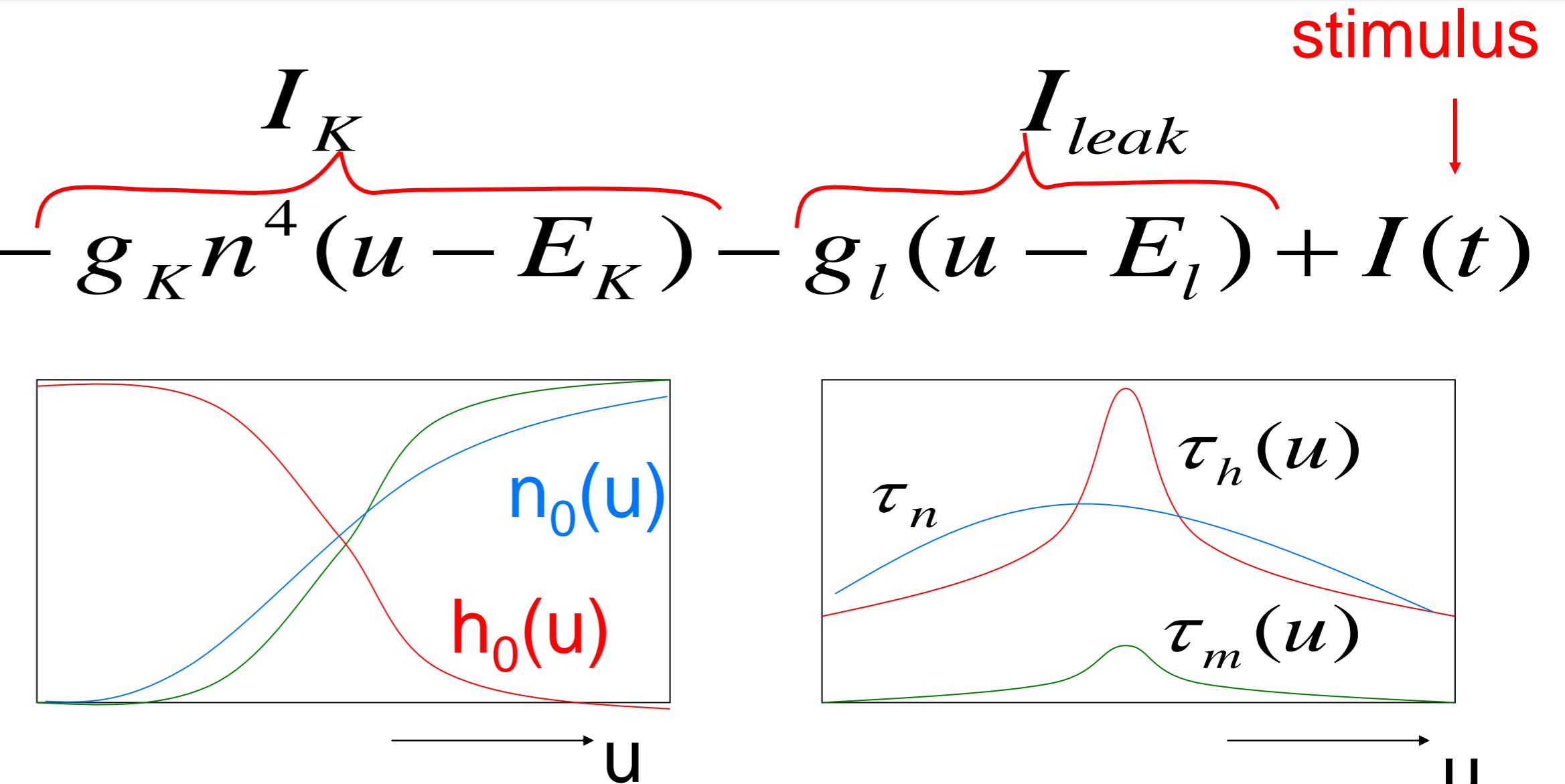
$$C \frac{du}{dt} = -g_{Na} m^3 h (u - E_{Na}) - g_K n^4 (u - E_K) - g_l (u - E_l) + I(t)$$

$$\frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n - n_0(u)}{\tau_n(u)}$$

dynamics of m is fast



$$\longrightarrow m(t) = m_0(u(t))$$

Fast compared to what?

Neuronal Dynamics – MathDetour 3.2: Separation of time scales

Two coupled differential equations

$$\tau_1 \frac{dx}{dt} = -x + h(y)$$

$$\tau_2 \frac{dy}{dt} = f(y) + g(x)$$

Exercise 1 (week 3)

even more general

Separation of time scales

$$\tau_1 \ll \tau_2 \rightarrow x = h(y)$$

Reduced 1-dimensional system

$$\tau_2 \frac{dy}{dt} = f(y) + g(h(y))$$

Neuronal Dynamics – Quiz 3.2.

A- Separation of time scales:

We start with two equations

$$\tau_1 \frac{dx}{dt} = -x + y + I(t)$$

$$\tau_2 \frac{dy}{dt} = -y + x^2 + A$$

[] If $\tau_1 \ll \tau_2$ then the system can be reduced to

$$\tau_2 \frac{dy}{dt} = -y + [y + I(t)]^2 + A$$

[] If $\tau_2 \ll \tau_1$ then the system can be reduced to

$$\tau_1 \frac{dx}{dt} = -x + x^2 + A + I(t)$$

[] None of the above is correct.

Attention
*I(t) can move rapidly,
therefore
choice [1]
not correct*

Neuronal Dynamics – Quiz 3.2-similar dynamics

Exploiting similarities:

A sufficient condition to replace two gating variables r, s by a single gating variable w is

- [] Both r and s have the same time constant (as a function of u)
- [] Both r and s have the same activation function
- [] Both r and s have the same time constant (as a function of u)
AND the same activation function
- [] Both r and s have the same time constant (as a function of u)
AND activation functions that are identical after some additive rescaling
- [] Both r and s have the same time constant (as a function of u)
AND activation functions that are identical after some multiplicative
rescaling

Neuronal Dynamics – 3.1. Reduction to 2 dimensions

2-dimensional equation

$$C \frac{du}{dt} = f(u(t), w(t)) + I(t)$$

$$\frac{dw}{dt} = g(u(t), w(t))$$

Enables graphical analysis!



Phase plane analysis

- Discussion of threshold
- Constant input current vs pulse input
- Type I and II
- Repetitive firing

Week 3 – part 1 : Reduction of the Hodgkin-Huxley Model



Biological Modeling of Neural Networks

Week 3 – Reducing detail:

Two-dimensional neuron models

Wulfram Gerstner

EPFL, Lausanne, Switzerland



3.1 From Hodgkin-Huxley to 2D

- Overview: From 4 to 2 dimensions
- MathDetour 1: Exploiting similarities
- MathDetour 2: Separation of time scales

3.2 Phase Plane Analysis

- Role of nullclines

3.3 Analysis of a 2D Neuron Model

- constant input vs pulse input
- MathDetour 3: Stability of fixed points

3.4 Type I and II Neuron Models

next week!

Neuronal Dynamics – 3.2. Reduced Hodgkin-Huxley model

$$C \frac{du}{dt} = -g_{Na} m_0^3 (1-w) (u - E_{Na}) - g_K \left(\frac{w}{a}\right)^4 (u - E_K) - g_l (u - E_l) + I(t)$$

$$\frac{dw}{dt} = -\frac{w - w_0(u)}{\tau_w(u)}$$

$$\tau \frac{du}{dt} = F(u, w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u, w)$$

stimulus



Neuronal Dynamics – 3.2. Phase Plane Analysis/nullclines

2-dimensional equation
stimulus

$$\tau \frac{du}{dt} = F(u, w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u, w)$$

u-nullcline

w-nullcline

Enables graphical analysis!
-Discussion of threshold
-Type I and II

Neuronal Dynamics – 3.2. FitzHugh-Nagumo Model

$$\tau \frac{du}{dt} = F(u, w) + RI(t)$$

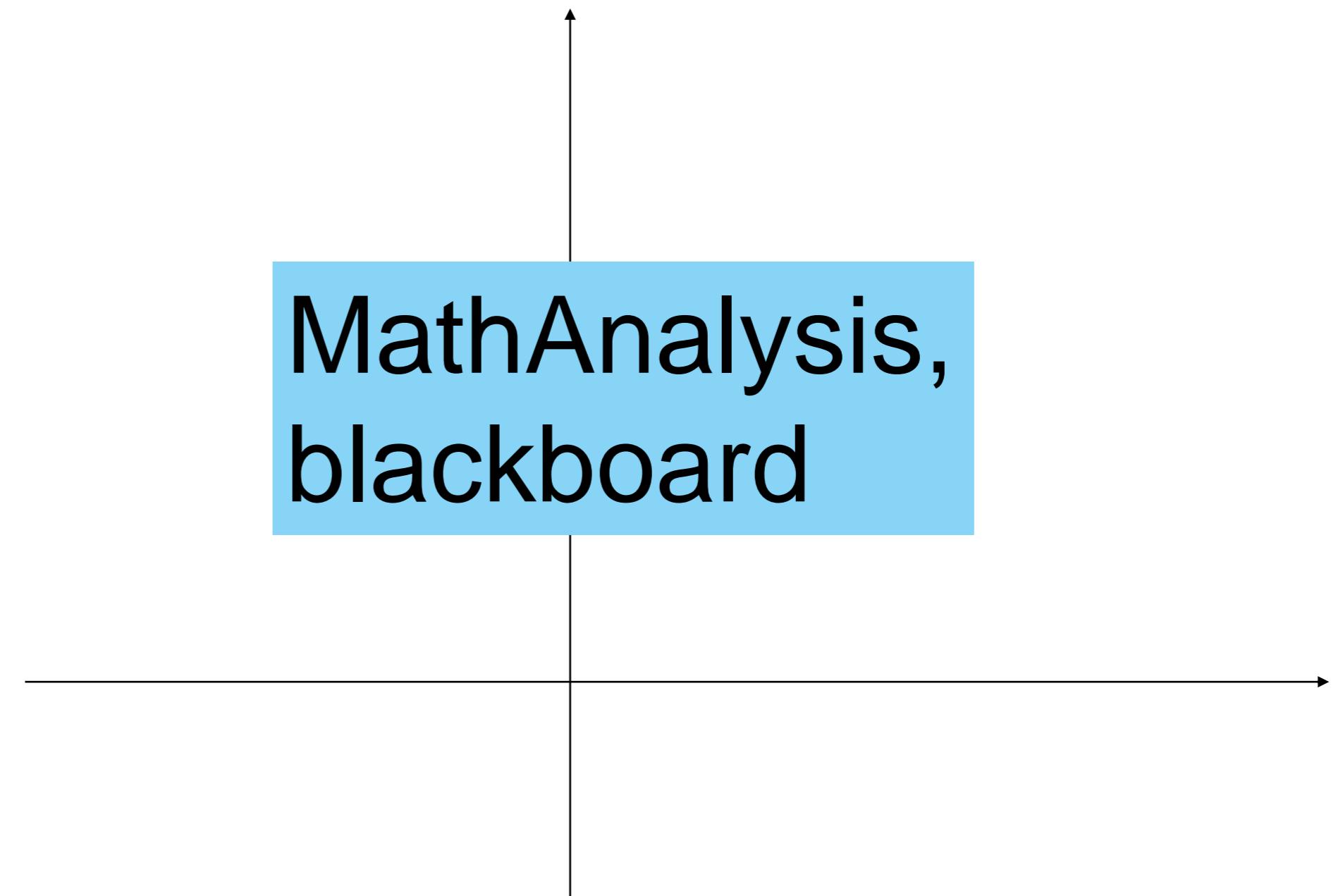
$$= u - \frac{1}{3}u^3 - w + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u, w) = b_0 + b_1 u - w$$

u-nullcline

w-nullcline

MathAnalysis,
blackboard



Neuronal Dynamics – 3.2. flow arrows

$$\tau \frac{du}{dt} = F(u, w) + RI(t)$$

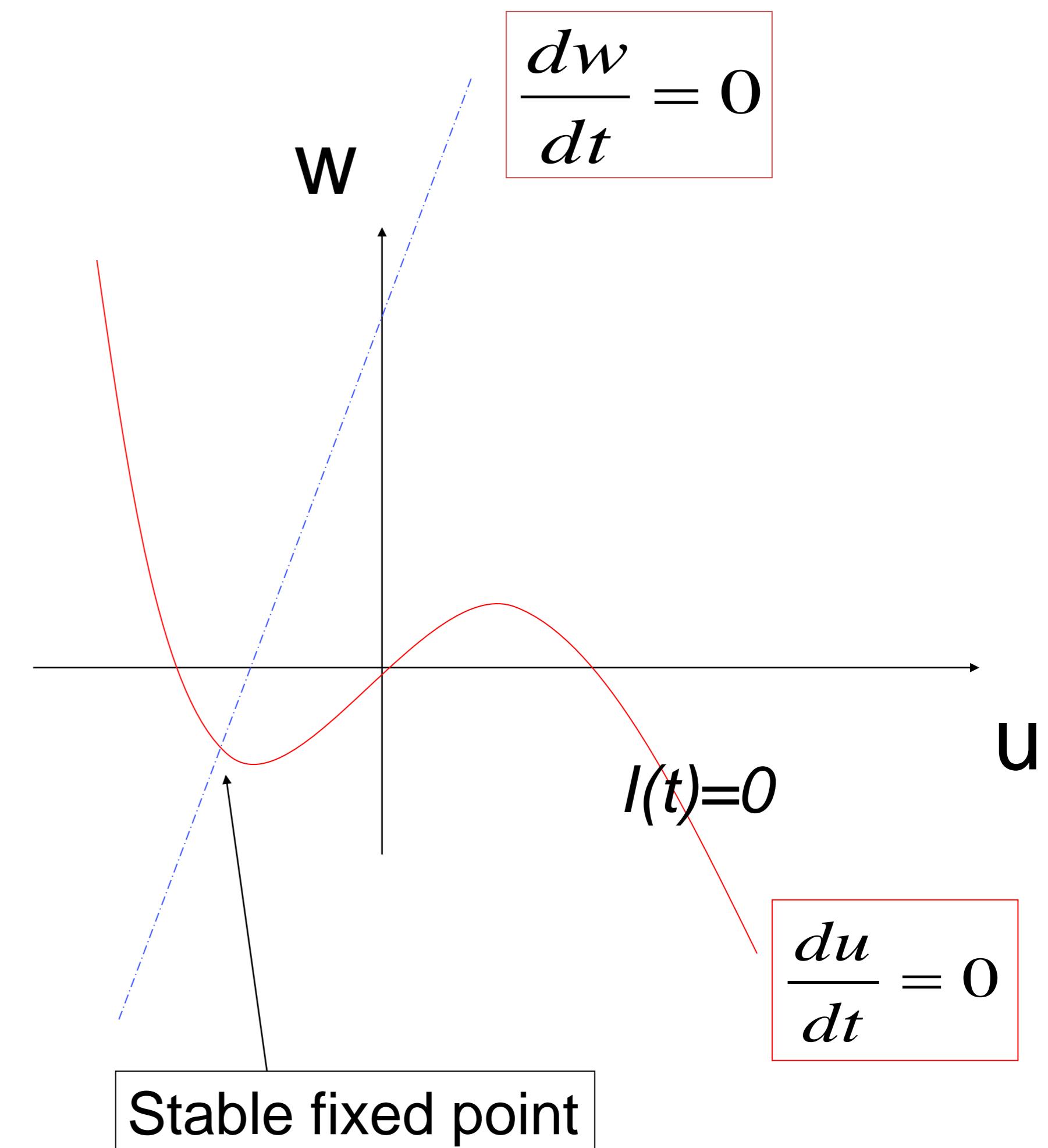
Stimulus $I=0$

$$\tau_w \frac{dw}{dt} = G(u, w)$$

Consider change in small time step

Flow on nullcline

Flow in regions between nullclines



Neuronal Dynamics – Quiz 3.3

A. u-Nullclines

- On the u-nullcline, arrows are always vertical
- On the u-nullcline, arrows point always vertically upward
- On the u-nullcline, arrows are always horizontal
- On the u-nullcline, arrows point always to the left
- On the u-nullcline, arrows point always to the right

Take 1 minute,
continue at 10:55

B. w-Nullclines

- On the w-nullcline, arrows are always vertical
- On the w-nullcline, arrows point always vertically upward
- On the w-nullcline, arrows are always horizontal
- On the w-nullcline, arrows point always to the left
- On the w-nullcline, arrows point always to the right
- On the w-nullcline, arrows can point in an arbitrary direction

Neuronal Dynamics – 4.2. FitzHugh-Nagumo Model

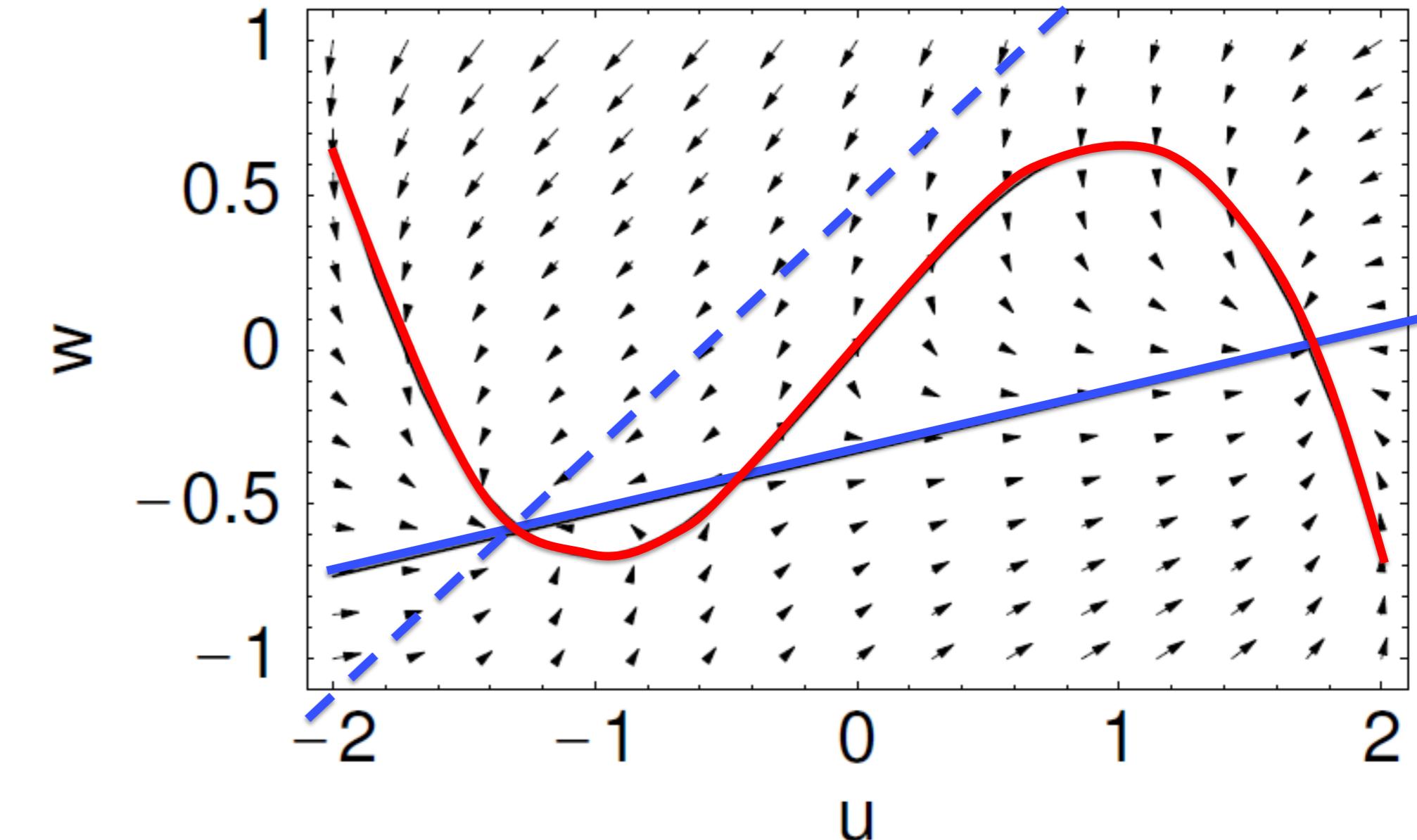
$$\tau \frac{du}{dt} = F(u, w) + RI(t)$$

$$= u - \frac{1}{3}u^3 + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u, w) = b_0 + b_1 u - w$$



change b_1



Neuronal Dynamics – 3.2. Nullclines of reduced HH model

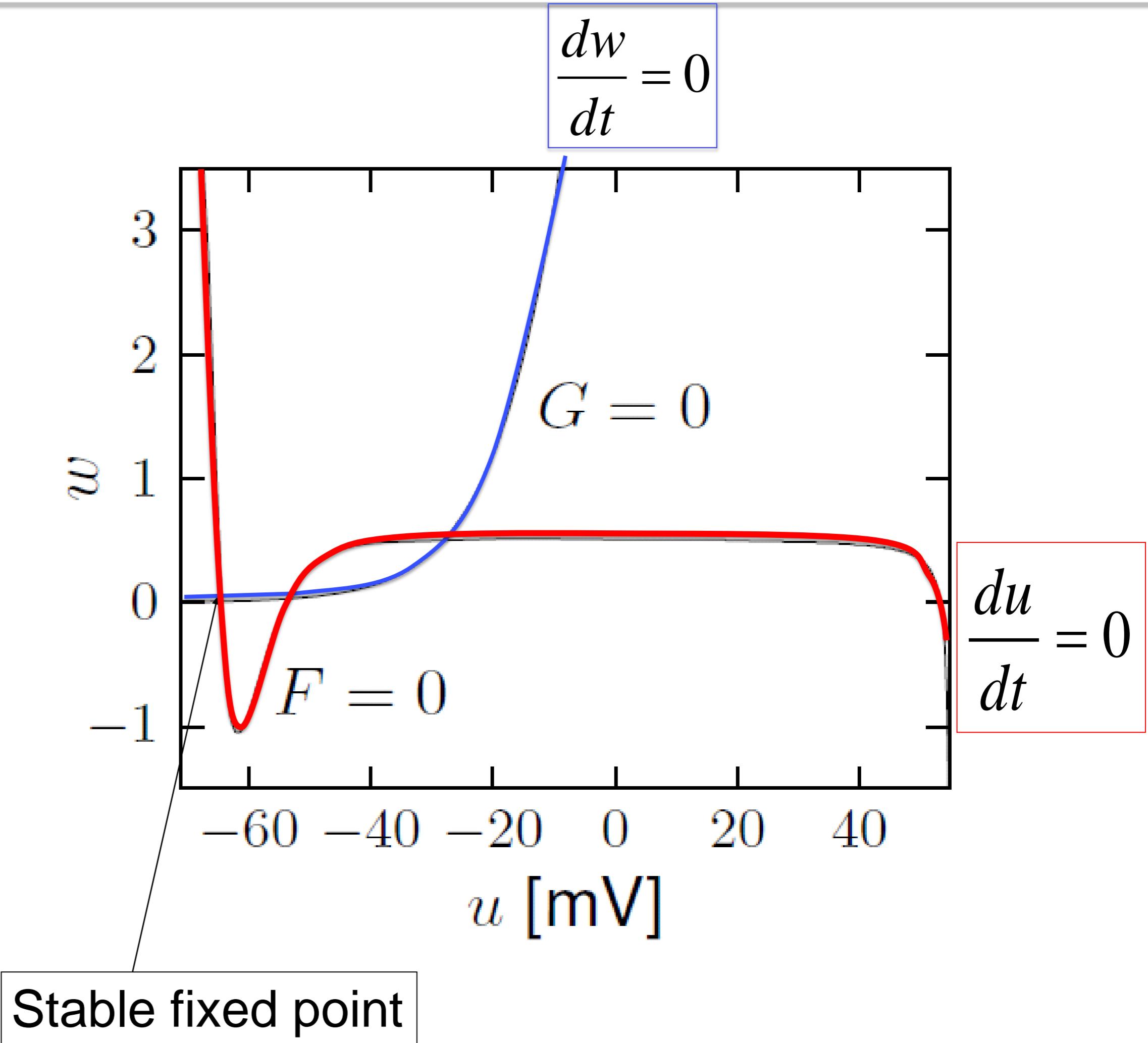
$$\tau \frac{du}{dt} = F(u, w) + RI(t)$$

stimulus

$$\tau_w \frac{dw}{dt} = G(u, w)$$

u-nullcline

w-nullcline



Neuronal Dynamics – 3.2. Phase Plane Analysis

2-dimensional equation

stimulus

$$\tau \frac{du}{dt} = F(u, w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u, w)$$

Enables graphical analysis!

Important role of

- nullclines
- flow arrows

→ Application to
neuron models

Week 3 – part 3: Analysis of a 2D neuron model



3.1 From Hodgkin-Huxley to 2D

3.2 Phase Plane Analysis

- Role of nullcline

3.3 Analysis of a 2D Neuron Model

- pulse input
- constant input
- MathDetour 3: Stability of fixed points

3.4 Type I and II Neuron Models (next week)

Neuronal Dynamics – 3.3. Analysis of a 2D neuron model

2-dimensional equation
stimulus

$$\tau \frac{du}{dt} = F(u, w) + RI(t)$$

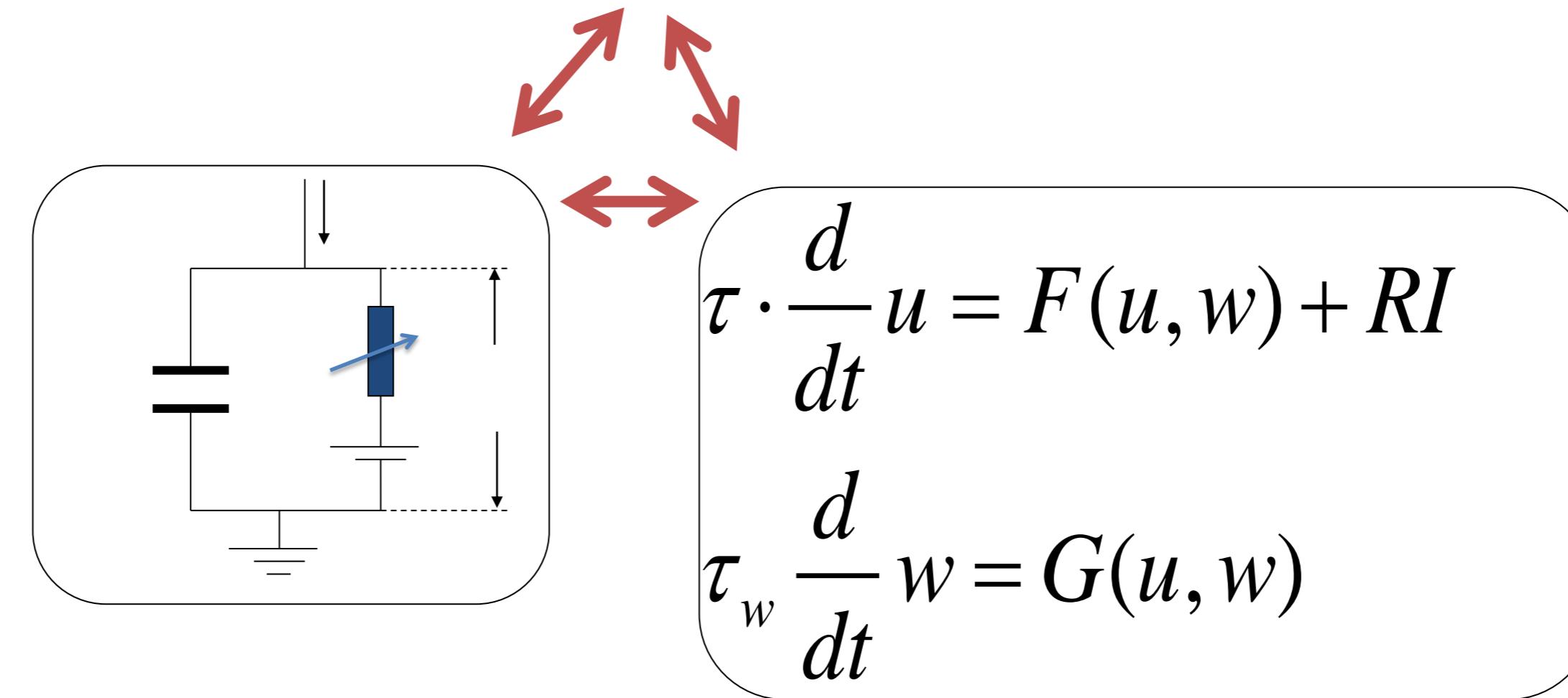
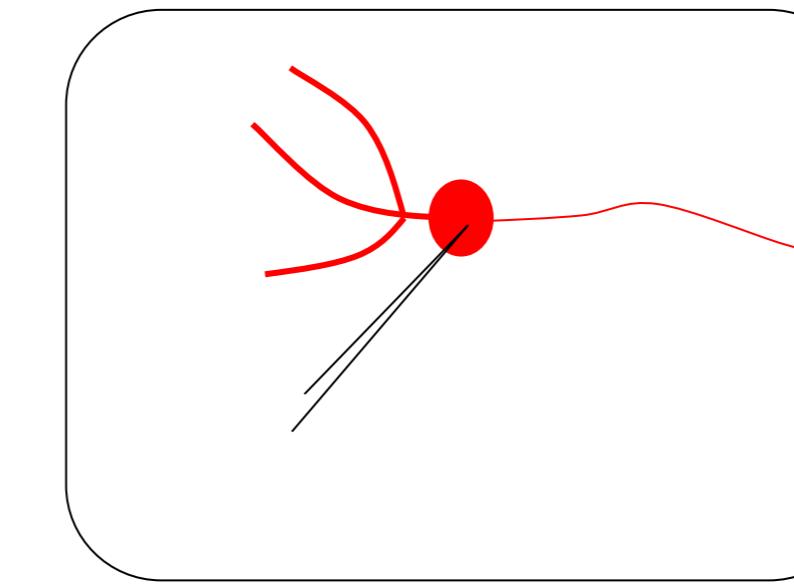
$$\tau_w \frac{dw}{dt} = G(u, w)$$

2 important input scenarios

- Pulse input
- Constant input

Enables graphical analysis!

Neuronal Dynamics – 3.3. 2D neuron model : Pulse input



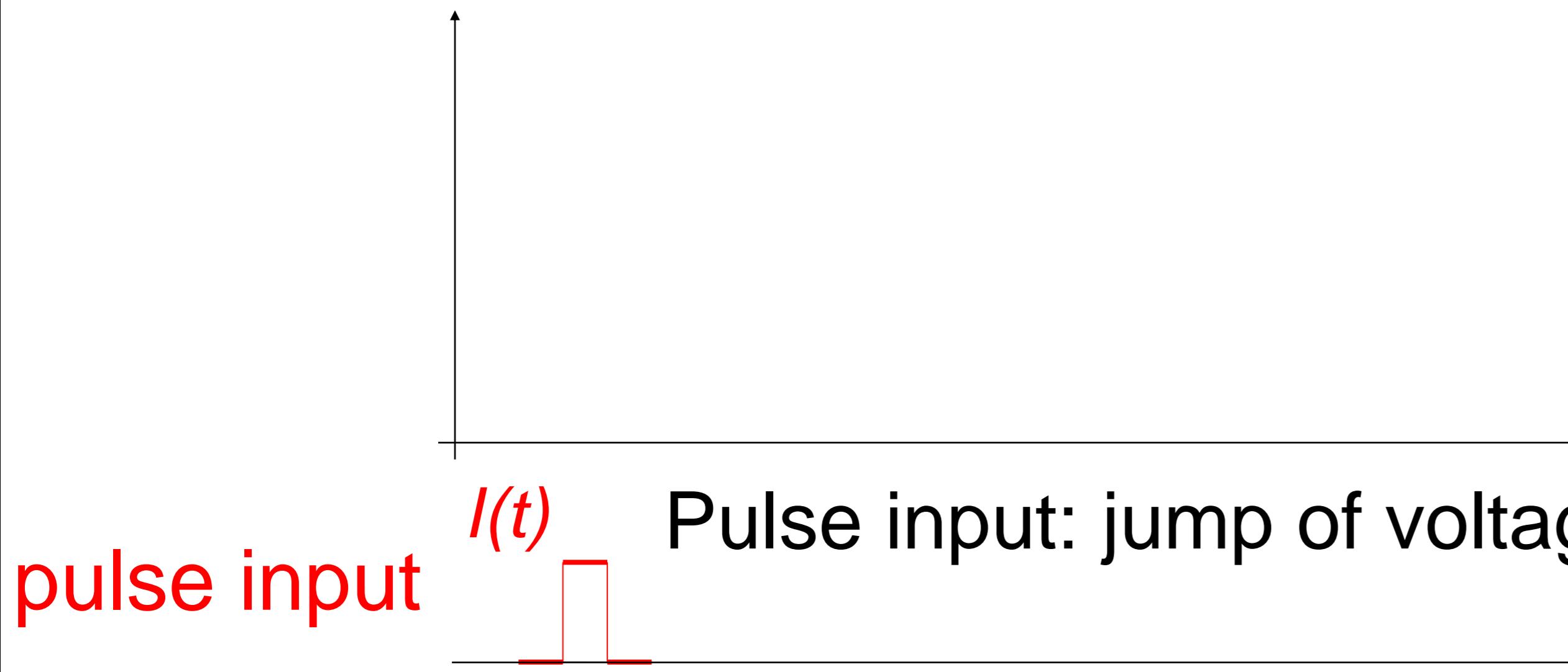
pulse input



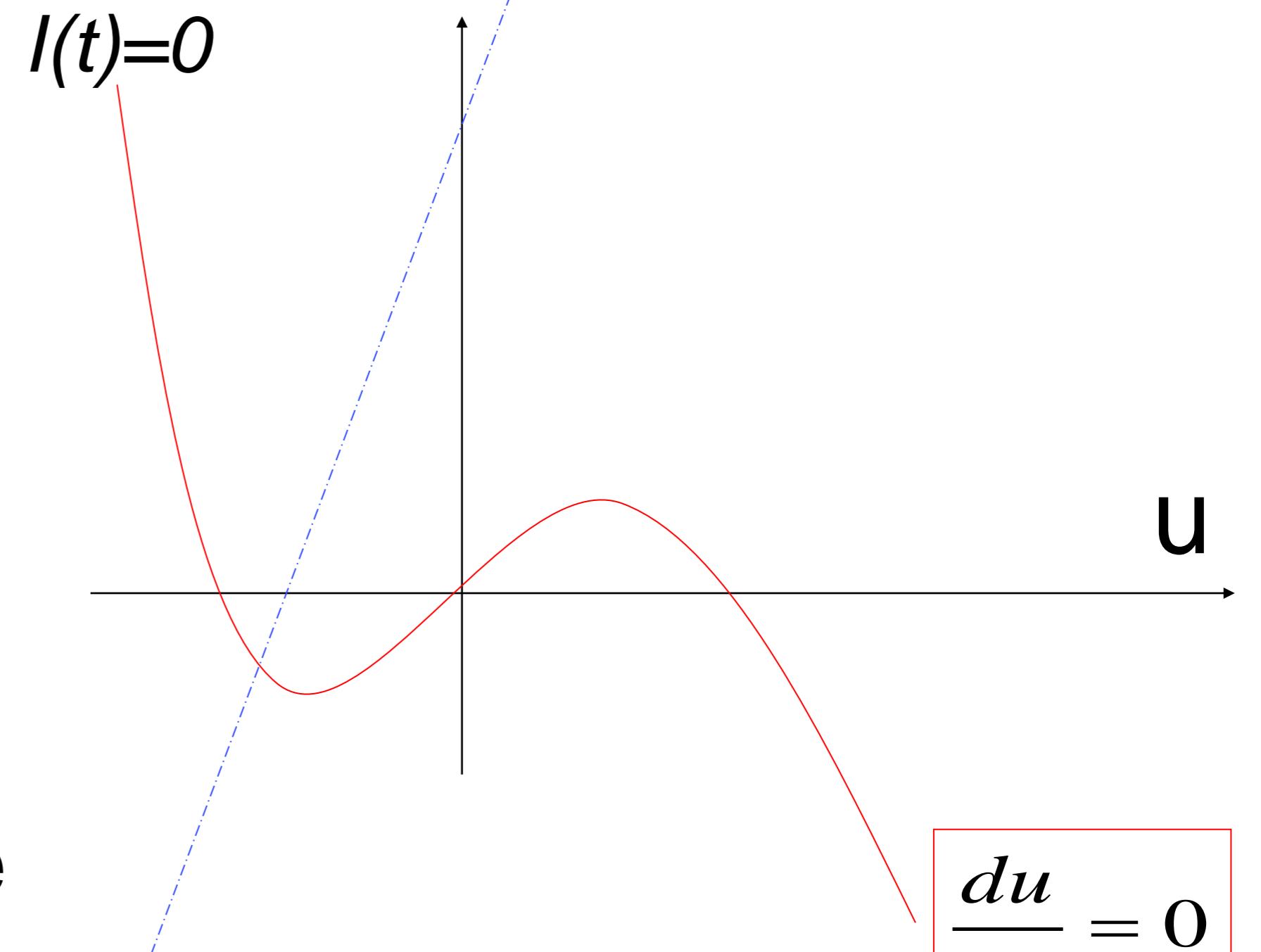
Neuronal Dynamics – 3.3. FitzHugh-Nagumo Model : Pulse input

$$\tau \frac{du}{dt} = F(u, w) + RI(t) = u - \frac{1}{3}u^3 - w + RI(t)$$

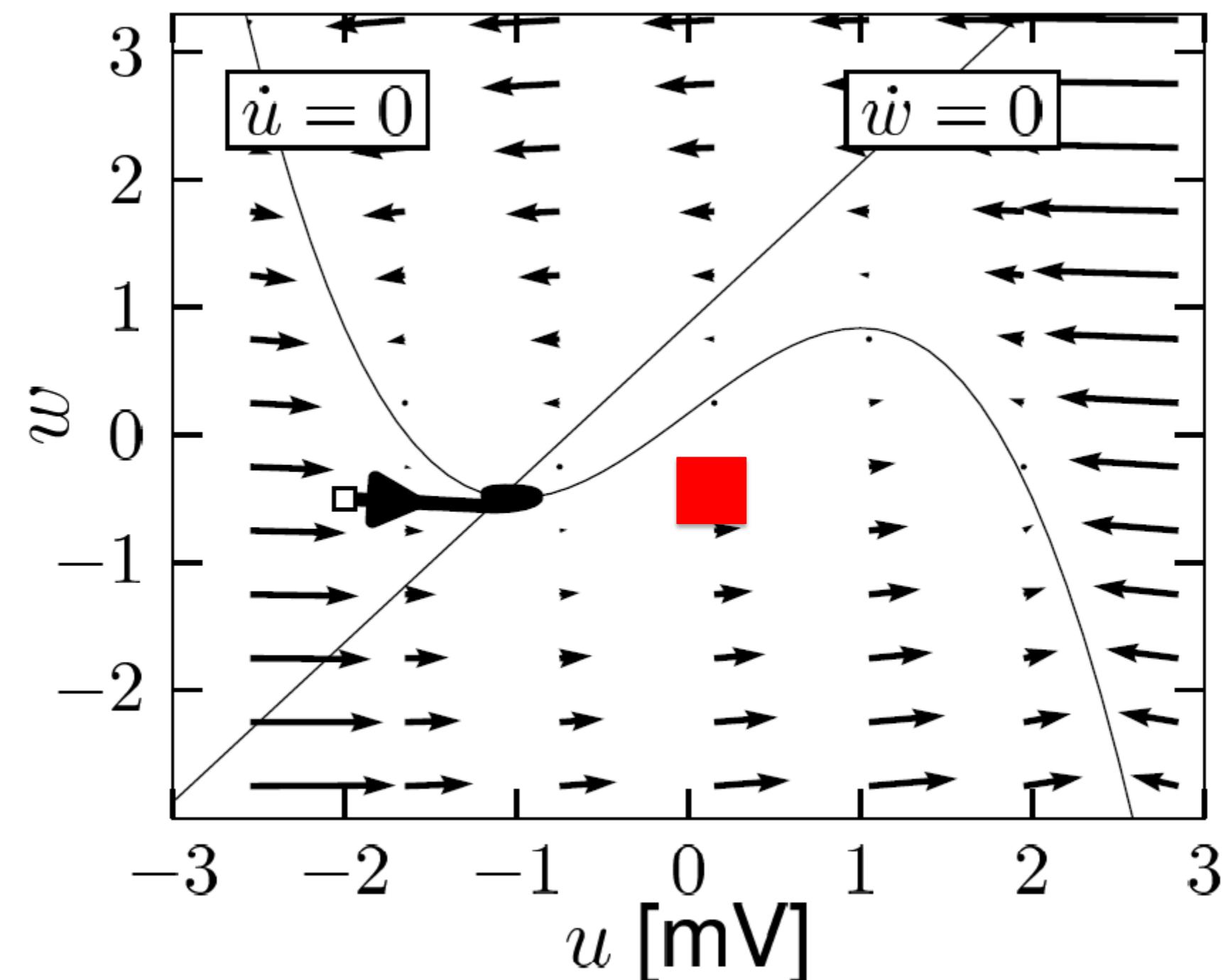
$$\tau_w \frac{dw}{dt} = G(u, w) = b_0 + b_1 u - w$$



$$\frac{dw}{dt} = 0$$

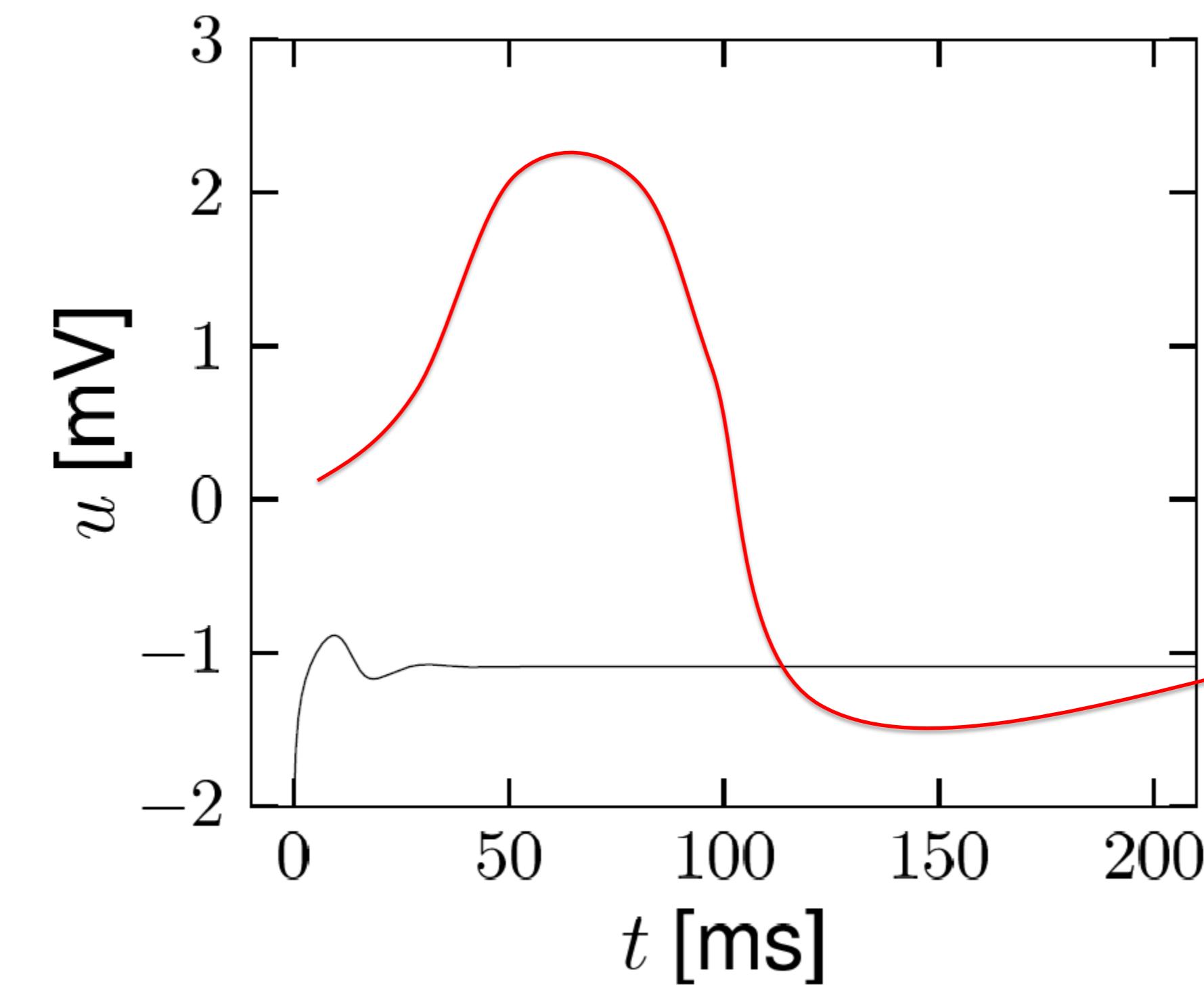


Neuronal Dynamics – 3.3. FitzHugh-Nagumo Model : Pulse input



FN model with $b_0 = 0.9; b_1 = 1.0$

Pulse input: jump of voltage/initial condition



*Image: Neuronal Dynamics,
Gerstner et al.,
Cambridge Univ. Press (2014)*

Neuronal Dynamics – 3.3. FitzHugh-Nagumo Model

Pulse input:

DONE!

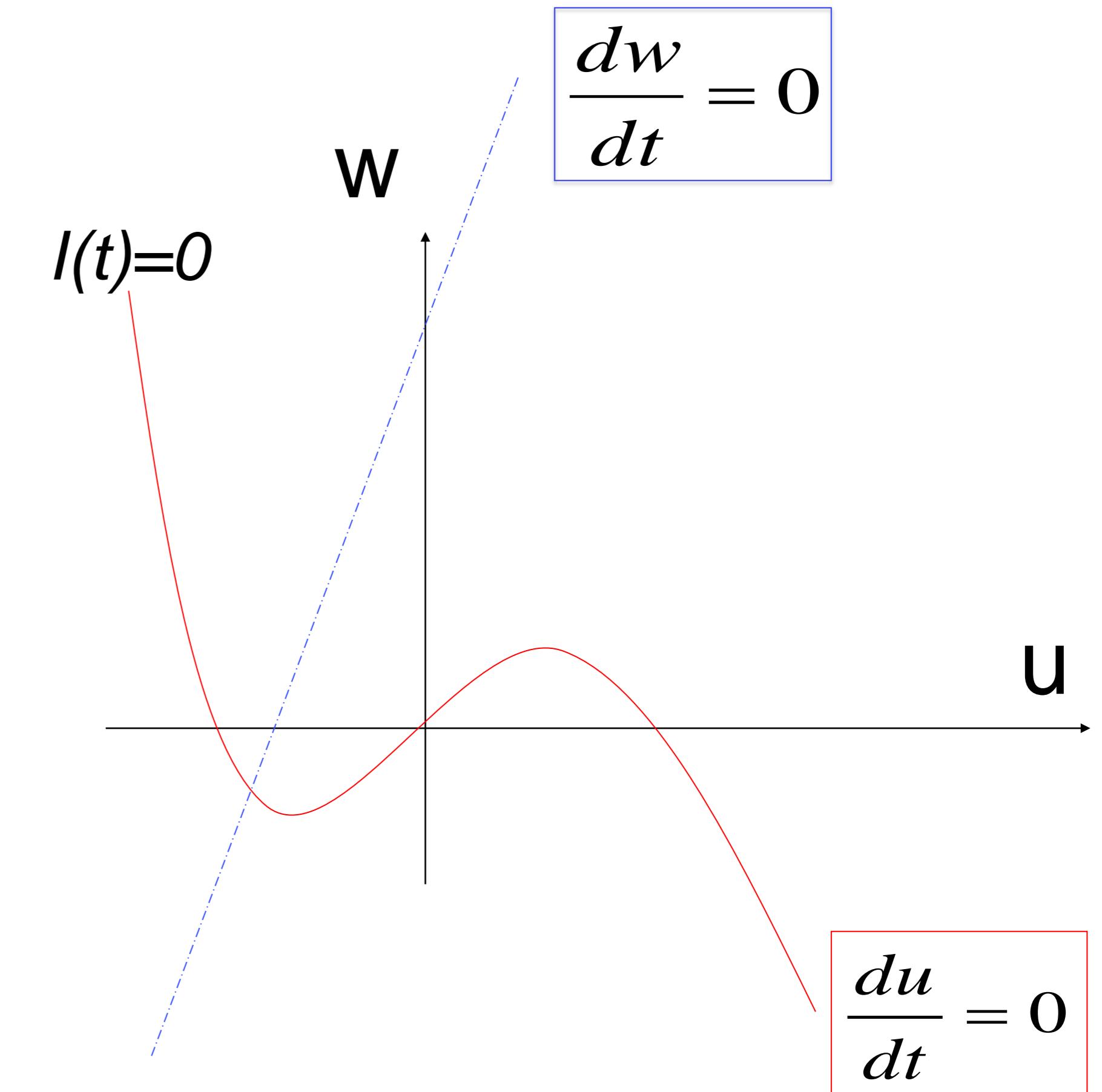
- jump of voltage
- ‘new initial condition’
- spike generation for large input pulses

2 important input scenarios

constant input:

- graphics?
- spikes?
- repetitive firing?

Now



Neuronal Dynamics – 3.3. FitzHugh-Nagumo Model: Constant input

$$\tau \frac{du}{dt} = F(u, w) + RI_0$$

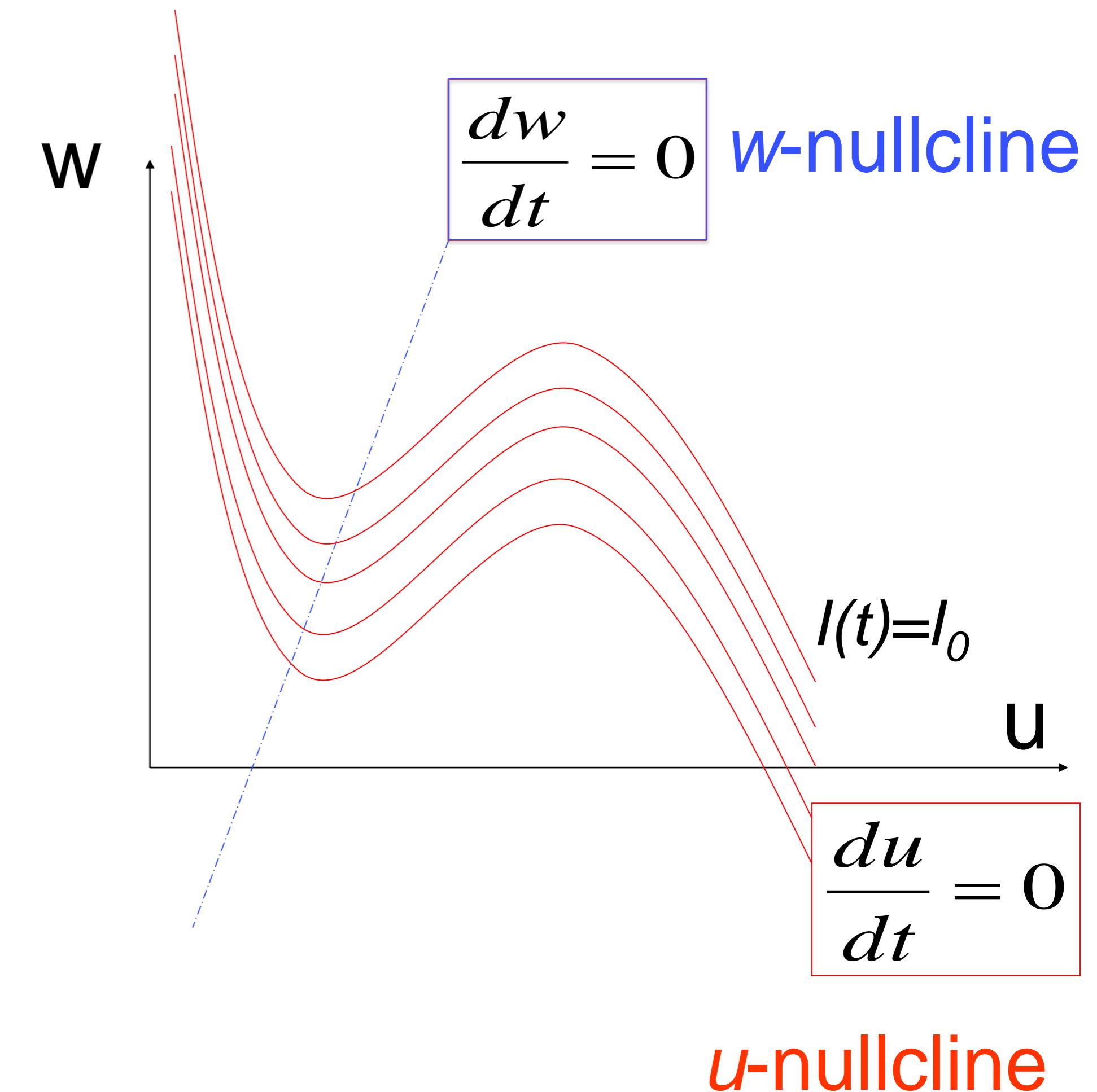
$$= u - \frac{1}{3}u^3 - w + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u, w) = b_0 + b_1 u - w$$

Intersection point (fixed point)

-moves

-changes Stability



NOW Exercise 2.1: Stability of Fixed Point in 2D

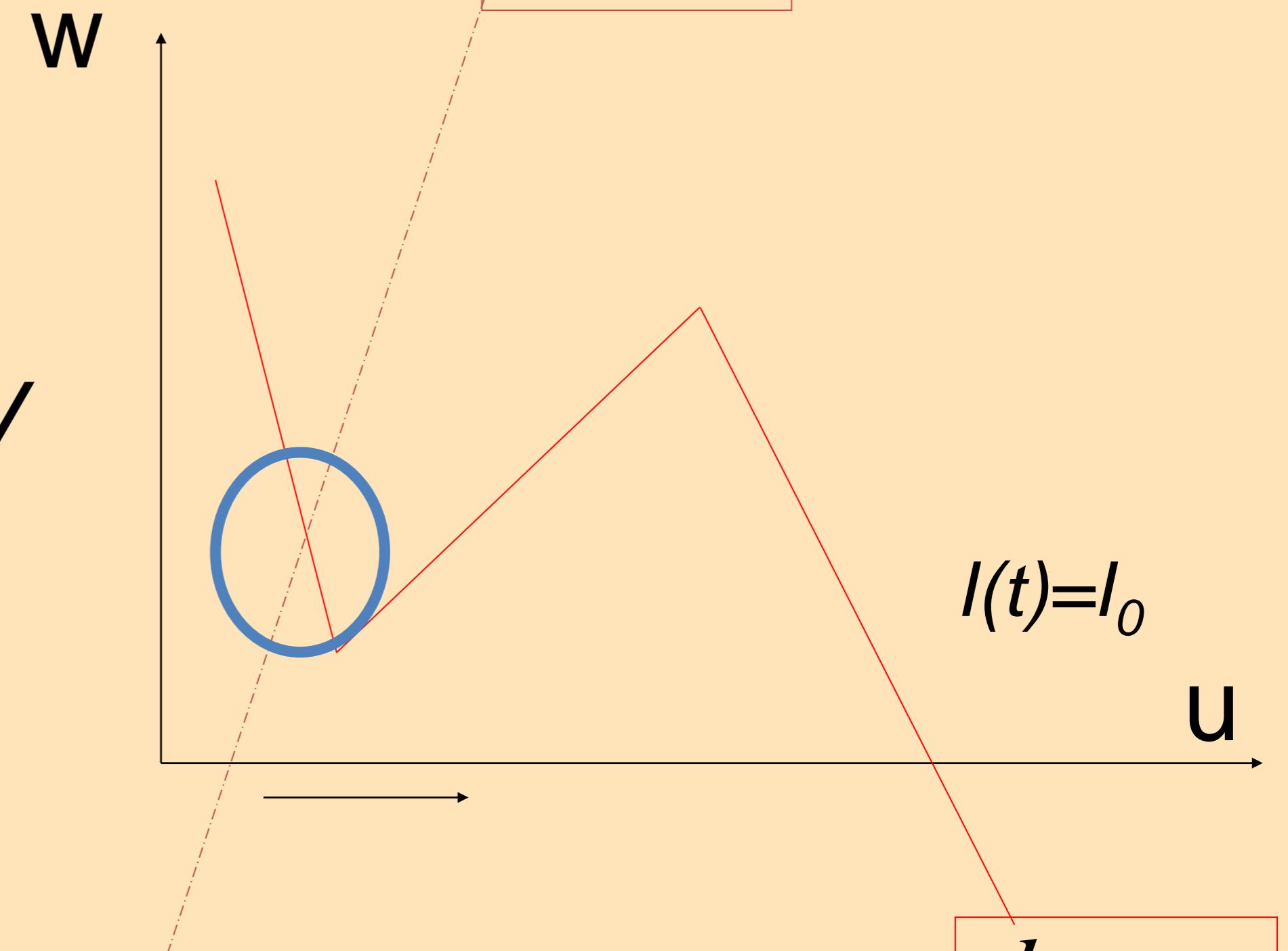
$$\frac{du}{dt} = \alpha u - w$$

$$\frac{dw}{dt} = \beta u - w$$

Exercises:
2.1 now!
2.2 homework

- calculate *stability*
- compare

$$\frac{dx}{dt} = -\frac{x}{\tau}$$



$$I(t) = I_0$$

$$\frac{du}{dt} = 0$$

Next lecture:
11:42

Week 3 – part 3: Analysis of a 2D neuron model



↓ 3.1 From Hodgkin-Huxley to 2D

↓ 3.2 Phase Plane Analysis

- Role of nullcline

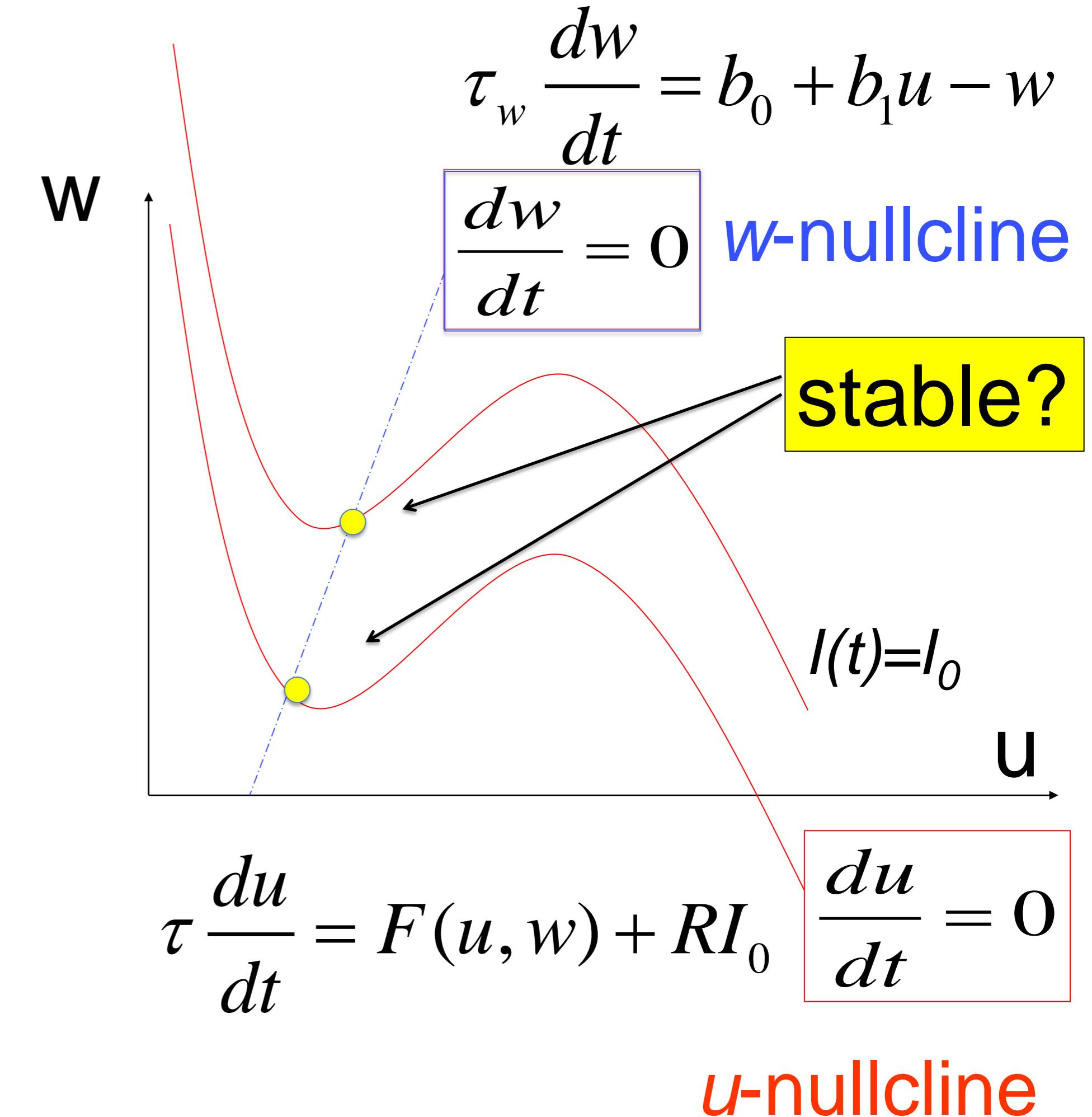
3.3 Analysis of a 2D Neuron Model

- ↓ - pulse input
- constant input

-MathDetour 3: Stability of fixed points

3.4 Type I and II Neuron Models (next week)

Neuronal Dynamics – Detour 3.3 : Stability of fixed points.



Neuronal Dynamics – 3.3 Detour. Stability of fixed points

2-dimensional equation
stimulus

$$\tau \frac{du}{dt} = F(u, w) + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u, w)$$

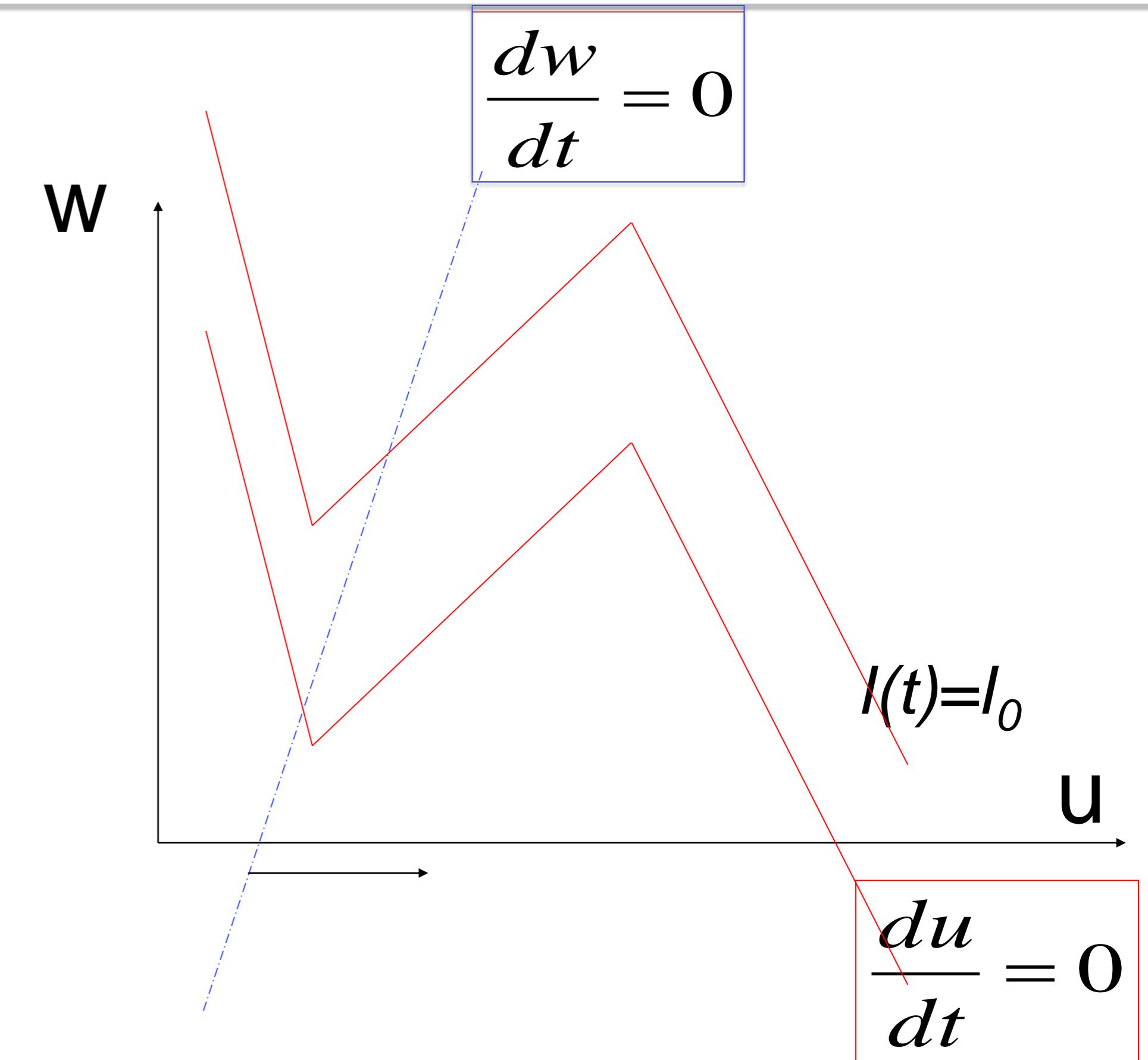
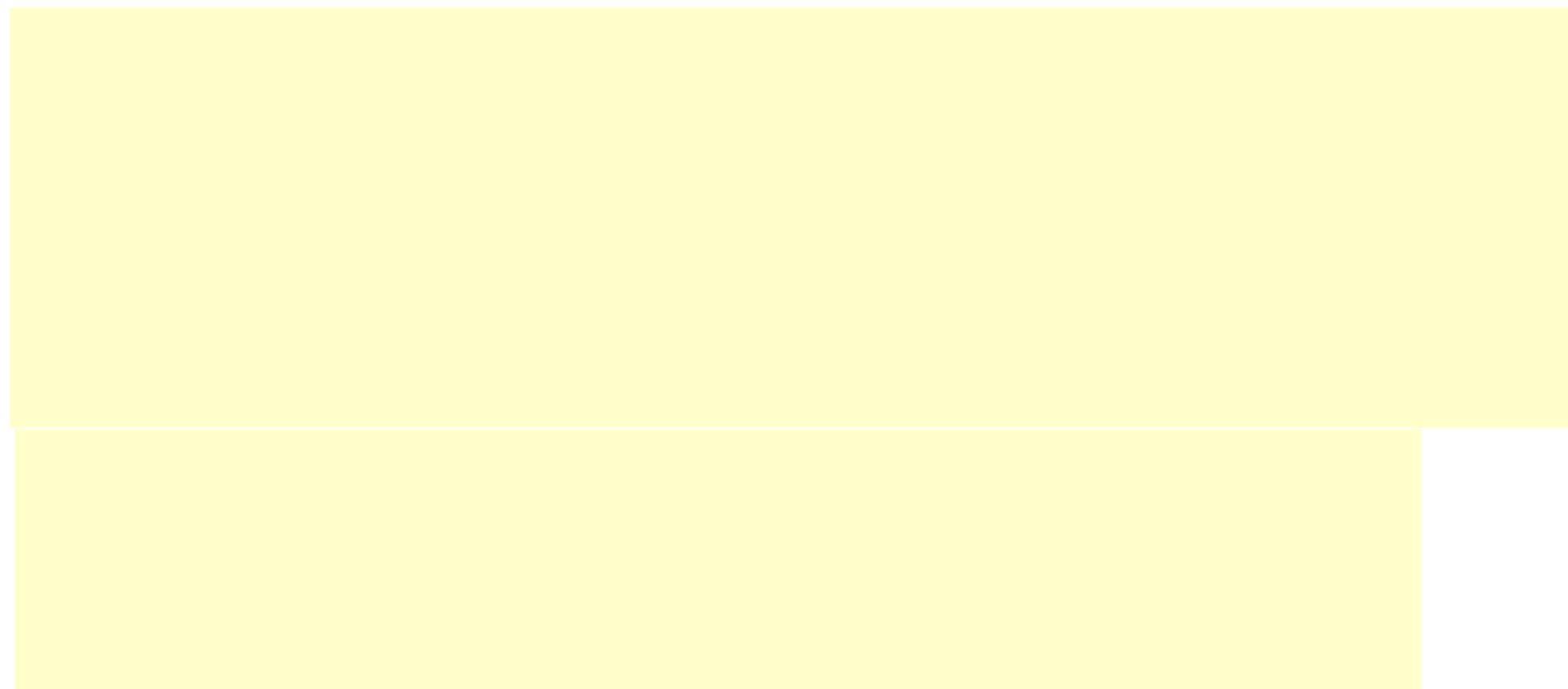
How to determine stability
of fixed point?

Neuronal Dynamics – 3.3 Detour. Stability of fixed points

stimulus

$$\tau \frac{du}{dt} = au - w + I_0$$

$$\tau_w \frac{dw}{dt} = c u - w$$

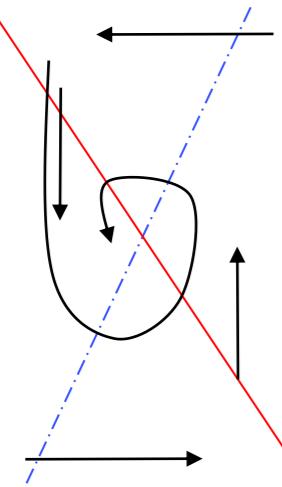


Neuronal Dynamics – 3.3 Detour. Stability of fixed points

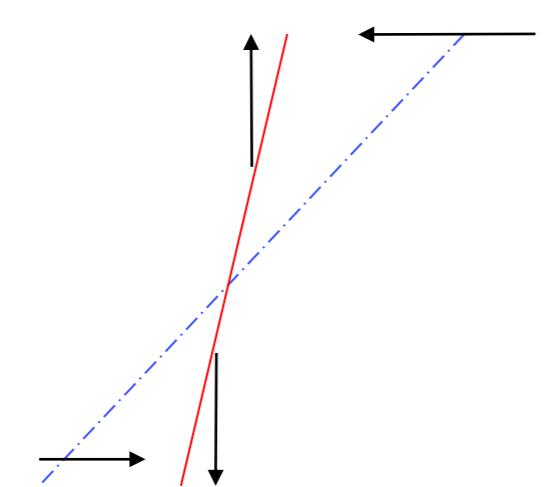
$$\tau \frac{du}{dt} = F(u, w) + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u, w)$$

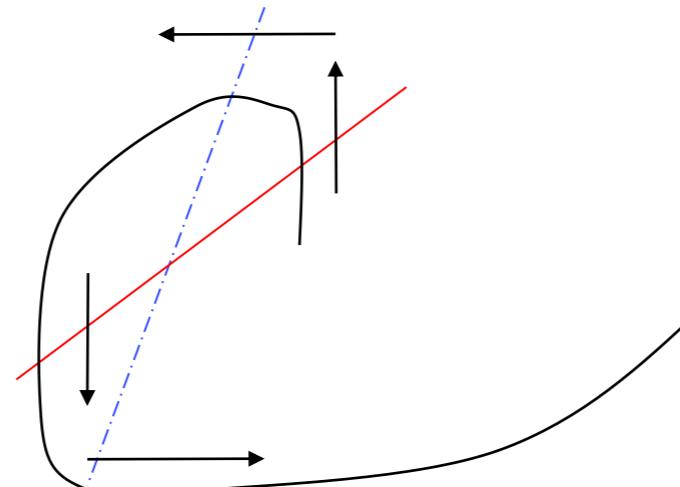
zoom in:



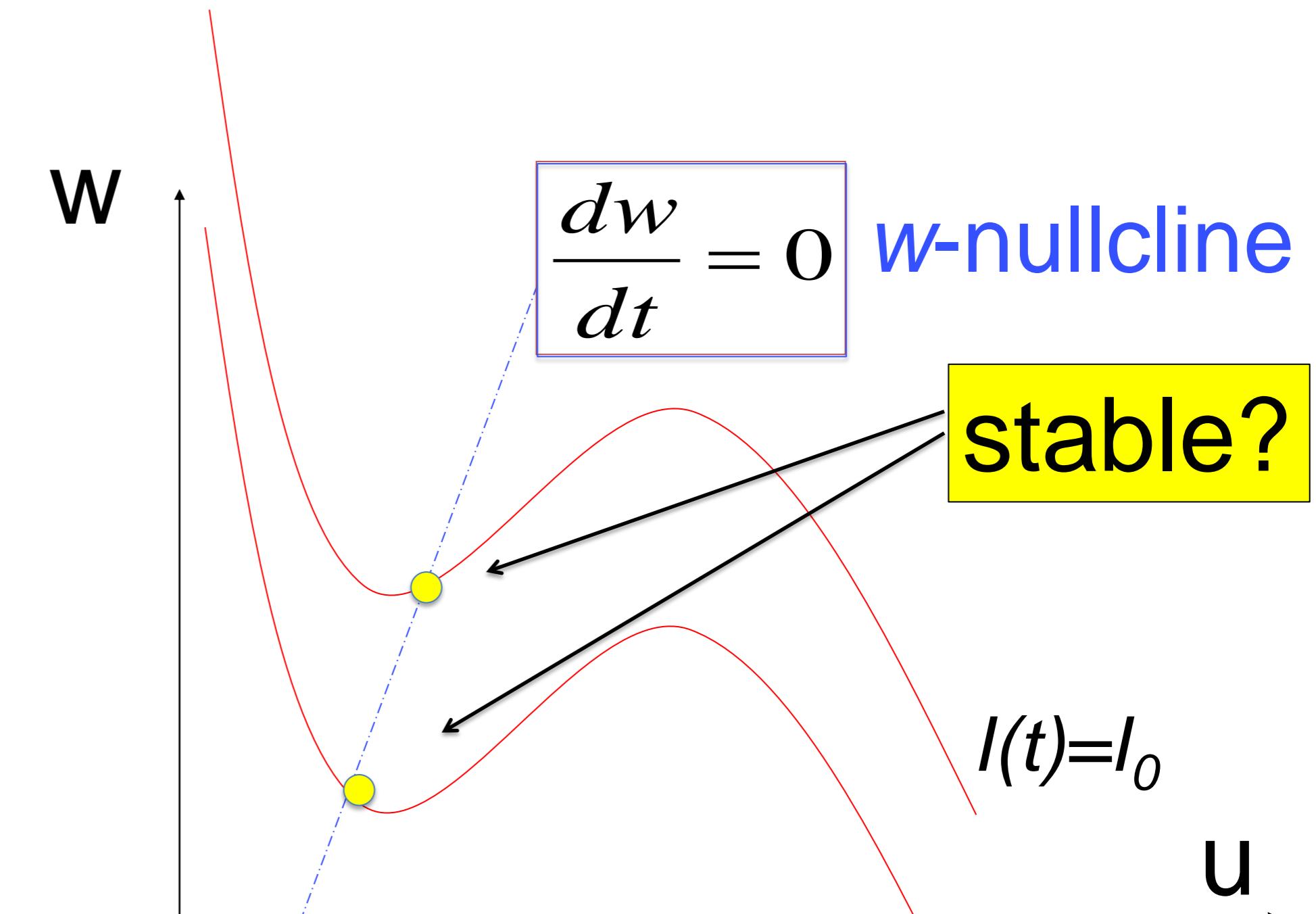
stable



saddle



unstable



Math derivation
now

u-nullcline

Neuronal Dynamics – 3.3 Detour. Stability of fixed points

$$\tau \frac{du}{dt} = F(u, w) + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u, w)$$

Fixed point at (u_0, w_0)

At fixed point

$$0 = F(u_0, w_0) + RI_0$$

$$0 = G(u_0, w_0)$$

zoom in:

$$x = u - u_0$$

$$y = w - w_0$$

Neuronal Dynamics – 3.3 Detour. Stability of fixed points

$$\tau \frac{du}{dt} = F(u, w) + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u, w)$$

zoom in:

$$x = u - u_0$$

$$y = w - w_0$$

$$\tau \frac{dx}{dt} = F_u x + F_w y$$

$$\tau_w \frac{dy}{dt} = G_u x + G_w y$$

Fixed point at (u_0, w_0)

At fixed point

$$0 = F(u_0, w_0) + RI_0$$

$$0 = G(u_0, w_0)$$

$$\frac{d}{dt} \mathbf{x} = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} \mathbf{x},$$

Neuronal Dynamics – 3.3 Detour. Stability of fixed points

Linear matrix equation

$$\frac{d}{dt} \boldsymbol{x} = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} \boldsymbol{x},$$

Search for solution

$$\boldsymbol{x}(t) = e^{\lambda t}$$

Two solution with Eigenvalues λ_+, λ_-

$$\lambda_+ + \lambda_- = F_u + G_w$$

$$\lambda_+ \lambda_- = F_u G_w - F_w G_u$$

Neuronal Dynamics – 3.3 Detour. Stability of fixed points

Linear matrix equation

$$\frac{d}{dt}x = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} x$$

Search for solution

$$x(t) = e^{\lambda t}$$

Two solution with Eigenvalues λ_+, λ_-

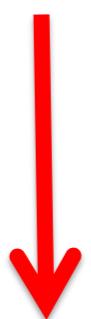
$$\lambda_+ + \lambda_- = F_u + G_w$$

$$\lambda_+ \lambda_- = F_u G_w - F_w G_u$$



Stability requires:

$$\lambda_+ < 0 \text{ and } \lambda_- < 0$$



$$F_u + G_w < 0$$

and

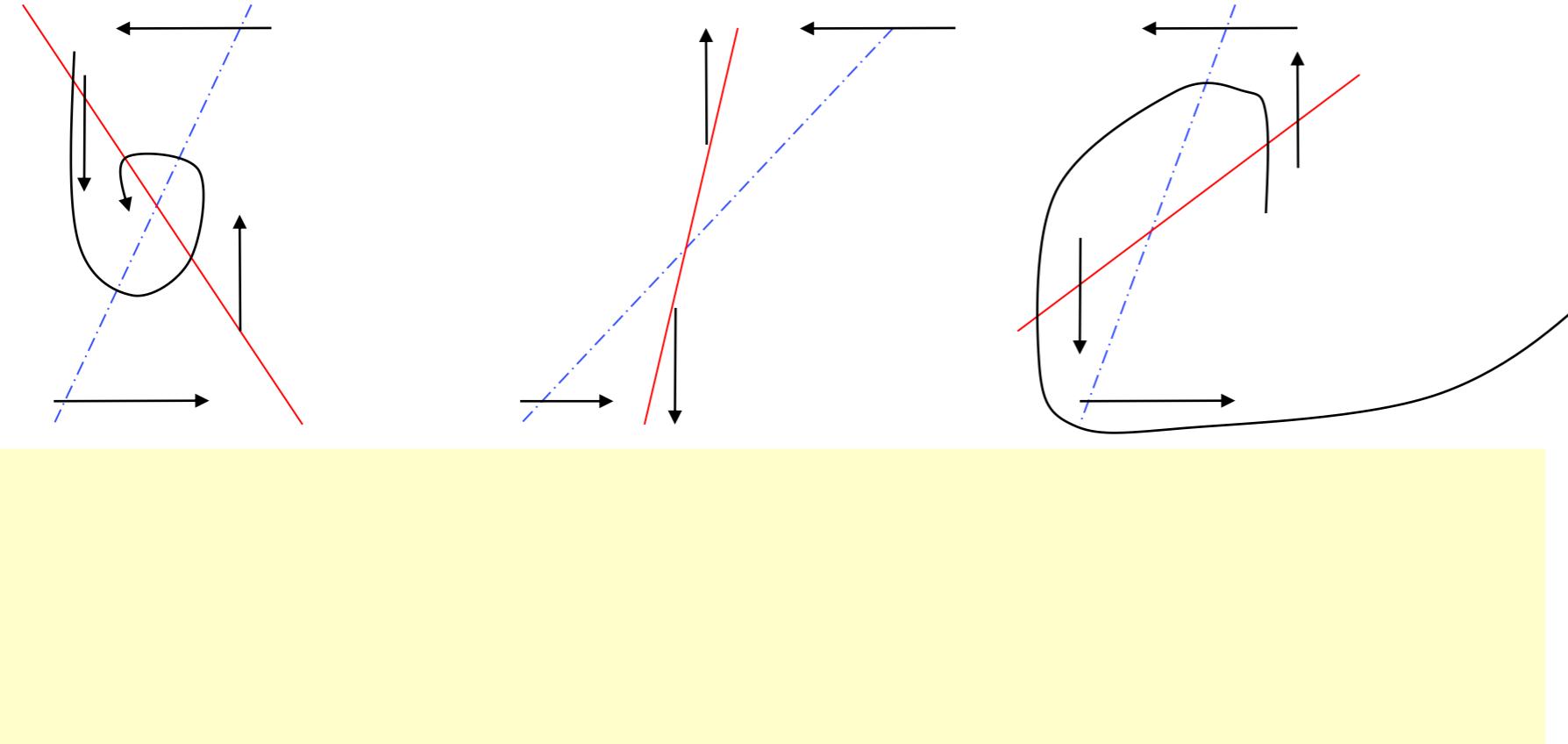
$$F_u G_w - F_w G_u > 0$$

Neuronal Dynamics – 3.3 Detour. Stability of fixed points

$$\tau \frac{du}{dt} = au - w + I_0$$

$$\tau_w \frac{dw}{dt} = cu - w$$

$$\lambda_{+/-} =$$



stimulus



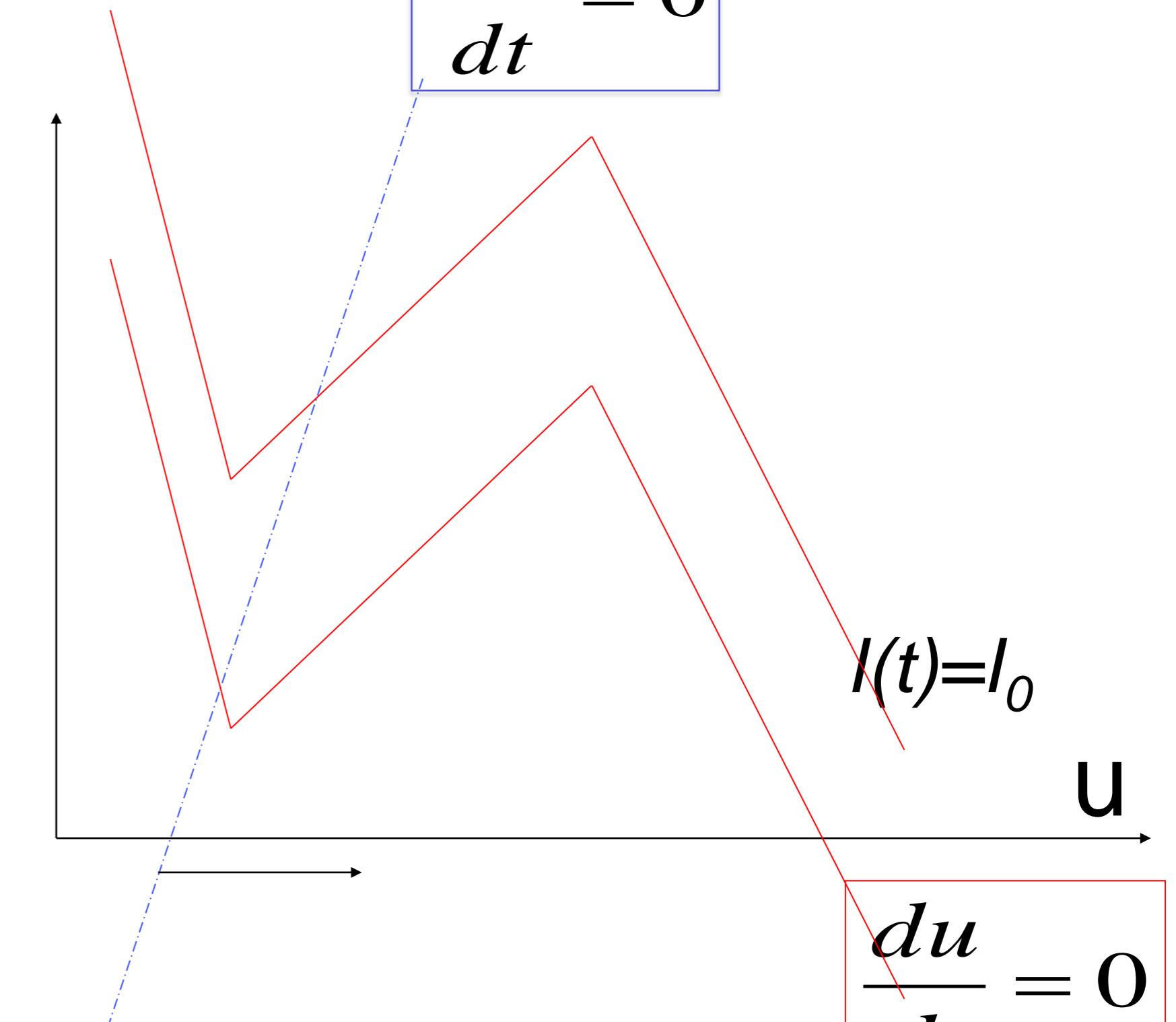
$$\boxed{\frac{dw}{dt} = 0}$$

w

$$I(t)=I_0$$

u

$$\boxed{\frac{du}{dt} = 0}$$



Neuronal Dynamics – 3.3 Detour. Stability of fixed points

2-dimensional equation

$$\tau \frac{du}{dt} = F(u, w) + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u, w)$$

Stability characterized
by Eigenvalues of
linearized equations

$$\frac{d}{dt} \mathbf{x} = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} \mathbf{x}$$

Now Back:

Application to our
neuron model

Neuronal Dynamics – 3.3. FitzHugh-Nagumo Model: Constant input

$$\tau \frac{du}{dt} = F(u, w) + RI_0$$

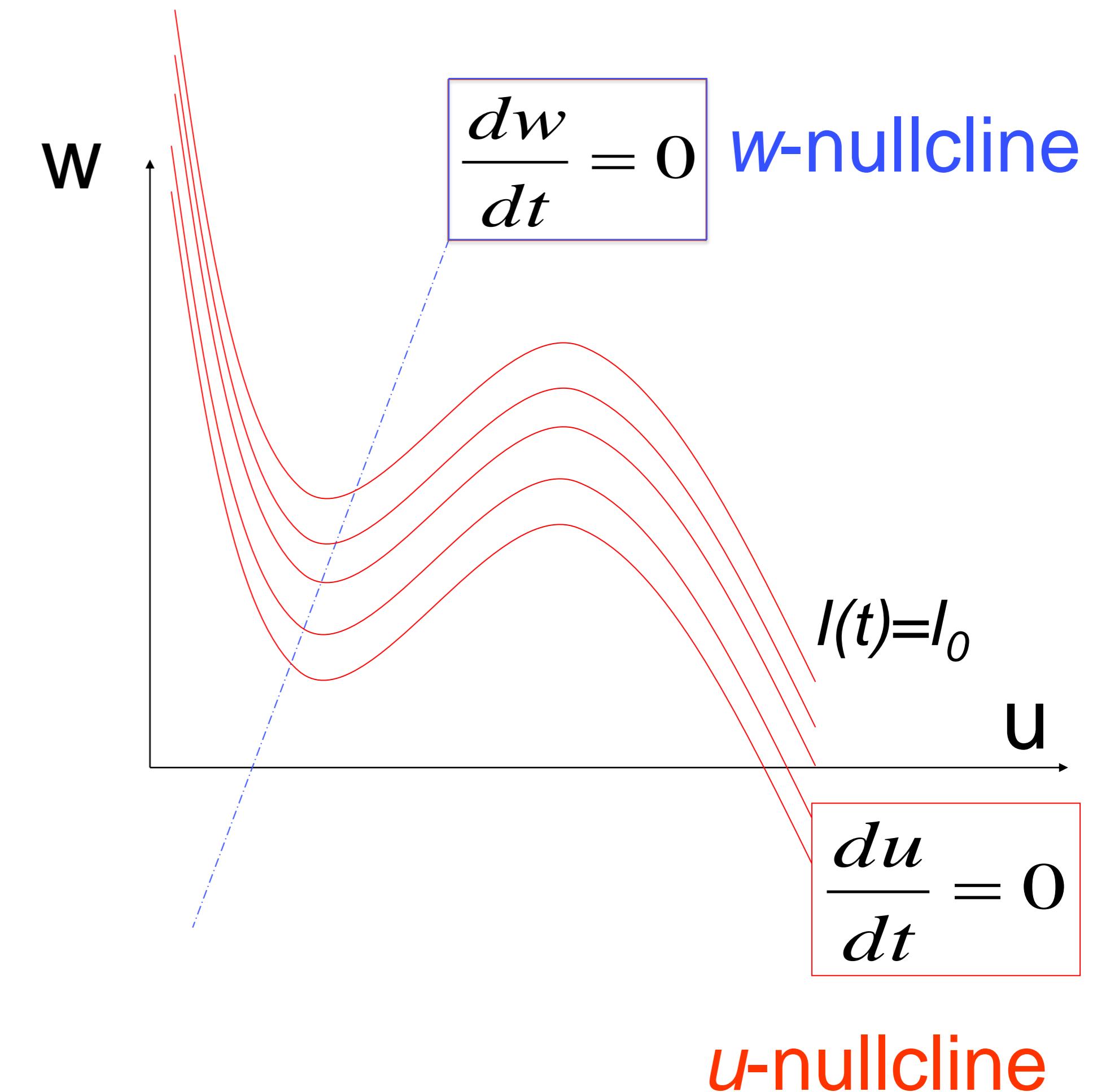
$$= u - \frac{1}{3}u^3 - w + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u, w) = b_0 + b_1 u - w$$

Intersection point (fixed point)

-moves

-changes Stability



Neuronal Dynamics – 3.3. FitzHugh-Nagumo Model: Constant input

$$\tau \frac{du}{dt} = F(u, w) + RI_0$$

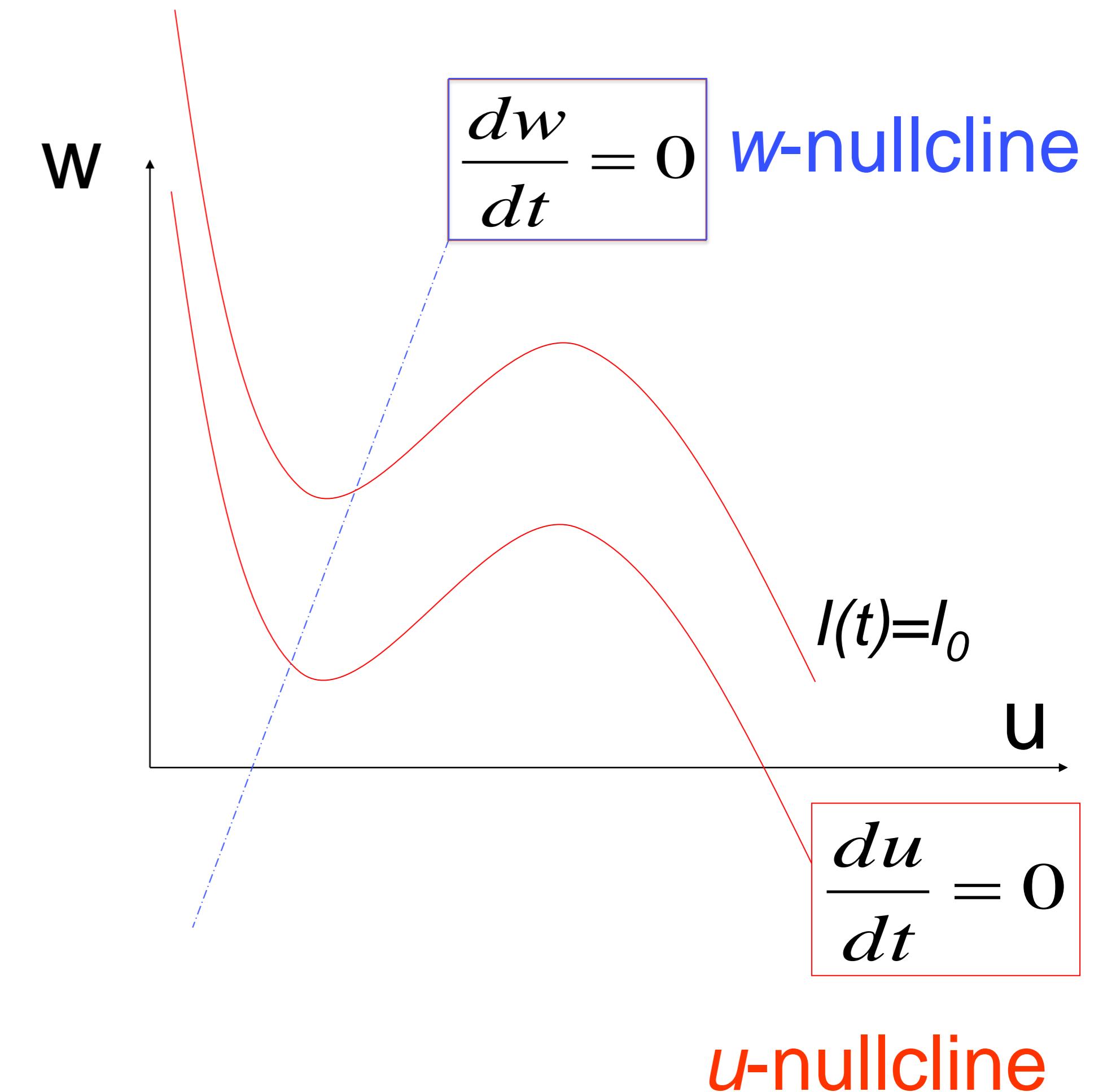
$$= u - \frac{1}{3}u^3 - w + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u, w) = b_0 + b_1 u - w$$

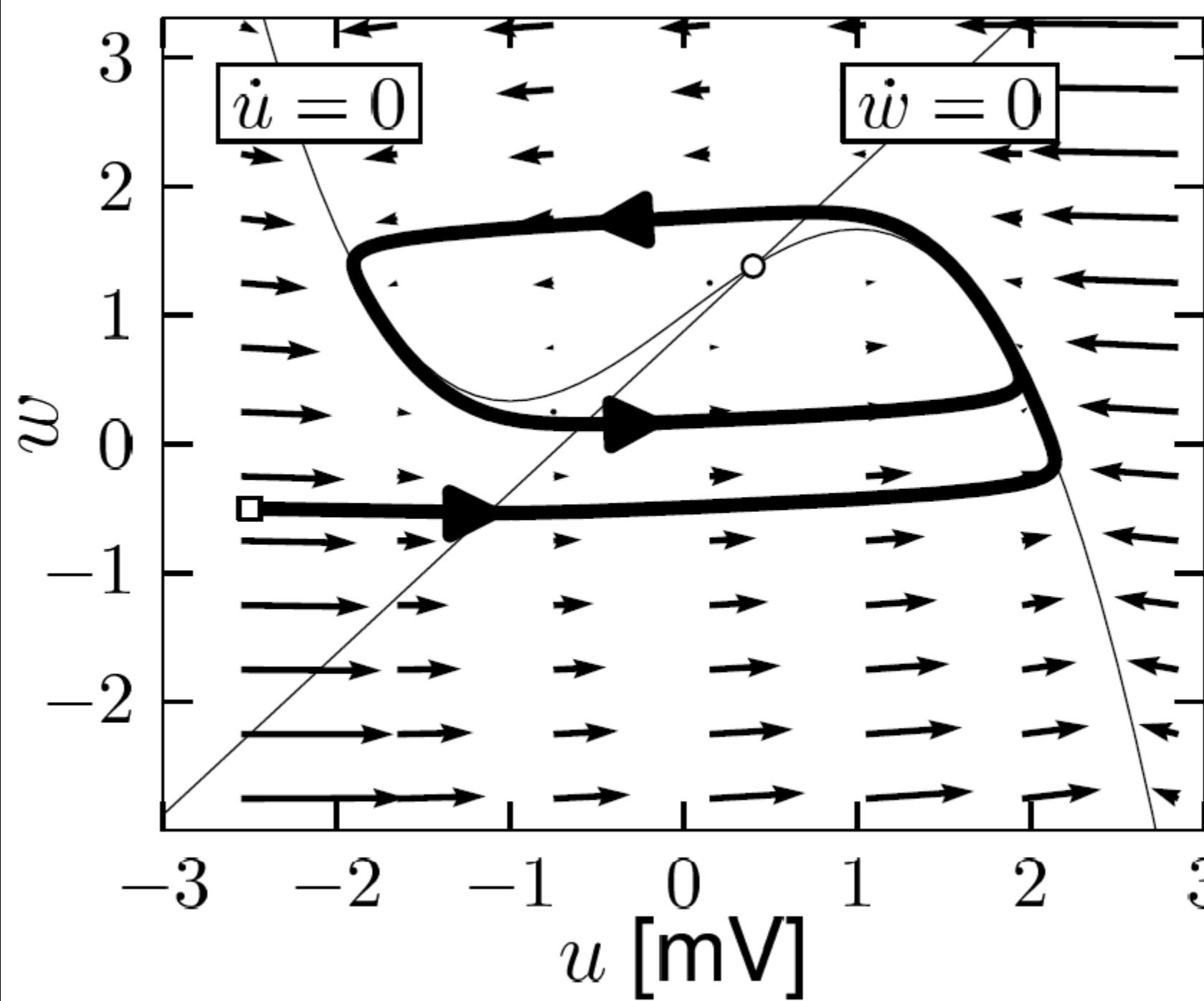
Intersection point (fixed point)

-moves

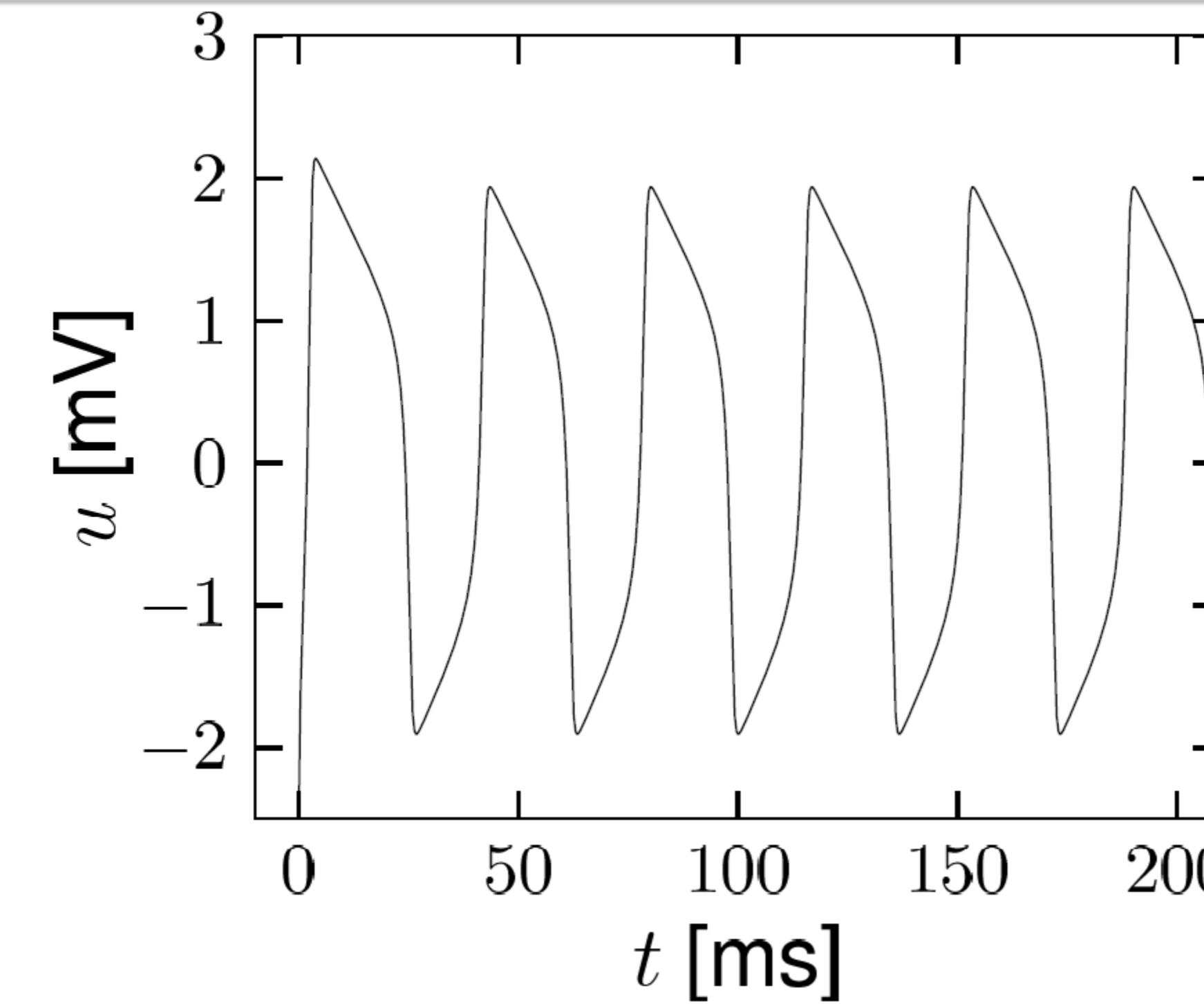
-changes Stability



Neuronal Dynamics – 3.3. FitzHugh-Nagumo Model : Constant input



D



FN model with $b_0 = 0.9; b_1 = 1.0; RI_0 = 2$
constant input: u -nullcline moves
limit cycle

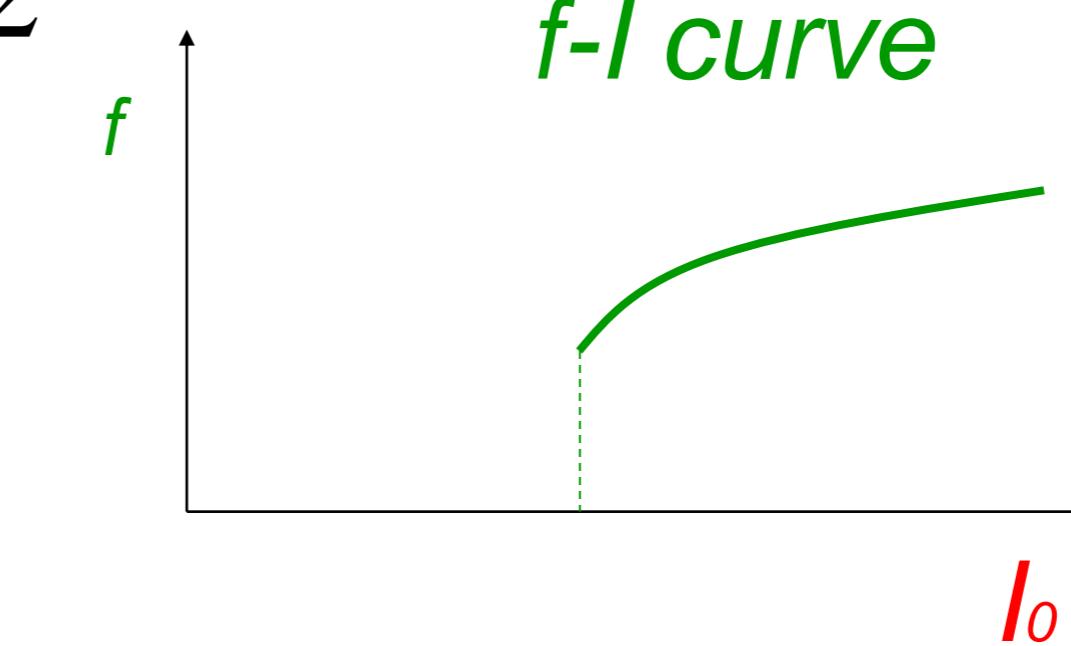


Image:
Neuronal Dynamics,
Gerstner et al.,
CUP (2014)

Neuronal Dynamics – Quiz 3.4.

A. **Short current pulses.** In a 2-dimensional neuron model, the effect of a delta current pulse can be analyzed

- [] By moving the u-nullcline vertically upward
- [] By moving the w-nullcline vertically upward
- [] As a potential change in the stability or number of the fixed point(s)
- [] As a new initial condition
- [] By following the flow of arrows in the appropriate phase plane diagram

B. **Constant current.** In a 2-dimensional neuron model, the effect of a constant current can be analyzed

- [] By moving the u-nullcline vertically upward
- [] By moving the w-nullcline vertically upward
- [] As a potential change in the stability or number of the fixed point(s)
- [] By following the flow of arrows in the appropriate phase plane diagram

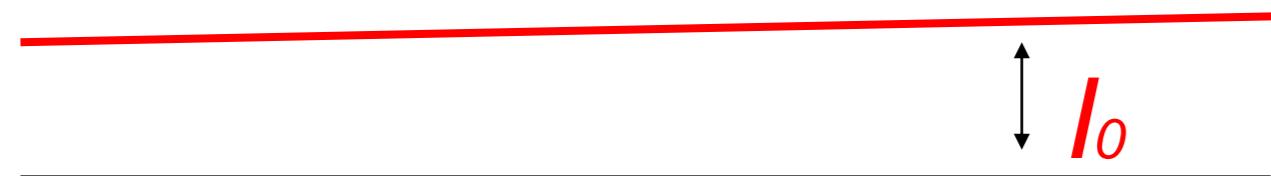
Computer exercise now

Can we understand the dynamics of the 2D model?

The END for today

Now: computer exercises

ramp input/
constant input



Type I and type II models

