Week 8 – part 1 :Variation of membrane potential

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks

Week 8 – Noisy input models: Barrage of spike arrivals

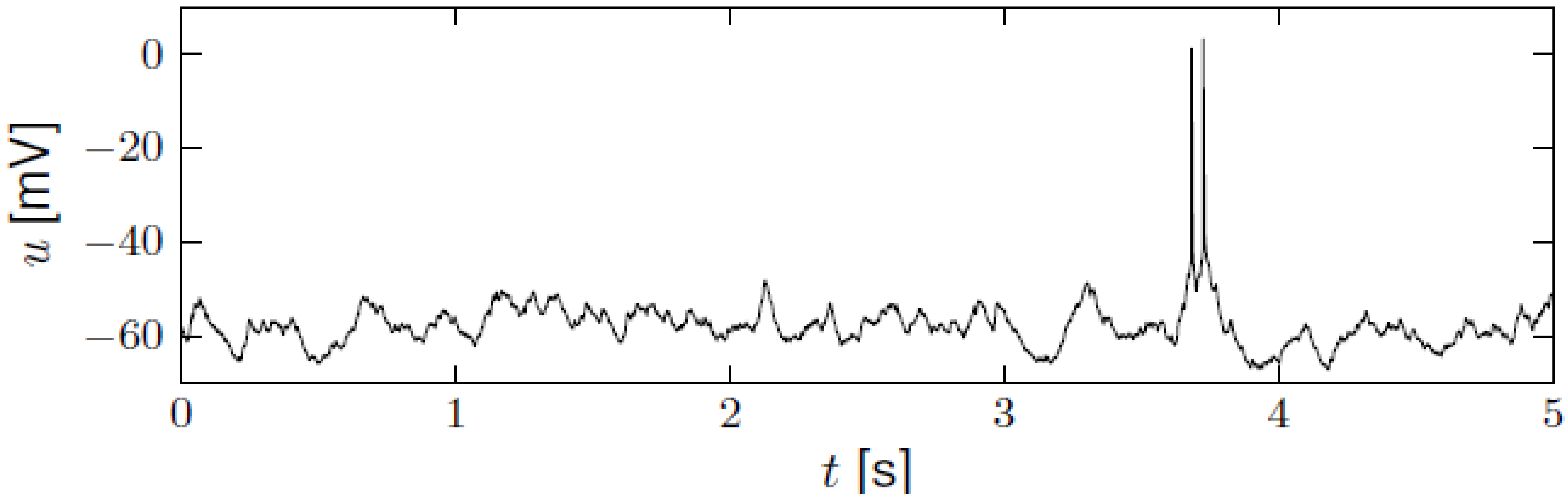
Wulfram Gerstner

EPFL, Lausanne, Switzerland

Spontaneous activity *in vivo*

Variability
- of membrane potential?
- of spike timing?

awake mouse, cortex, freely whisking,



Crochet et al., 2011

Fluctuations

-of membrane potential

-of spike times

In vivo data

→ looks ‘noisy’

fluctuations=noise?

relevance for coding?

In vitro data

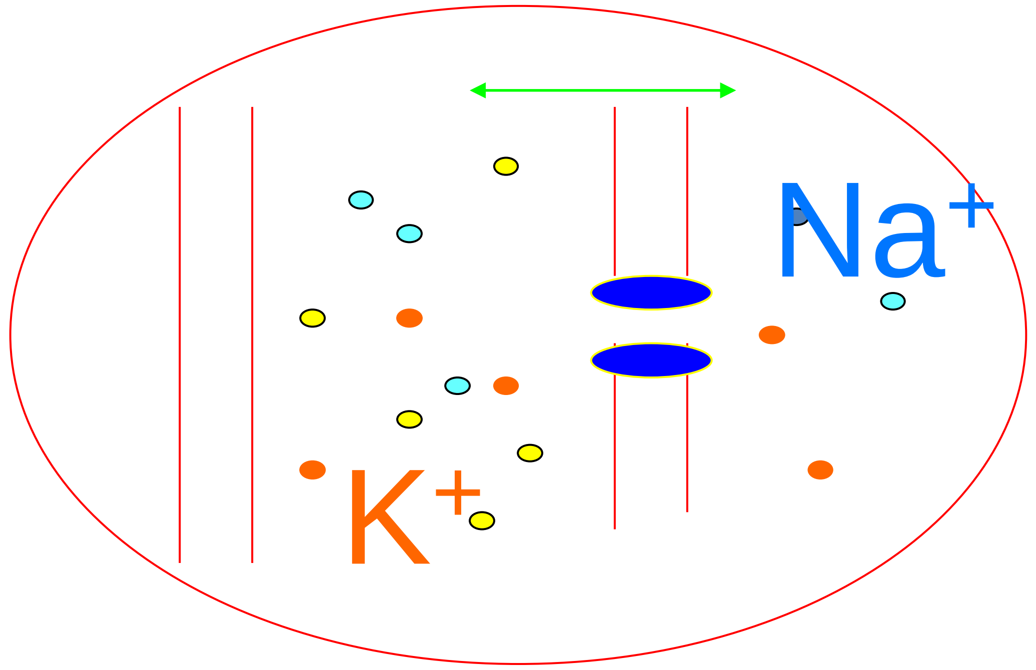
→ fluctuations

source of fluctuations?

model of fluctuations?

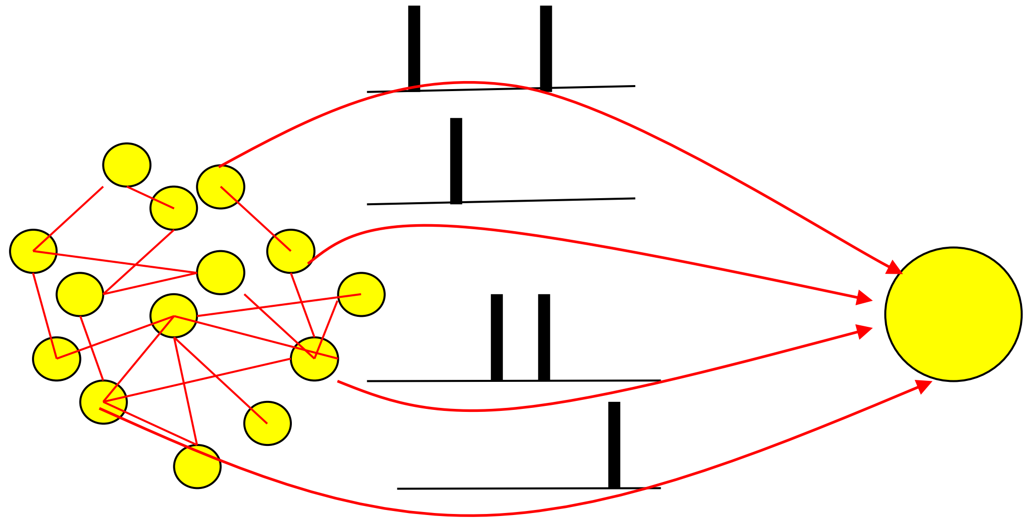
Neuronal Dynamics – 8.1. Review Sources of Variability

- Intrinsic noise (ion channels)



-Finite number of channels
-Finite temperature

-Network noise (background activity)



-Spike arrival from other neurons
-Beyond control of experimentalist

→ Check intrinisic noise by removing the network

Neuronal Dynamics – 8.1. Review: Sources of Variability

In vivo data

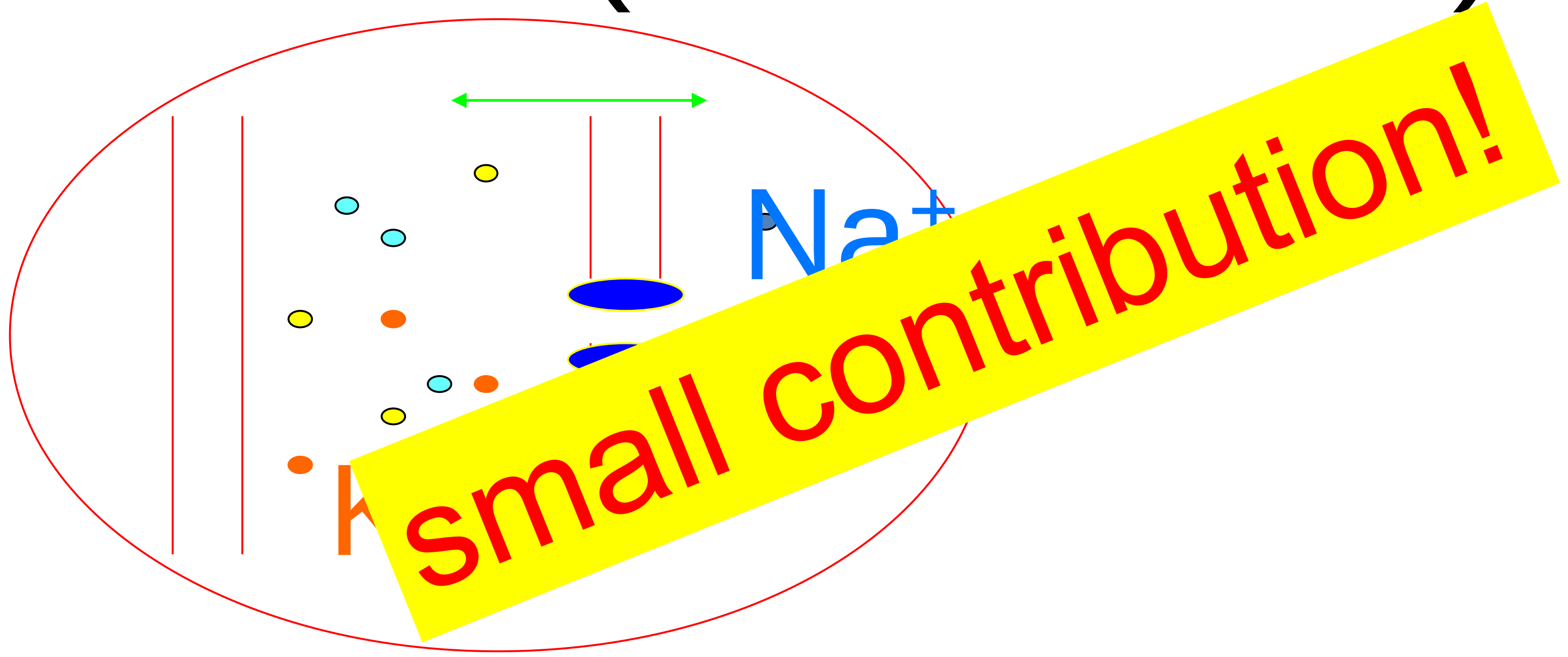
→ looks ‘noisy’

In vitro data

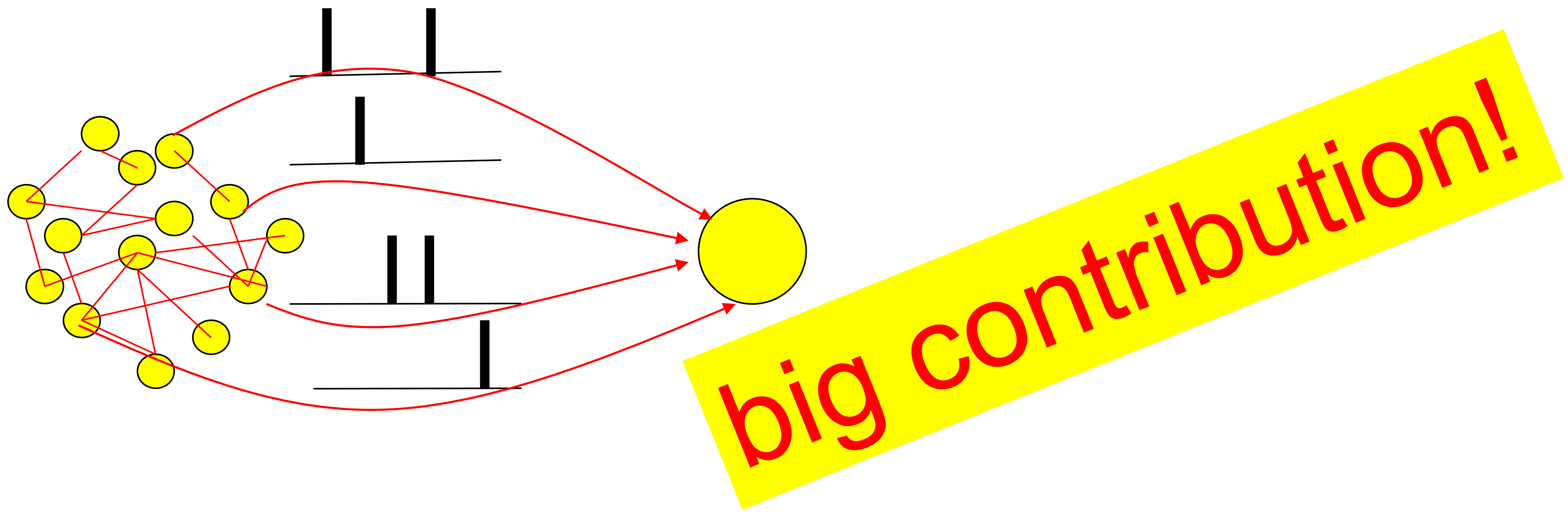
→small fluctuations

→nearly deterministic

- Intrinsic noise (ion channels)

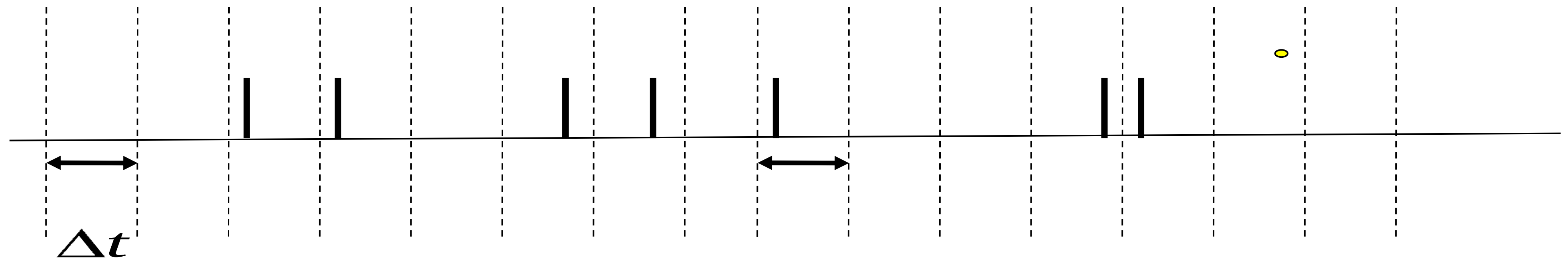


-Network noise



Neuronal Dynamics – 8.1 Review: Calculating the mean

$$RI^{syn}(t) = \sum_k w_k \sum_f \alpha(t - t_k^f)$$



$$I^{syn}(t) = \frac{1}{R} \sum_k w_k \sum_f \int dt' \alpha(t - t') \delta(t' - t_k^f)$$

$$x(t) = \sum_f \int dt' f(t - t') \delta(t' - t_k^f)$$

mean: assume Poisson process

$$I_0 = \langle I^{syn}(t) \rangle = \frac{1}{R} \sum_k w_k \int dt' \alpha(t - t') \left\langle \sum_f \delta(t' - t_k^f) \right\rangle$$

$$\langle x(t) \rangle = \int dt' f(t - t') \left\langle \sum_f \delta(t' - t_k^f) \right\rangle$$

use for exercises

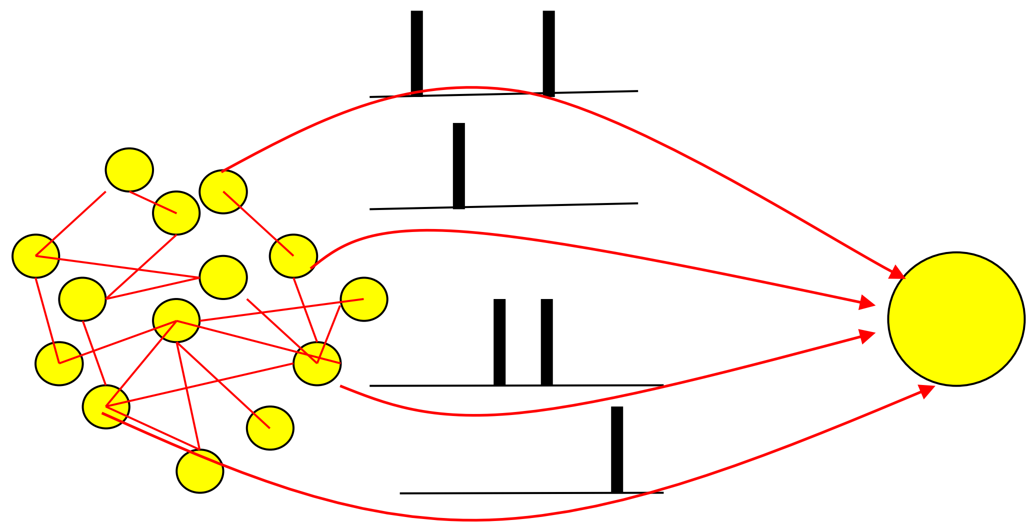
$$I_0 = \frac{1}{R} \sum_k w_k \int dt' \alpha(t - t') \; \nu_k$$

$$\langle x(t) \rangle = \int dt' f(t - t') \; \rho(t')$$

rate of inhomogeneous Poisson process

Neuronal Dynamics – 8.1. Fluctuation of potential

for a passive membrane, predict
-mean
-variance
of membrane potential fluctuations

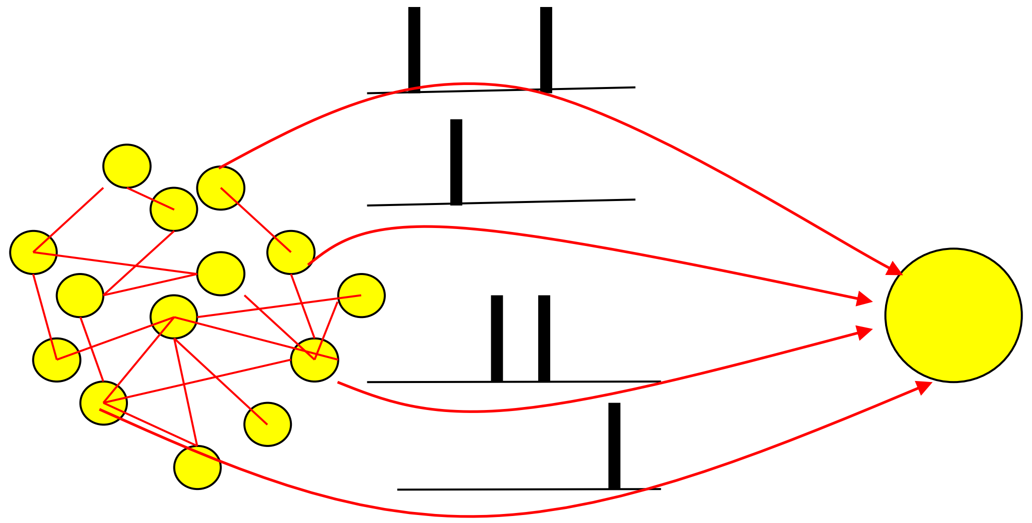


Passive membrane

$$\tau \frac{d}{dt} u = -(u - u_{rest}) \quad + R I^{syn}(t)$$

Passive membrane
=Leaky integrate-and-fire
without threshold

Neuronal Dynamics – 8.1. Fluctuation of current/potential



Synaptic current pulses of shape α

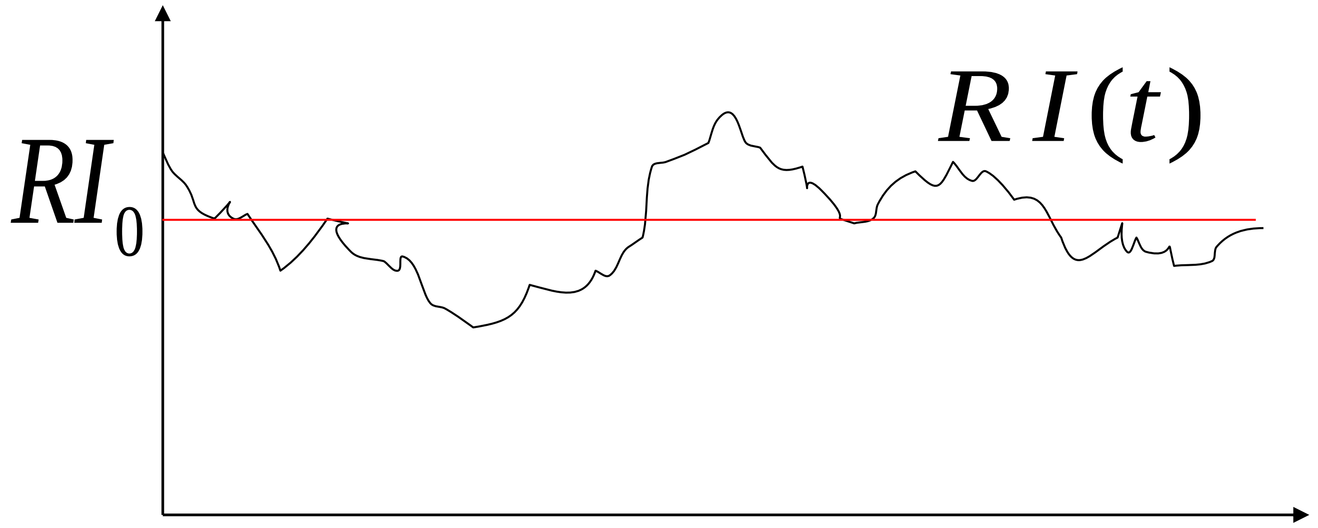
$$RI^{syn}(t) = \sum_k w_k \sum_f \alpha(t - t_k^f)$$

EPSC

Blackboard, Math detour: White noise

Passive membrane

$$\tau \frac{d}{dt} u = -(u - u_{rest}) \quad + RI^{syn}(t)$$

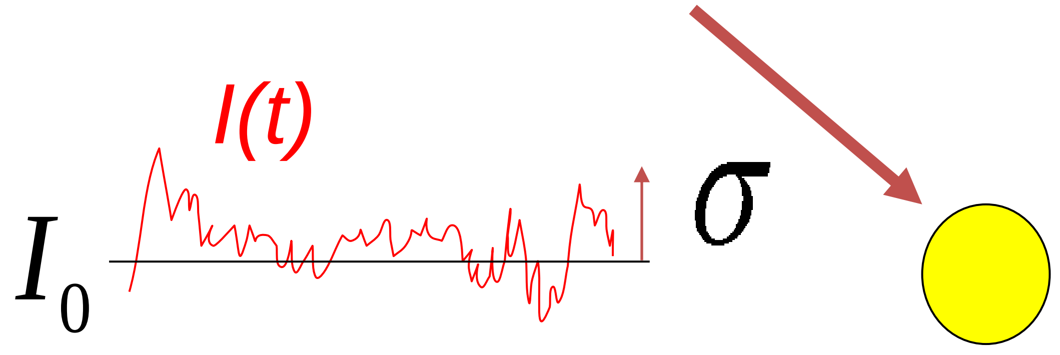


$$I^{syn}(t) = I_0 + I^{fluct}(t)$$

$$RI^{syn}(t) = RI_0(t) + \xi(t)$$

$$\langle \xi(t) \rangle = 0$$

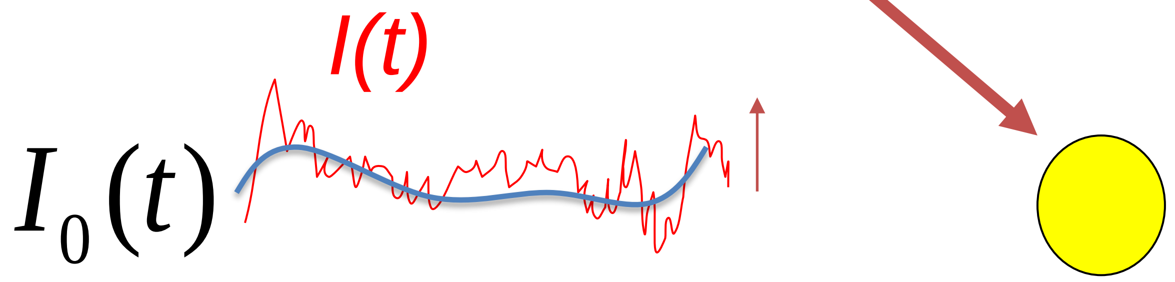
$$\langle \xi(t)\xi(t') \rangle = a^2 \tau\, \delta(t - t')$$



Fluctuating input current

Neuronal Dynamics – 8.1 Calculating autocorrelations

$$I(t) = I_0(t) + \xi(t)$$



Fluctuating input current

Autocorrelation

$$\langle x(t)x(t')\rangle =$$

Blackboard, Math detour

$$x(t) = \int dt' f(t-t') I(t)'$$

$$x(t) = \int ds\, f(s) I(t-s)$$

Mean:

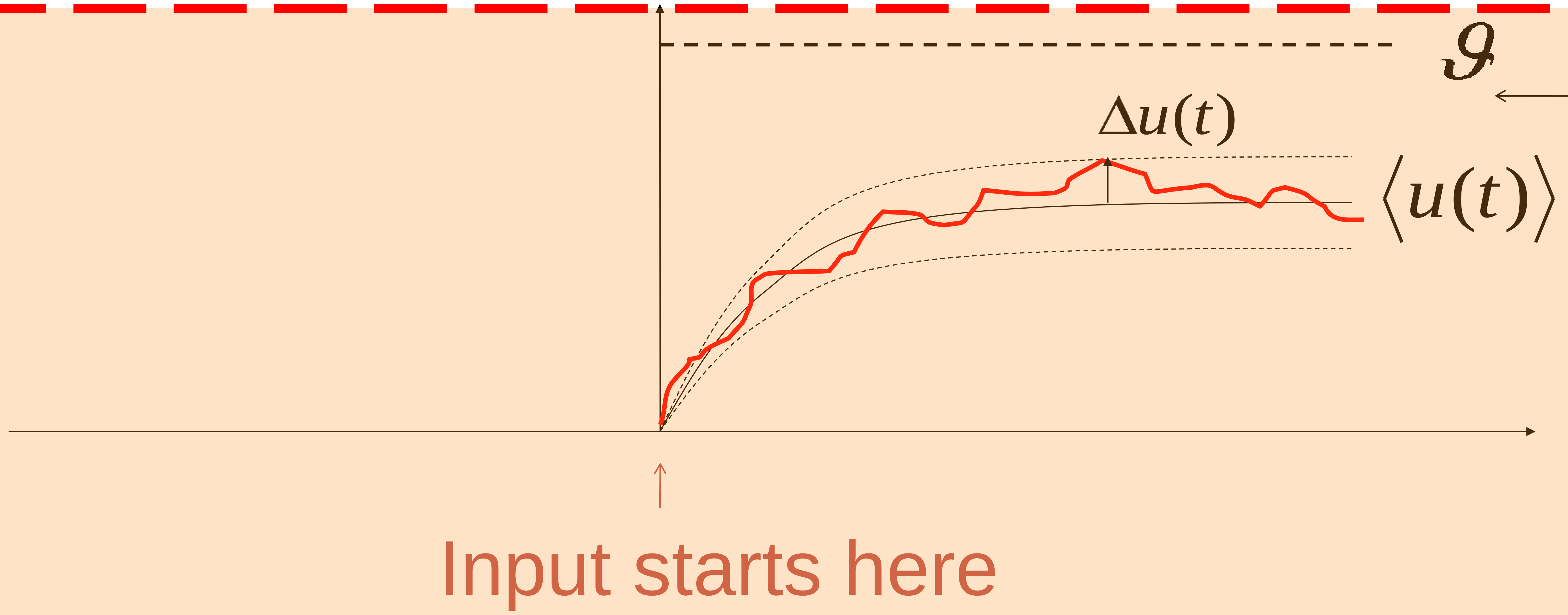
$$\langle x(t)\rangle = \int ds\, f(s) \langle I(t-s)\rangle$$

$$\langle x(t)\rangle = \int ds\, f(s)\, [I_0(t-s) + \langle \xi(t-s)\rangle]$$

$$\langle x(t)\rangle = \int ds\, f(s)\, I_0(t-s)$$

$$\langle x(t)x(\hat{t})\rangle = \int dt' \int dt'' f(t-t') f(\hat{t}-t'') \langle I(t')I(t'')\rangle$$

White noise: Exercise 1.1-1.2 now



Assumption:
far away from theshold

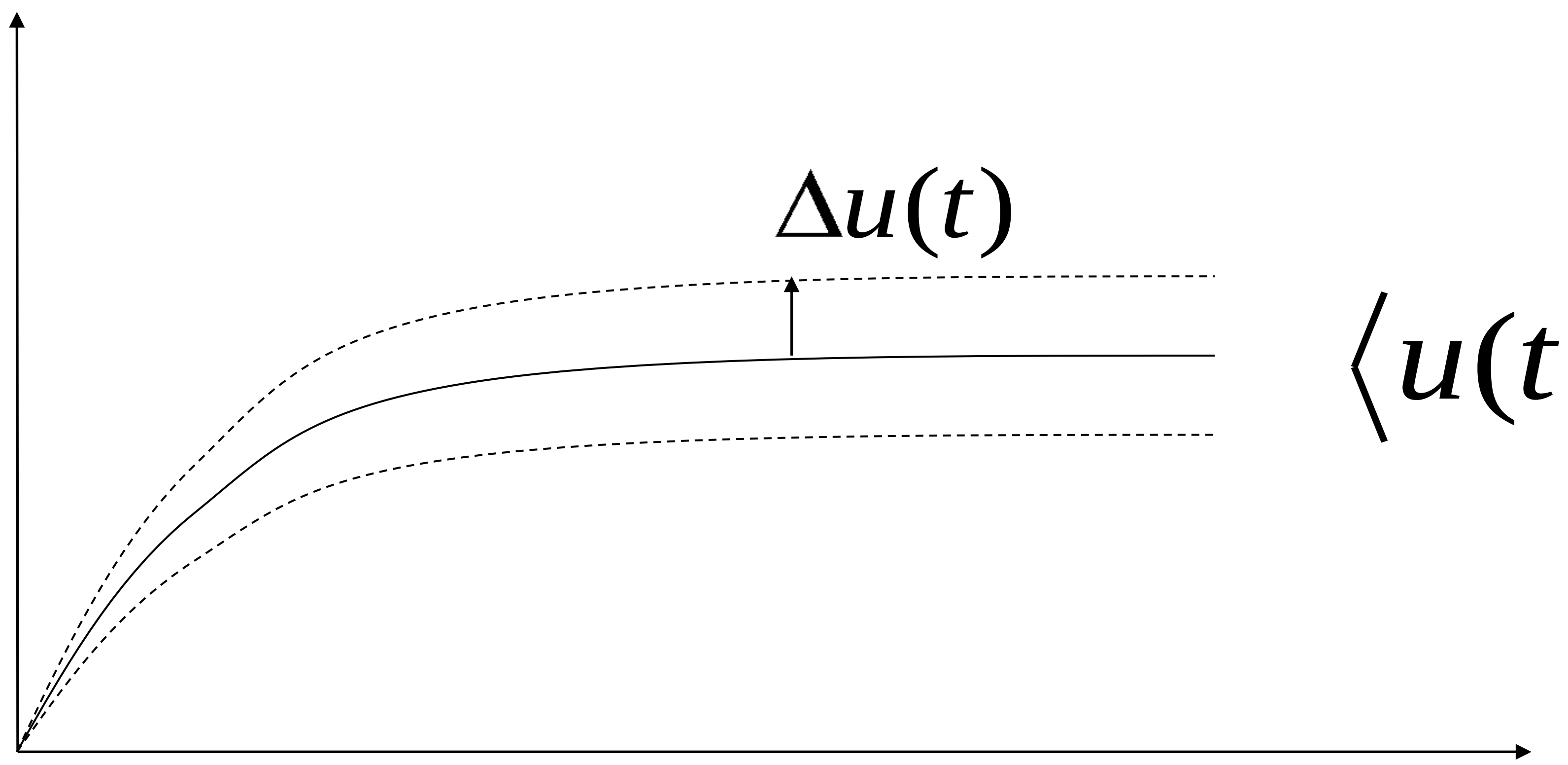
Next lecture:
9:56

Expected voltage at time *t* $\langle u(t) \rangle = ?$

Variance of voltage at time *t*

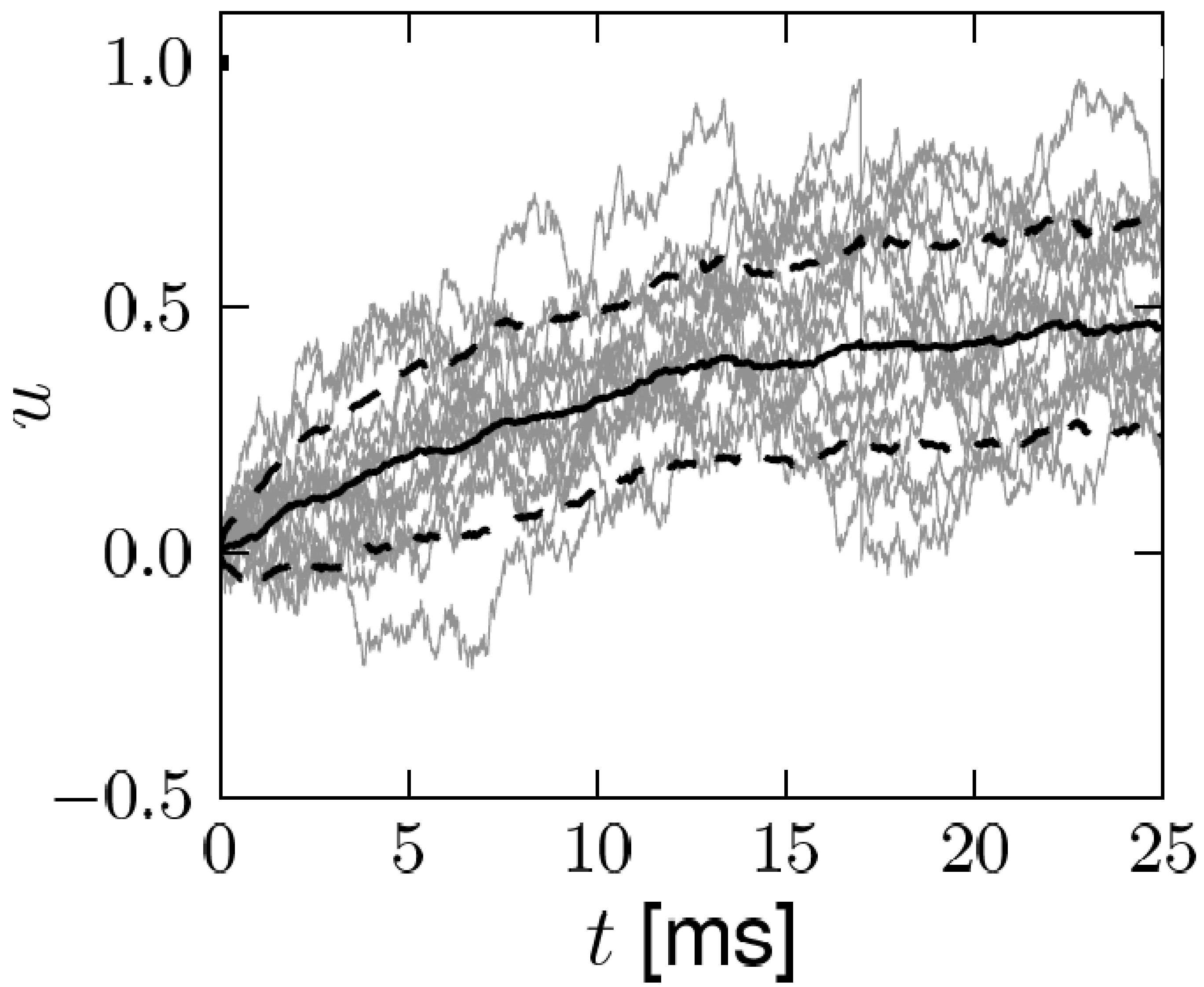
$$\langle \Delta u(t) \Delta u(t) \rangle = \langle u(t) u(t) \rangle - \langle u(t) \rangle^2 =$$

Diffusive noise (stochastic spike arrival)



$$\langle u(t) \rangle = u_0(t)$$

$$\tau \frac{d}{dt} u = -(u - u_{rest}) + R\, I(t) + \xi(t)$$



$$\langle \Delta u(t) \Delta u(t) \rangle = \langle u(t) u(t) \rangle - \langle u(t) \rangle^2 =$$

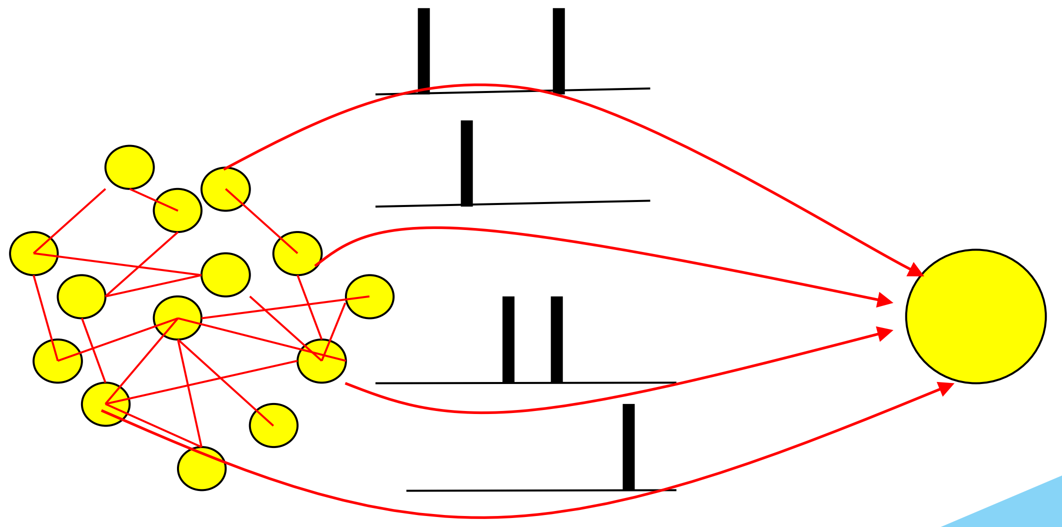
$$\langle \Delta u(t') \Delta u(t) \rangle = \langle u(t) u(t') \rangle - \langle u(t) \rangle \langle u(t') \rangle =$$

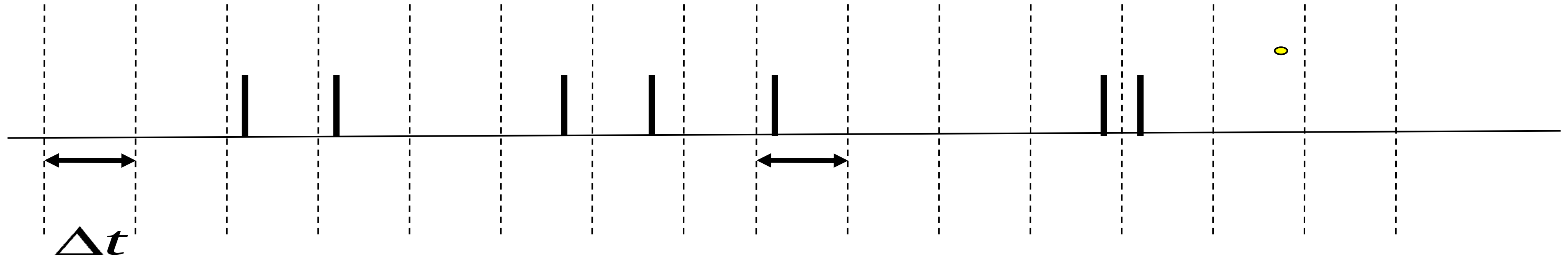
Math argument

$$p(u,t) = \frac{1}{\sqrt{2\pi \langle \Delta u^2(t) \rangle}} \exp\left\{ -\frac{[u(t|\hat{t}) - u_0(t)]^2}{2 \langle \Delta u^2(t) \rangle} \right\}$$

$$\langle [\Delta u(t)]^2 \rangle = \sigma_u^2 [1 - \exp(-2t/\tau)]$$

Neuronal Dynamics – 8.1 Calculating autocorrelations





Blackboard, Math detour

$$x(t) = \sum_f \int dt' \, f(t-t')\, \delta(t' - t_k^f)$$

$$= \int dt' \, f(t-t')\, S(t')$$

Autocorrelation

$$\langle x(t) x(t') \rangle =$$

$$\langle x(t) x(\hat{t}) \rangle = \int dt' \int dt'' f(t-t')\, f(\hat{t}-t'') \langle S(t') S(t'') \rangle$$

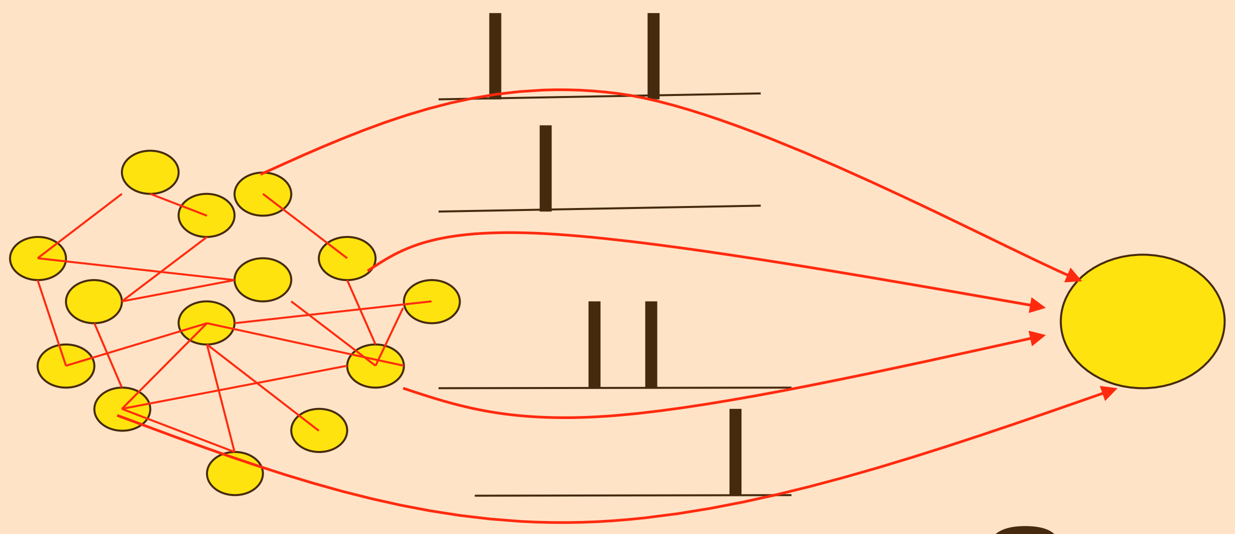
Mean:

$$\langle x(t) \rangle = \int dt' \, f(t-t') \langle S(t') \rangle$$

$$\langle x(t) \rangle = \int ds \, f(s) \; \nu(t-s)$$

rate of inhomogeneous Poisson process

Exercise 2.1-2.3 now: Diffusive noise (stochastic spike arrival)



Stochastic spike arrival:

excitation, total rate $\langle S(t) \rangle = \nu$

Next lecture: 9h58

Synaptic current pulses

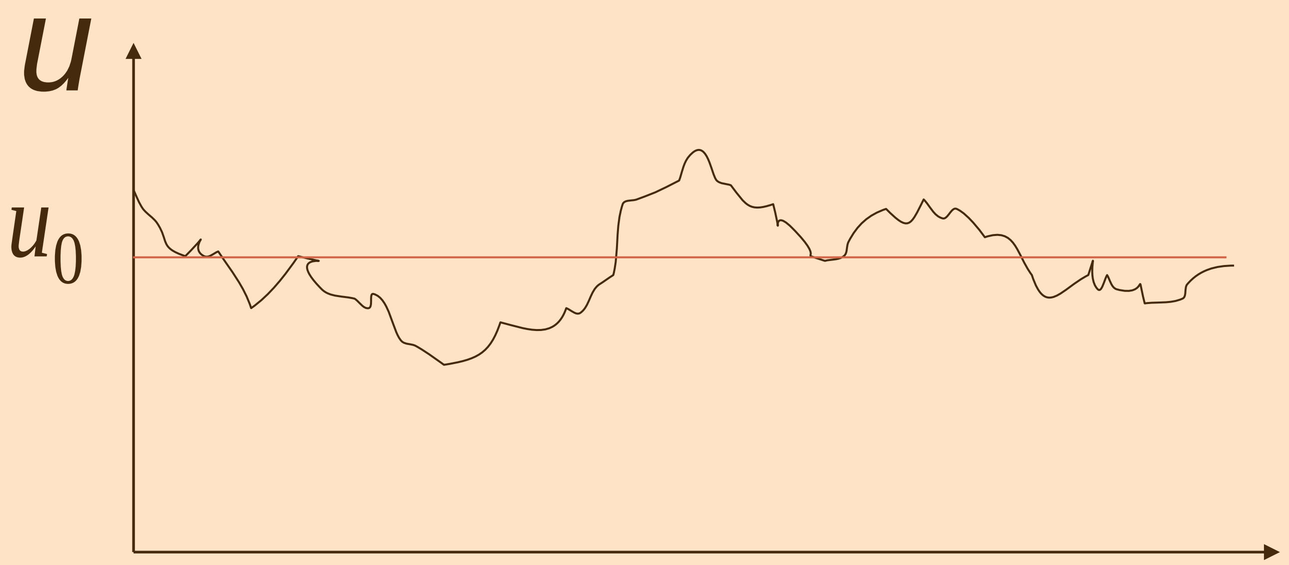
$$\tau \frac{d}{dt} u = -(u - u_{rest}) + R\, S(t)$$

$$S(t) = q_e \sum_f \delta(t - t^f)$$

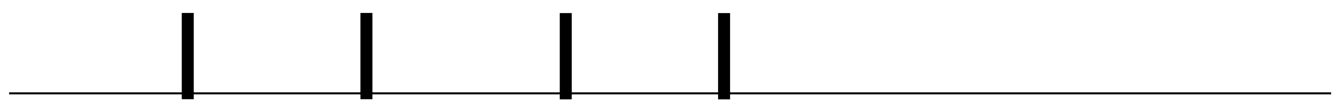
1. Assume that for t>0 spikes arrive stochastically with rate ν
   - Calculate mean voltage

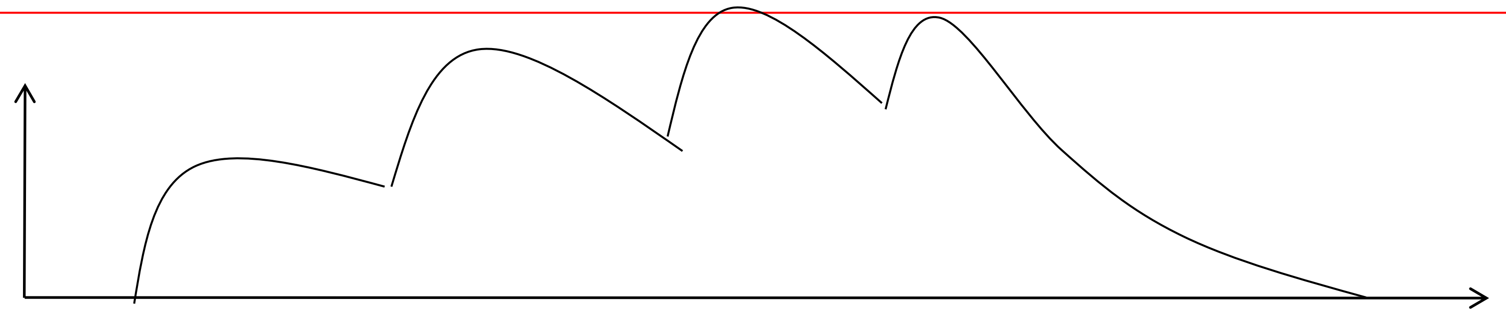
2. Assume autocorrelation $\langle S(t) S(t') \rangle = \nu\, \delta(t - t') + \nu^2$

   - Calculate $\langle u(t) u(t) \rangle = ?$



Poisson spike arrival: **Mean and autocorrelation of filtered signal**





$$S(t) = \sum_f \delta(t - t^f)$$

$$F(s)$$

Filter

$$x(t) = \int F(s) S(t-s) ds$$

Assumption: stochastic spiking

rate $\nu(t)$

$$\langle x(t) \rangle = \int F(s) \langle S(t-s) \rangle ds$$

mean

$$\langle x(t) \rangle = \int F(s) \langle \nu(t-s) \rangle ds$$

Autocorrelation of output

$$\langle x(t) x(t') \rangle = \left\langle \int F(s) S(t-s) ds \int F(s') S(t'-s') ds' \right\rangle$$

$$\langle x(t) x(t') \rangle = \int F(s) F(s') \underline{\langle S(t-s) S(t'-s') \rangle} ds ds'$$

Autocorrelation of input

Week 8 – part 2 : Autocorrelation of Poisson Process

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Biological Modeling of Neural Networks

Week 8 – Variability and Noise: Autocorrelation

Wulfram Gerstner

EPFL, Lausanne, Switzerland

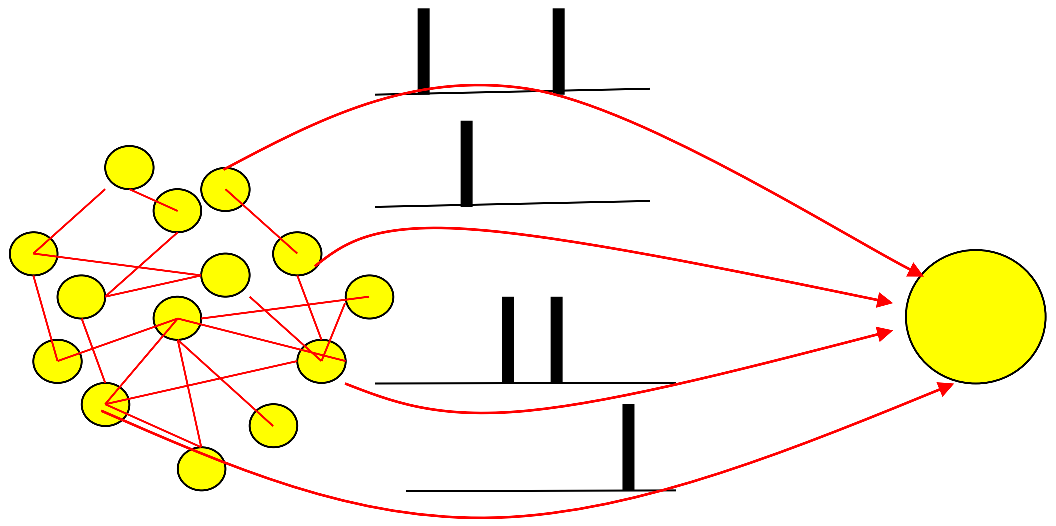
8.1 Variation of membrane potential
- white noise approximation

8.2 Autocorrelation of Poisson

8.3 Noisy integrate-and-fire
- superthreshold and subthreshold

8.4 Escape noise
-renewal model

Justify autocorrelation of spike input:
Poisson process in discrete time



Stochastic spike arrival:

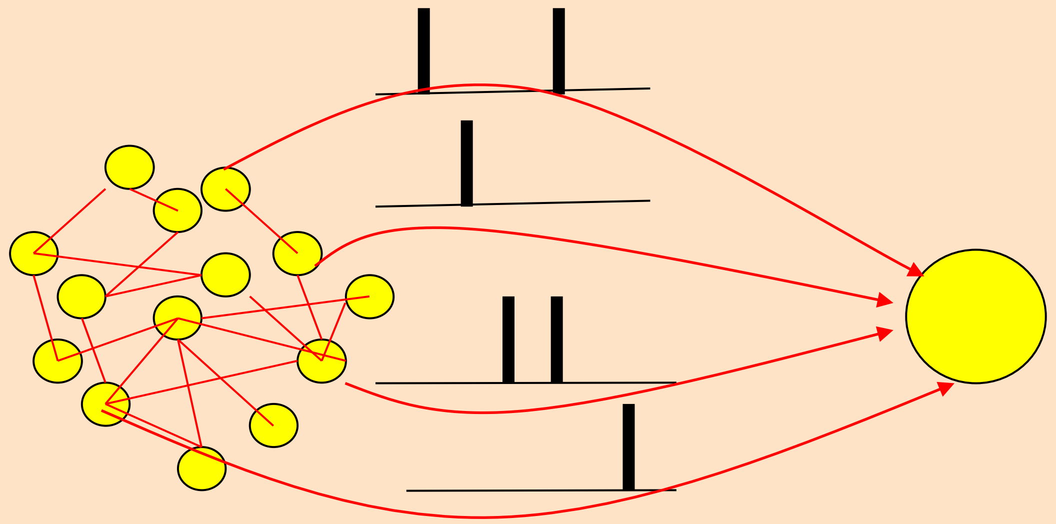
Blackboard

In each small time step Δt

Prob. Of firing $p = \nu \Delta t$

Firing independent between one time step and the next

Exercise 2 now: Poisson process in discrete time



Stochastic spike arrival:
excitation, total rate

Next lecture: 10:46

In each small time step Δt

Prob. Of firing $p = \nu \Delta t$

Firing independent between one time step and the next

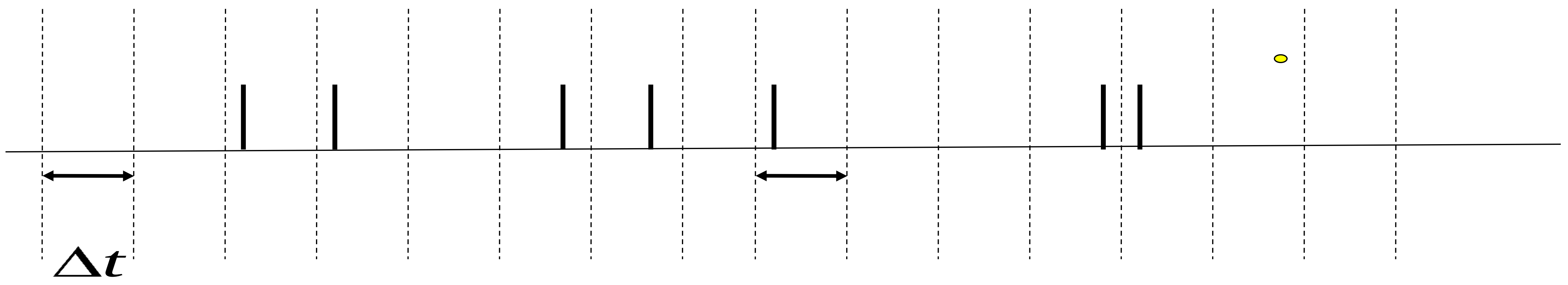
Show that autocorrelation for $\Delta t \rightarrow 0$ $\langle S(t) S(t') \rangle = \nu \delta(t - t') + \nu^2$

Show that in an a long interval of duration T, the expected number of spikes is $\langle N(T) \rangle = \nu T$

Neuronal Dynamics – 8.2. Autocorrelation of Poisson

math detour now!

Probability of spike
in step *n* **AND** step *k*



spike train

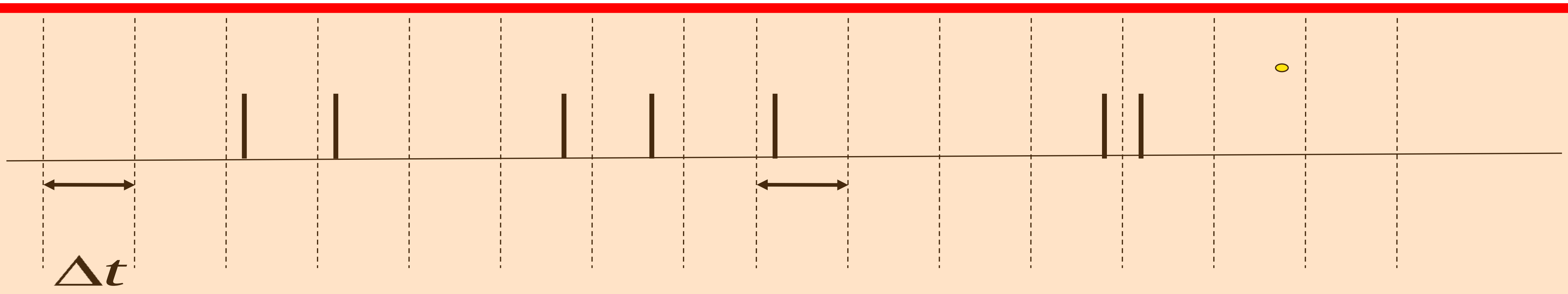
Probability of spike in time step:

$$P_F = \nu_0 \, \Delta t$$

Autocorrelation (continuous time)

$$\langle S(t)S(t')\rangle = \nu_0 \, \delta(t-t') + [\nu_0]^2$$

Quiz – 8.1. Autocorrelation of Poisson



The Autocorrelation (continuous time)

spike train

$$\langle S(t)S(t')\rangle$$

Has units

[ ] probability (unit-free)
[ ] probability squared (unit-free)
[ ] rate (1 over time)
[ ] (1 over time)-squred

Week 8 – part 3 : Noisy Integrate-and-fire

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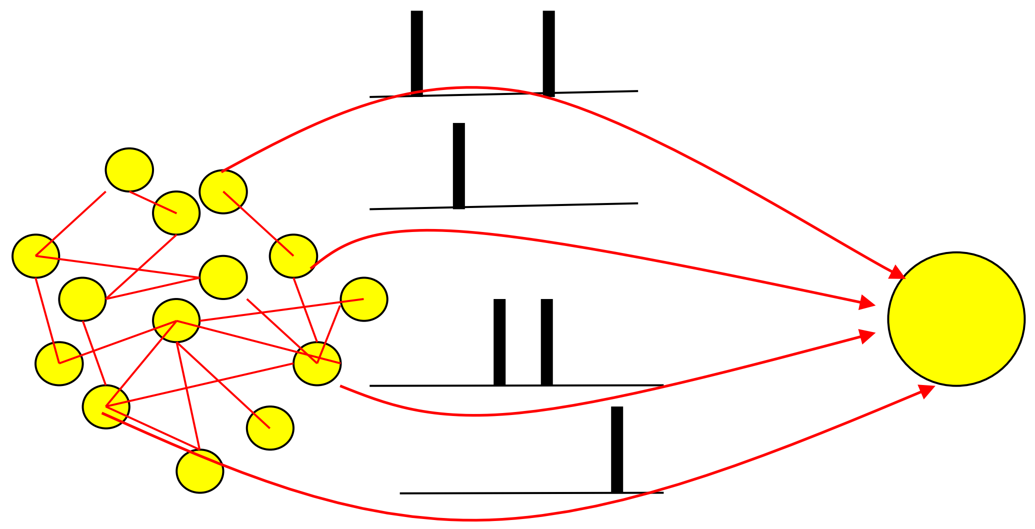
Biological Modeling of Neural Networks

Week 8 – Variability and Noise: Autocorrelation

Wulfram Gerstner

EPFL, Lausanne, Switzerland



for a passive membrane, we can analytically predict the mean of membrane potential fluctuations

Passive membrane

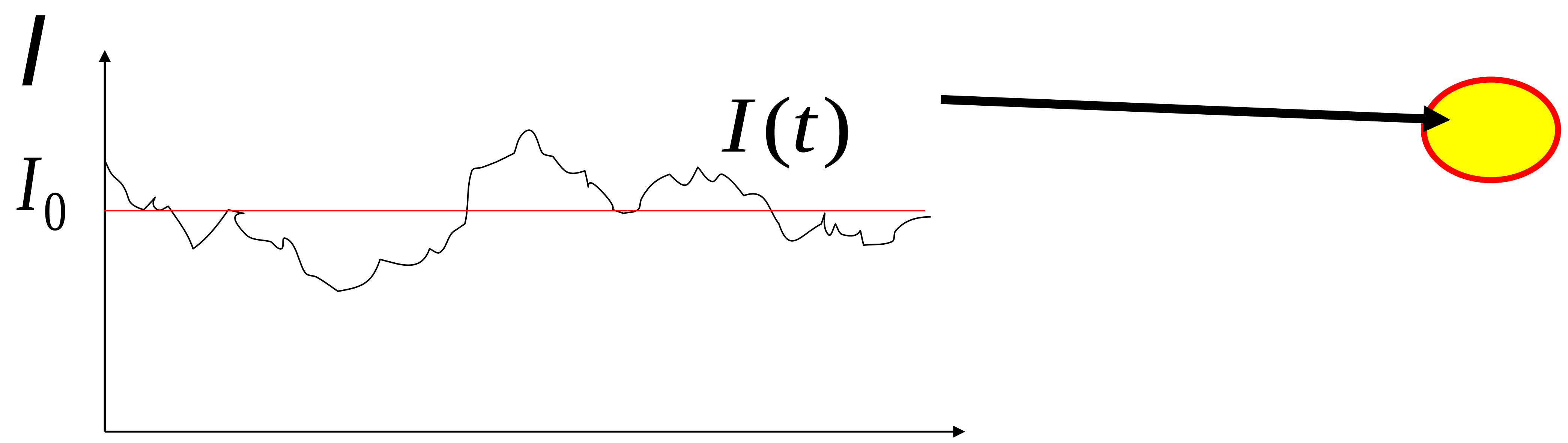
$$\tau \frac{d}{dt} u = -(u - u_{rest}) \quad + R I^{syn}(t)$$

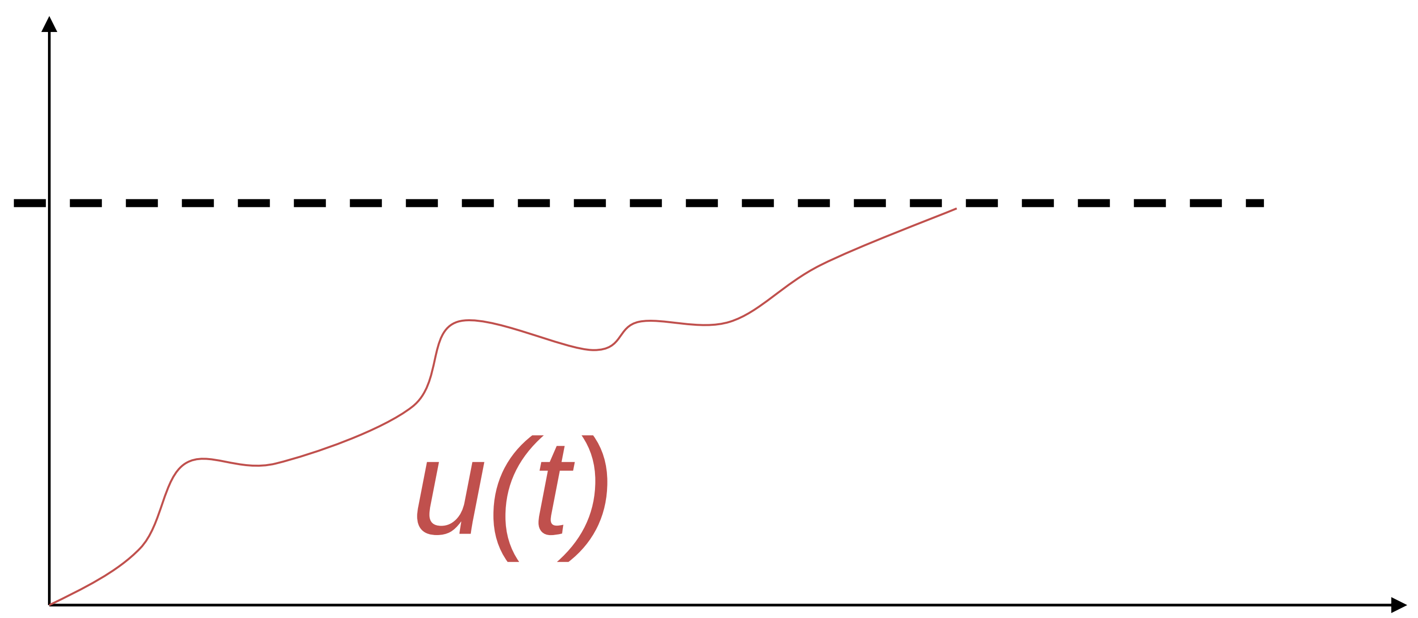
Passive membrane =Leaky integrate-and-fire without threshold

ADD THRESHOLD
→ Leaky Integrate-and-Fire

Neuronal Dynamics – 8.3 Noisy Integrate-and-fire

effective noise current





LIF

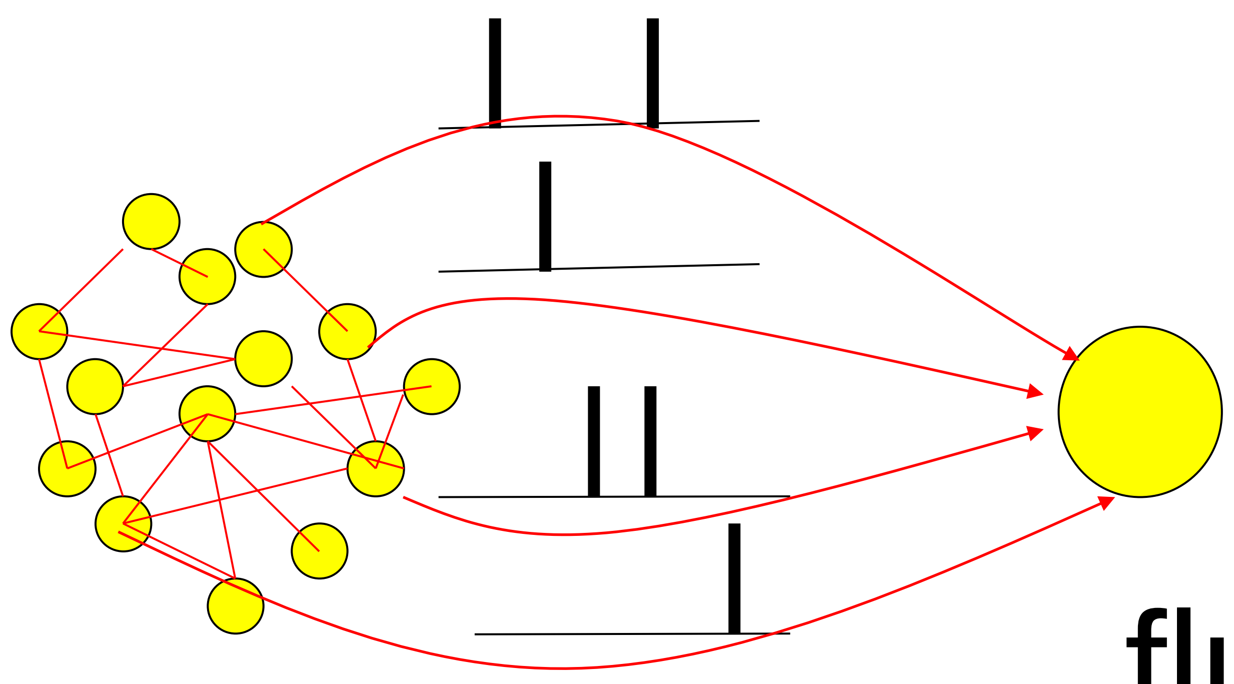
$$\tau \frac{d}{dt} u = -(u - u_{rest}) \quad + R\, I(t)$$

$$I(t) = I_o + I_{noise}$$

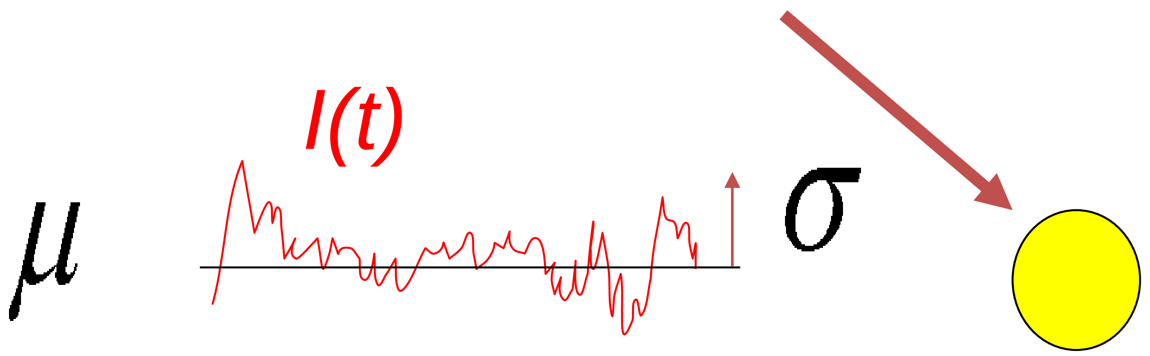
$$IF \;\; u(t) = \vartheta \;\; THEN \;\; u(t+\Delta) = u_r$$

noisy input/
diffusive noise/
stochastic spike
arrival

Neuronal Dynamics – 8.3 Noisy Integrate-and-fire



fluctuating input current

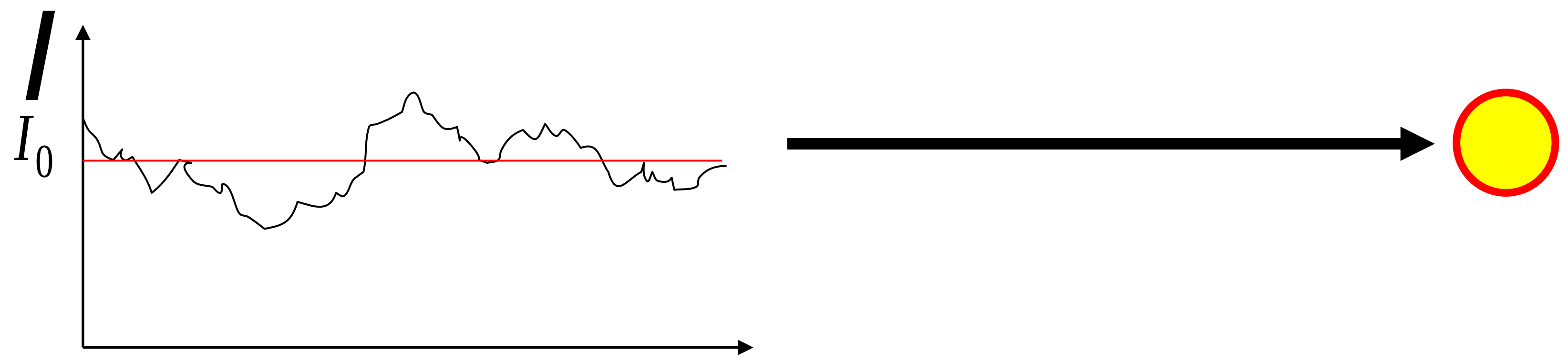


Random spike arrival

fluctuating potential

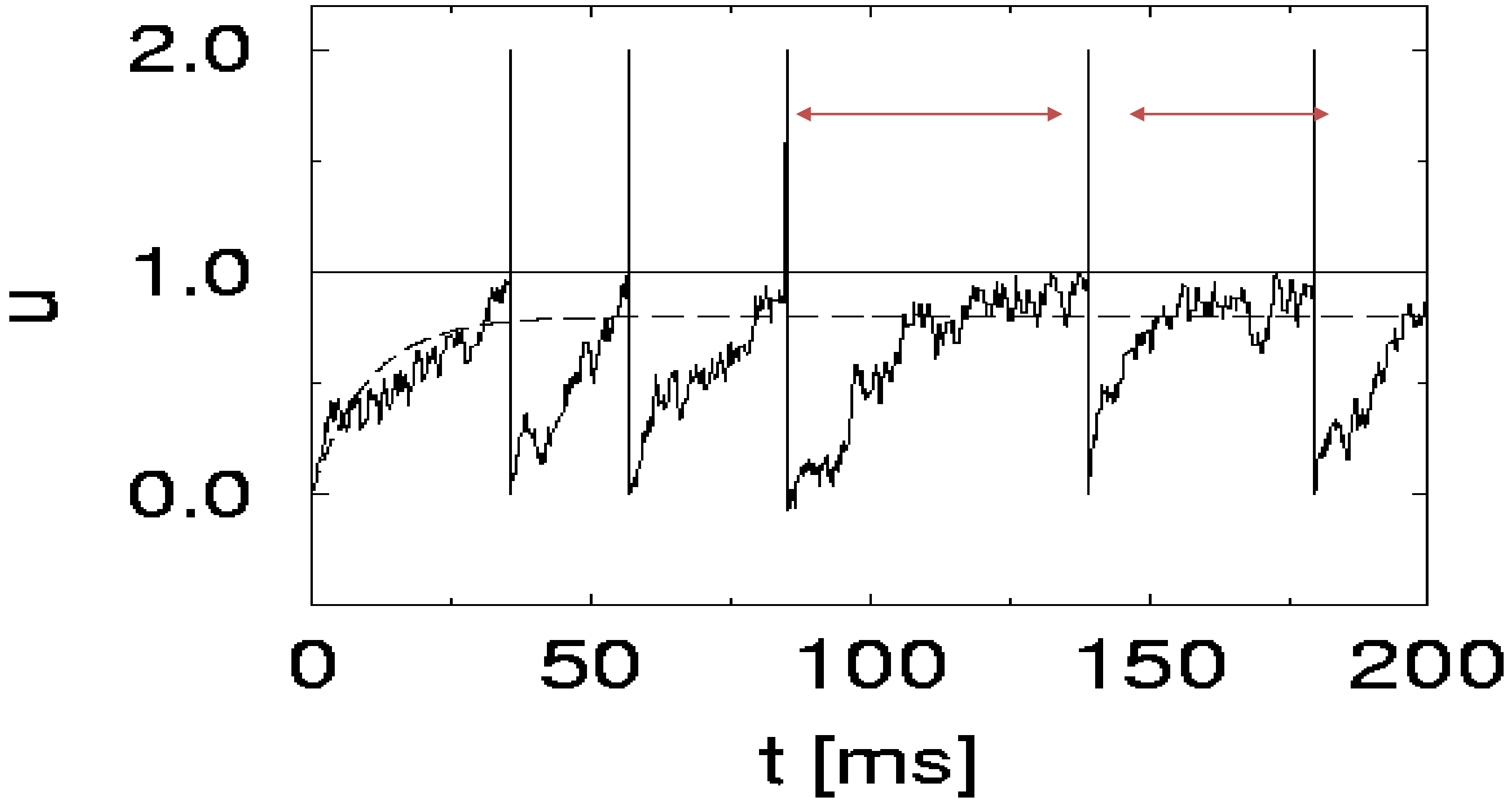
Neuronal Dynamics – 8.3 Noisy Integrate-and-fire

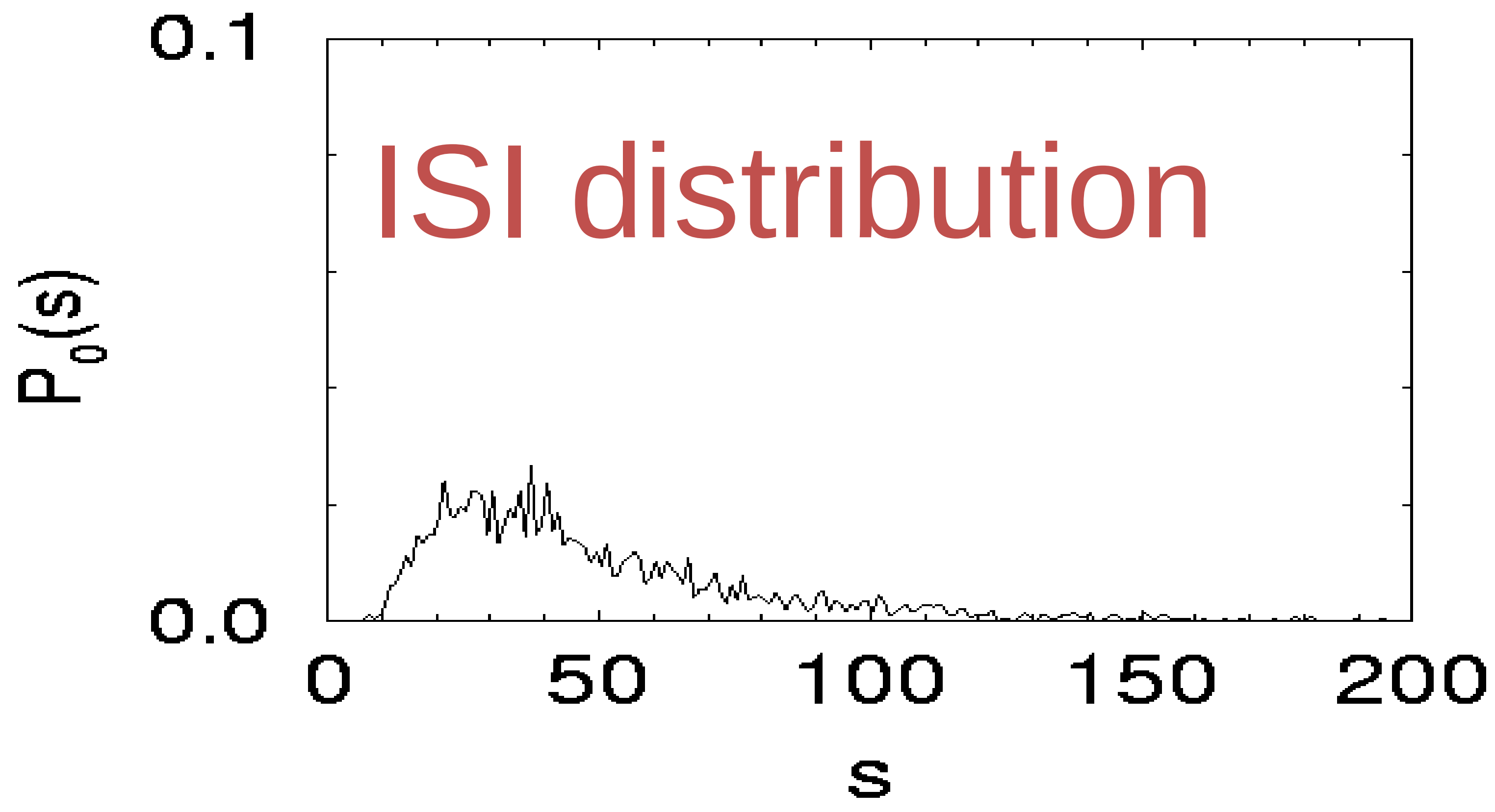
stochastic spike arrival in I&F – interspike intervals



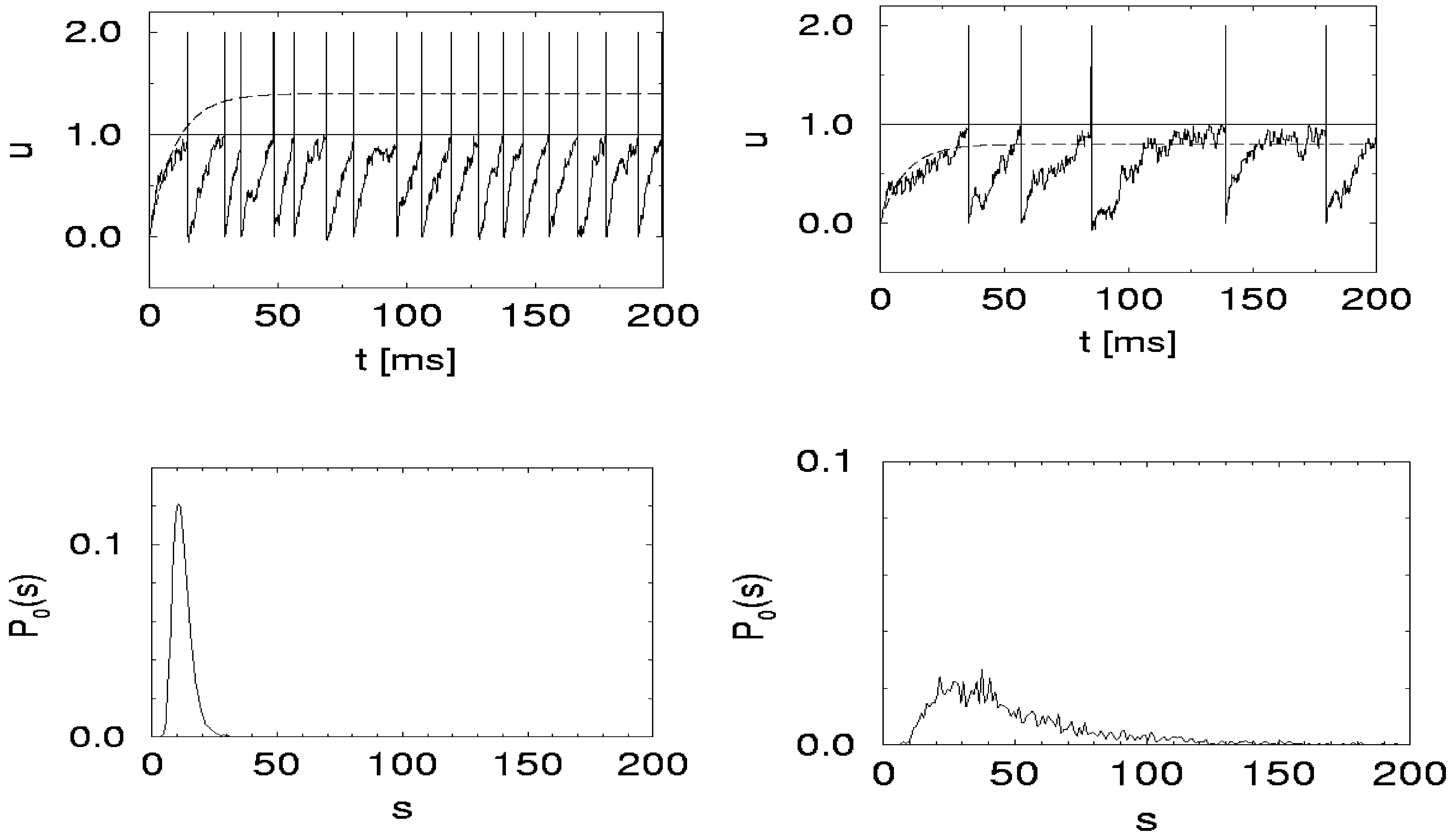
$$RI^{syn}(t) = RI_0(t) + \xi(t)$$

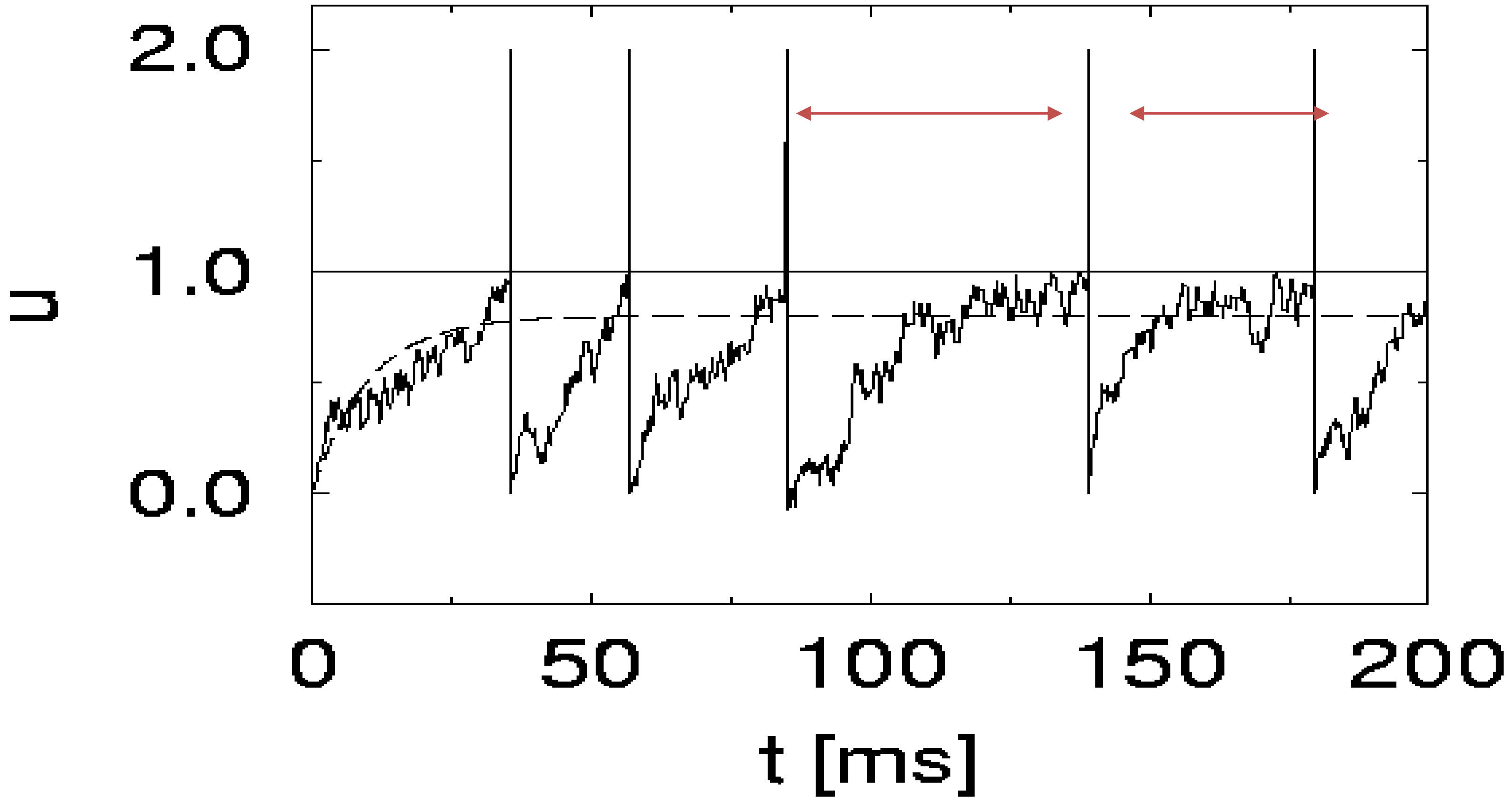
white noise

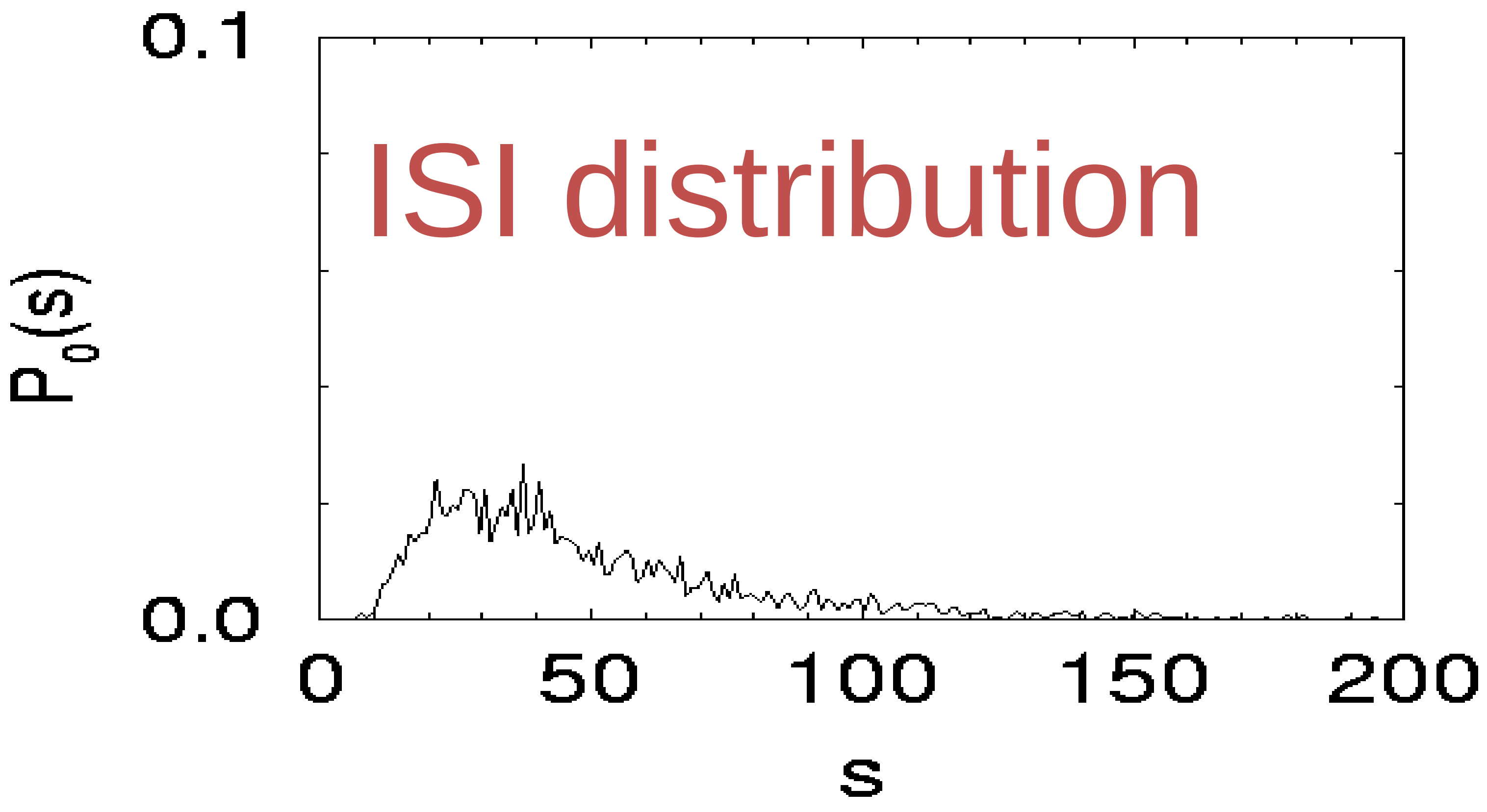




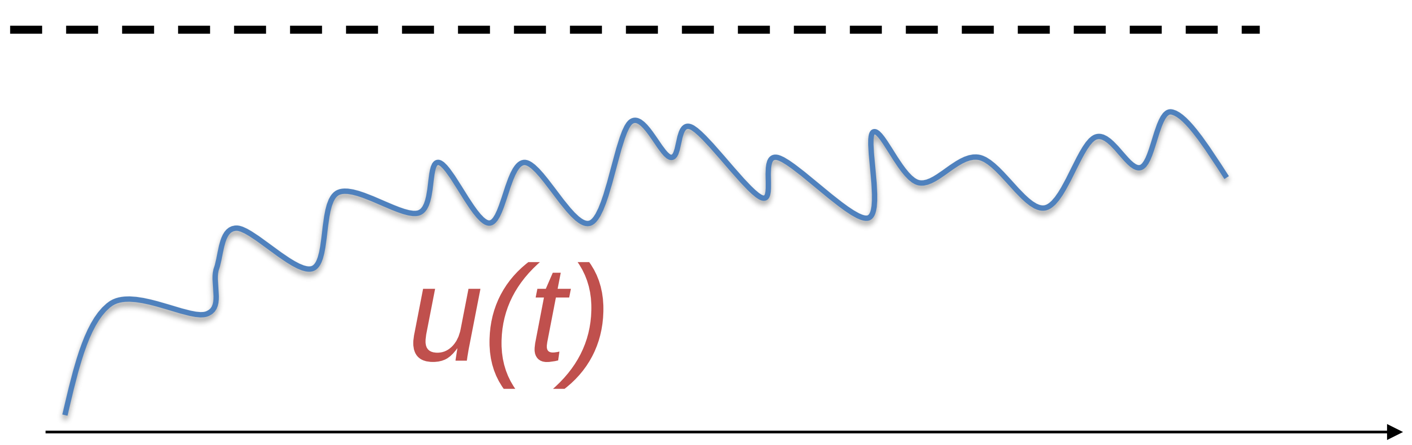
Superthreshold vs. Subthreshold regime







noisy input/ diffusive noise/ stochastic spike arrival



subthreshold regime:

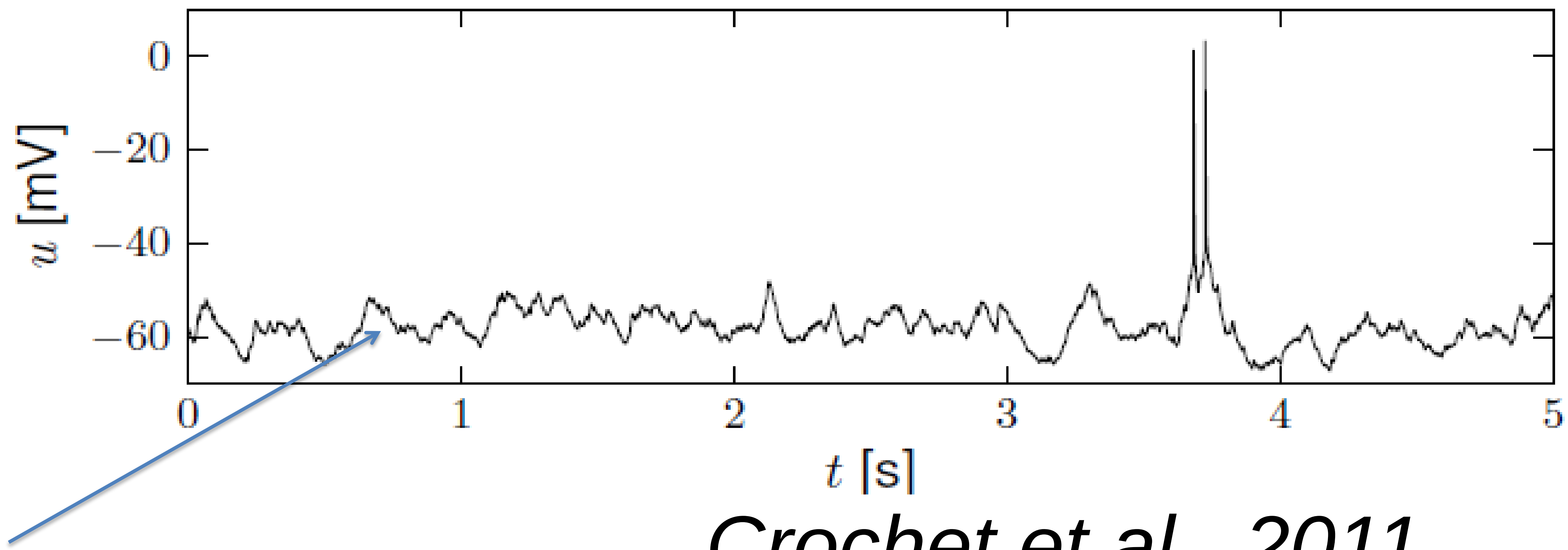
- firing driven by fluctuations
- **broad ISI distribution**
- *in vivo* like

Neuronal Dynamics – review- Variability in vivo

Spontaneous activity *in vivo*

Variability
of membrane potential?

awake mouse, freely whisking,

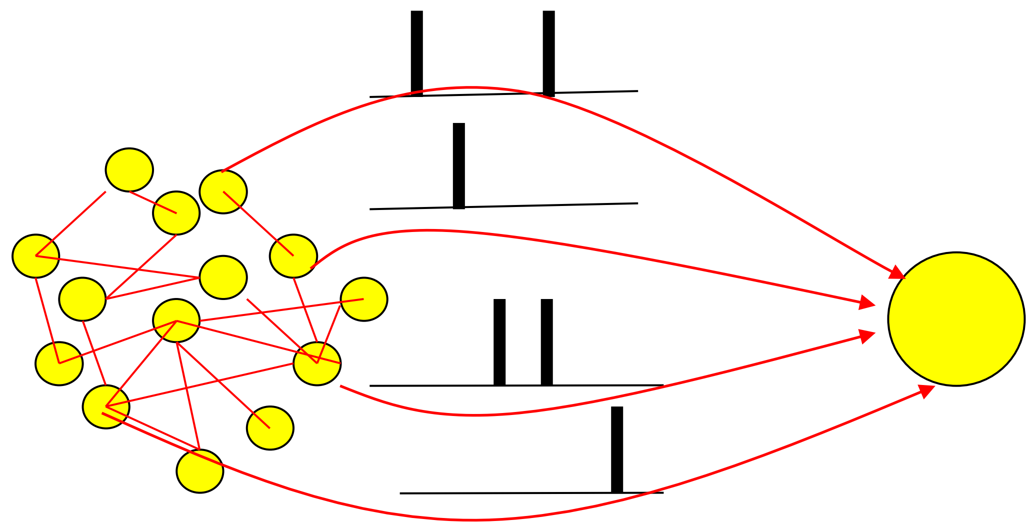


Crochet et al., 2011

Subthreshold regime

Stochastic spike arrival:

for a passive membrane, we can analytically predict the amplitude of membrane potential fluctuations



Passive membrane

$$u(t) = \sum_k w_k \sum_f \varepsilon(t' - t_k^f)$$

$$= \sum_k w_k \int dt' \varepsilon(t - t') S_k(t')$$

fluctuating potential

$$\langle \Delta u(t) \Delta u(t) \rangle = \langle [u(t)]^2 \rangle - \langle u(t) \rangle^2$$

Leaky integrate-and-fire in ***subthreshold*** *regime*
→ In vivo like

Week 8 – Noisy input models: barrage of spike arrivals

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Biological Modeling of Neural Networks

8.1 Variation of membrane potential
- white noise approximation

8.2 Autocorrelation of Poisson

8.3 Noisy integrate-and-fire
- superthreshold and subthreshold

THE END

Week 8 – part 4 : Escape noise

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Biological Modeling of Neural Networks

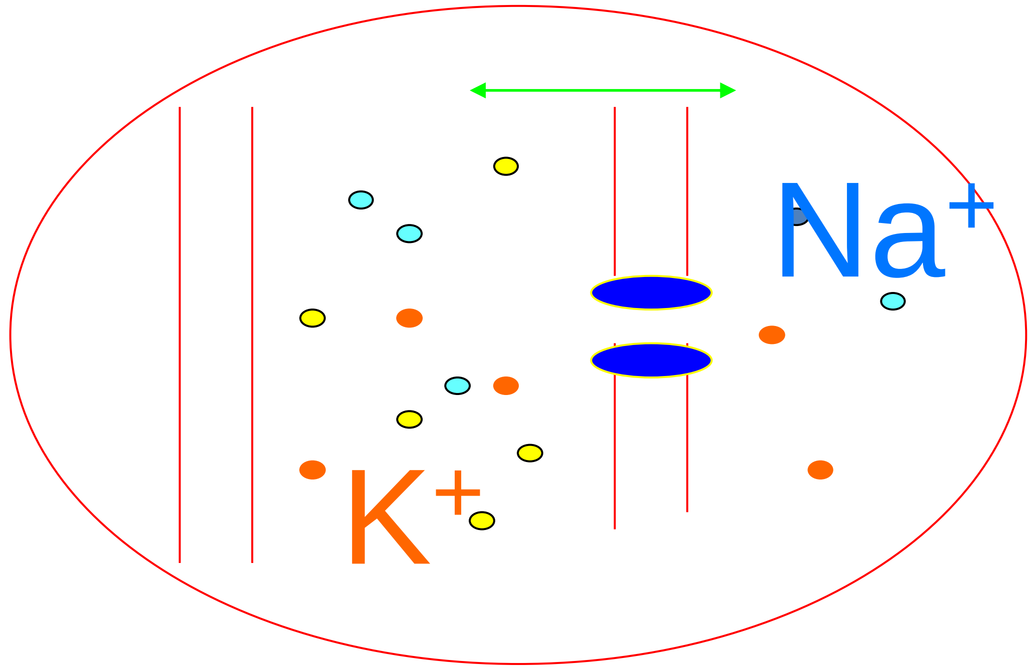
Week 8 – Noisy input models: Barrage of spike arrivals

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Neuronal Dynamics – Review: Sources of Variability

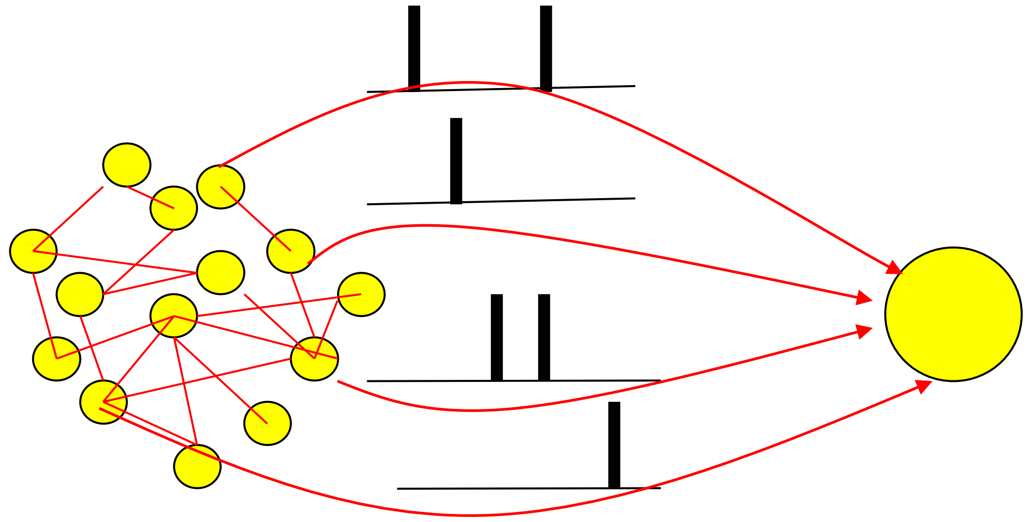
- Intrinsic noise (ion channels)



-Finite number of channels
-Finite temperature

small contribution!

-Network noise (background activity)



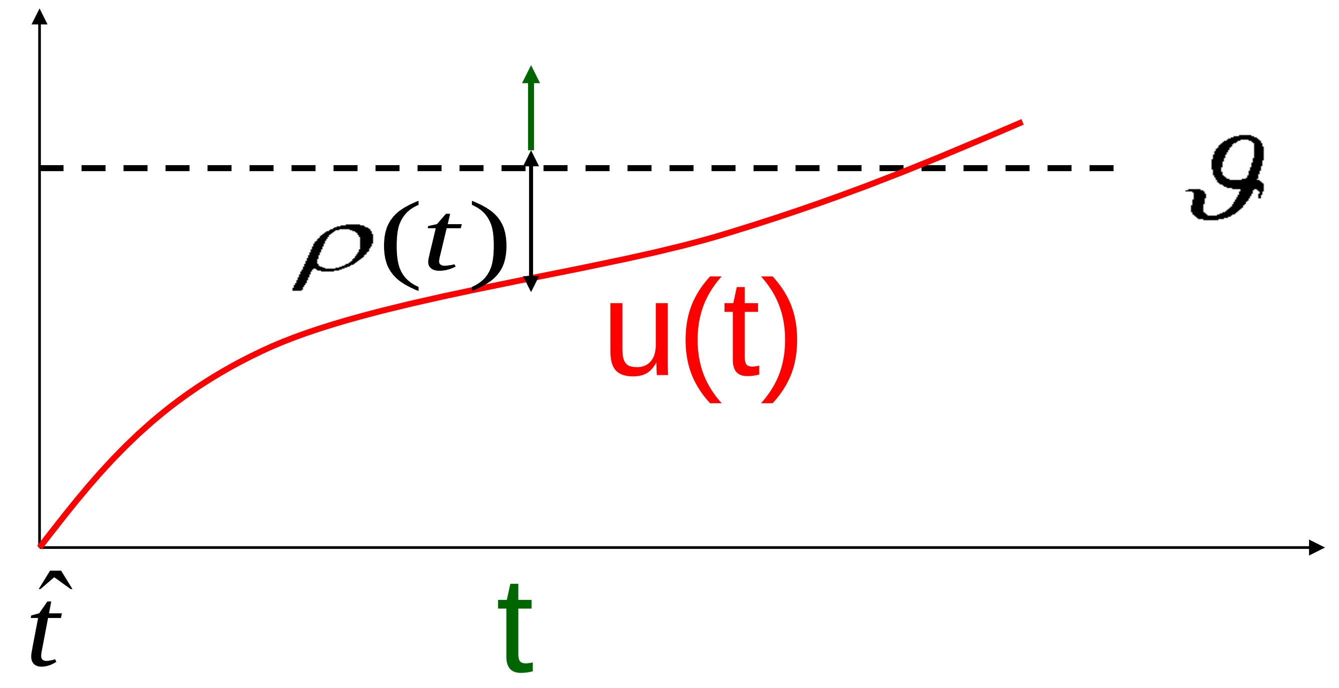
-Spike arrival from other neurons
-Beyond control of experimentalist

big contribution!

Noise models?

Noise models

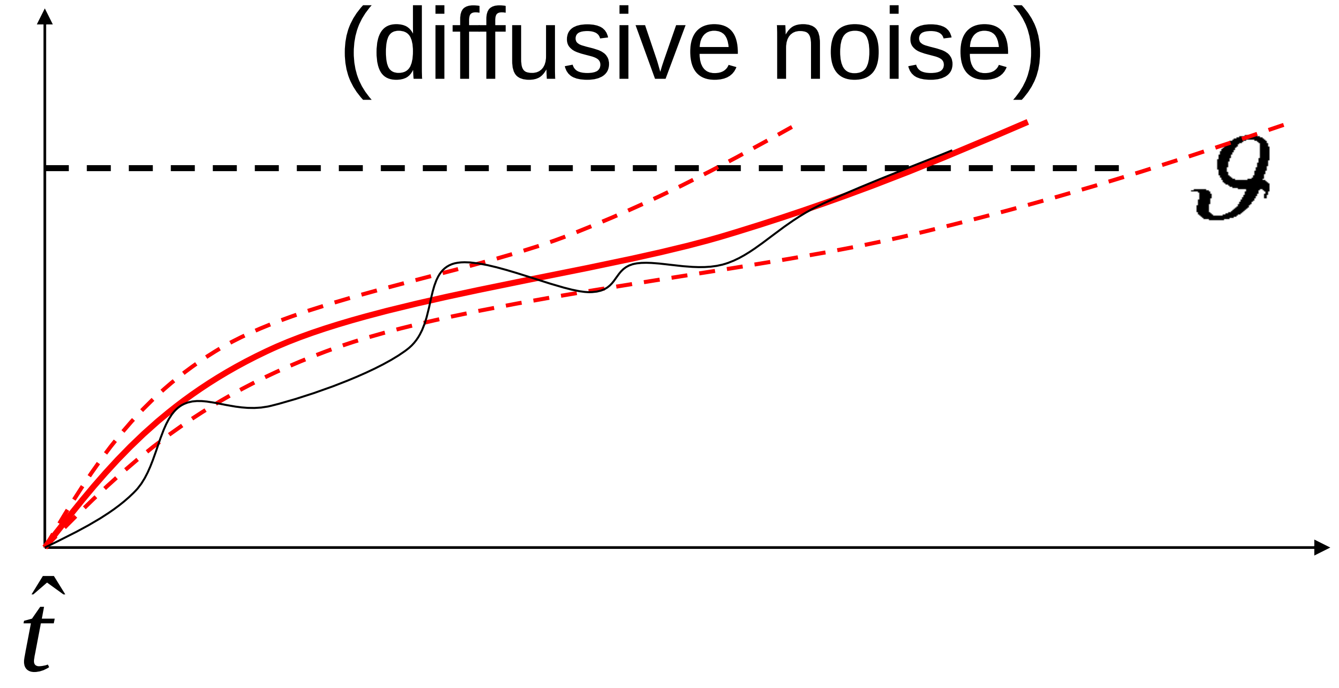
escape process,
stochastic intensity



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$

stochastic spike arrival
(diffusive noise)



noisy integration

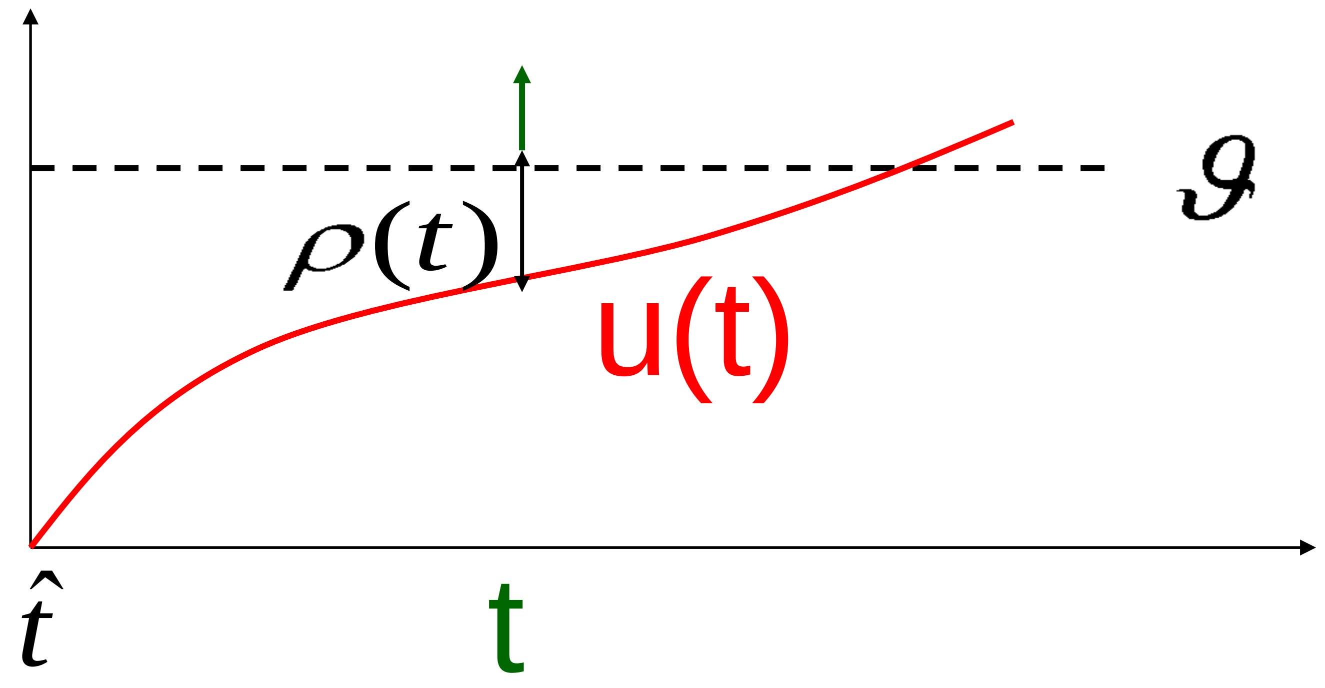
$$\tau \cdot \frac{du_i}{dt} = -u_i + RI + \xi(t)$$

Now:
Escape noise!

Relation between the two models:
later

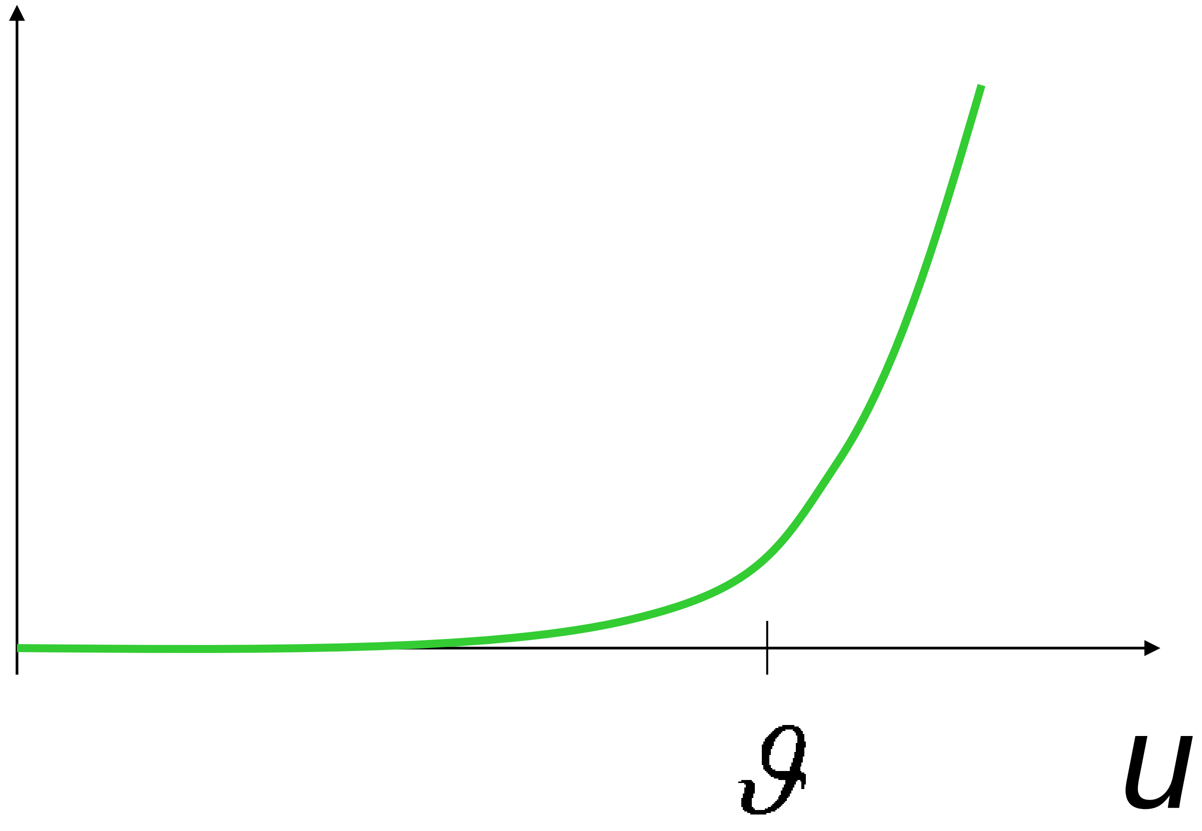
Neuronal Dynamics – 8.4 Escape noise

escape process



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$



escape rate

$$\rho(t) = \frac{1}{\Delta} \exp\left(\frac{u(t) - \vartheta}{\Delta}\right)$$

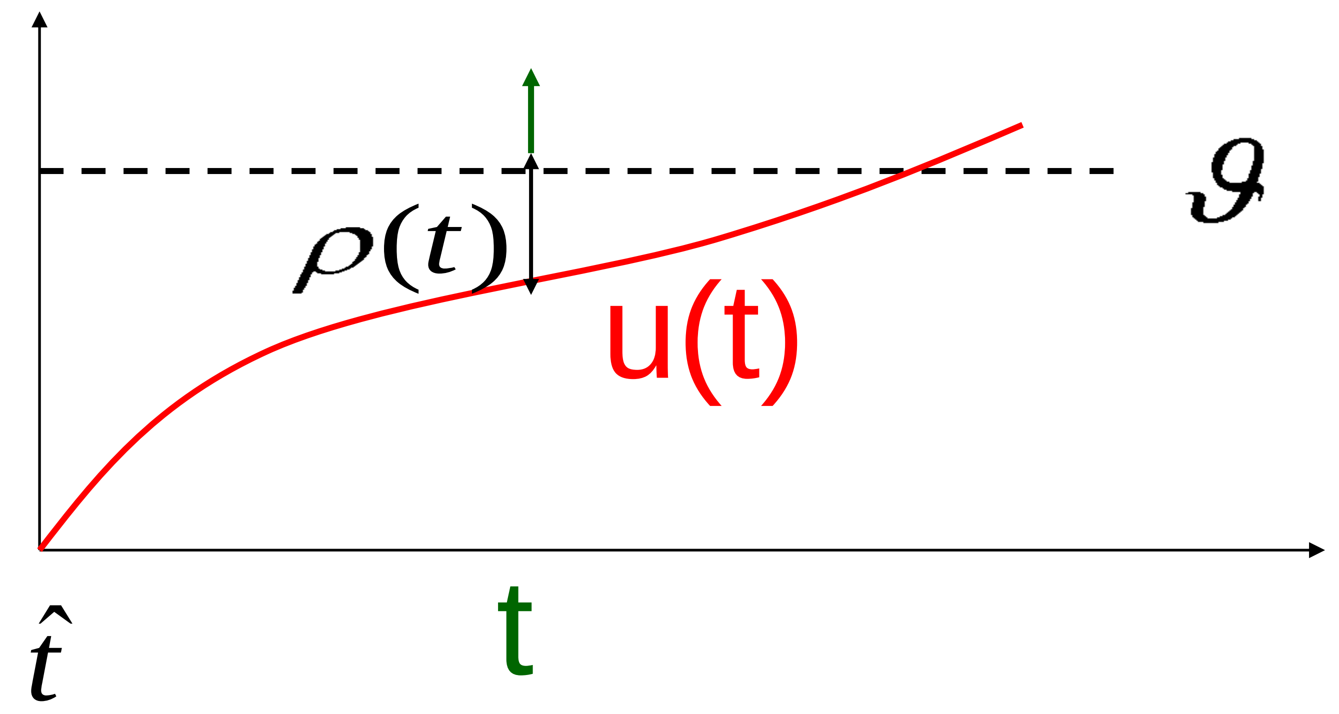
Example: leaky integrate-and-fire model

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

$$\text{if spike at } t^f \Rightarrow u(t^f + \delta) = u_r$$

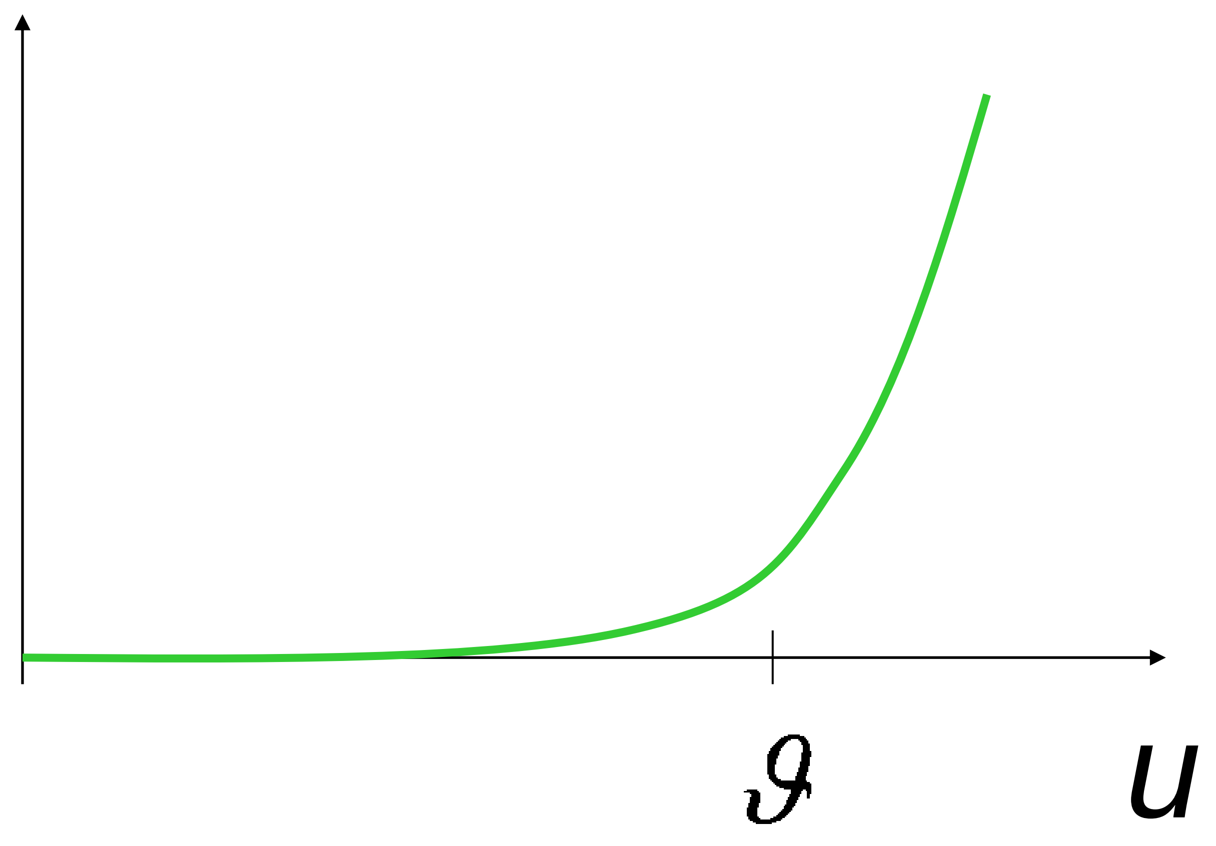
Neuronal Dynamics – 8.4 stochastic intensity

escape process



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$



Escape rate = stochastic intensity of point process

$$\rho(t) = f(u(t))$$

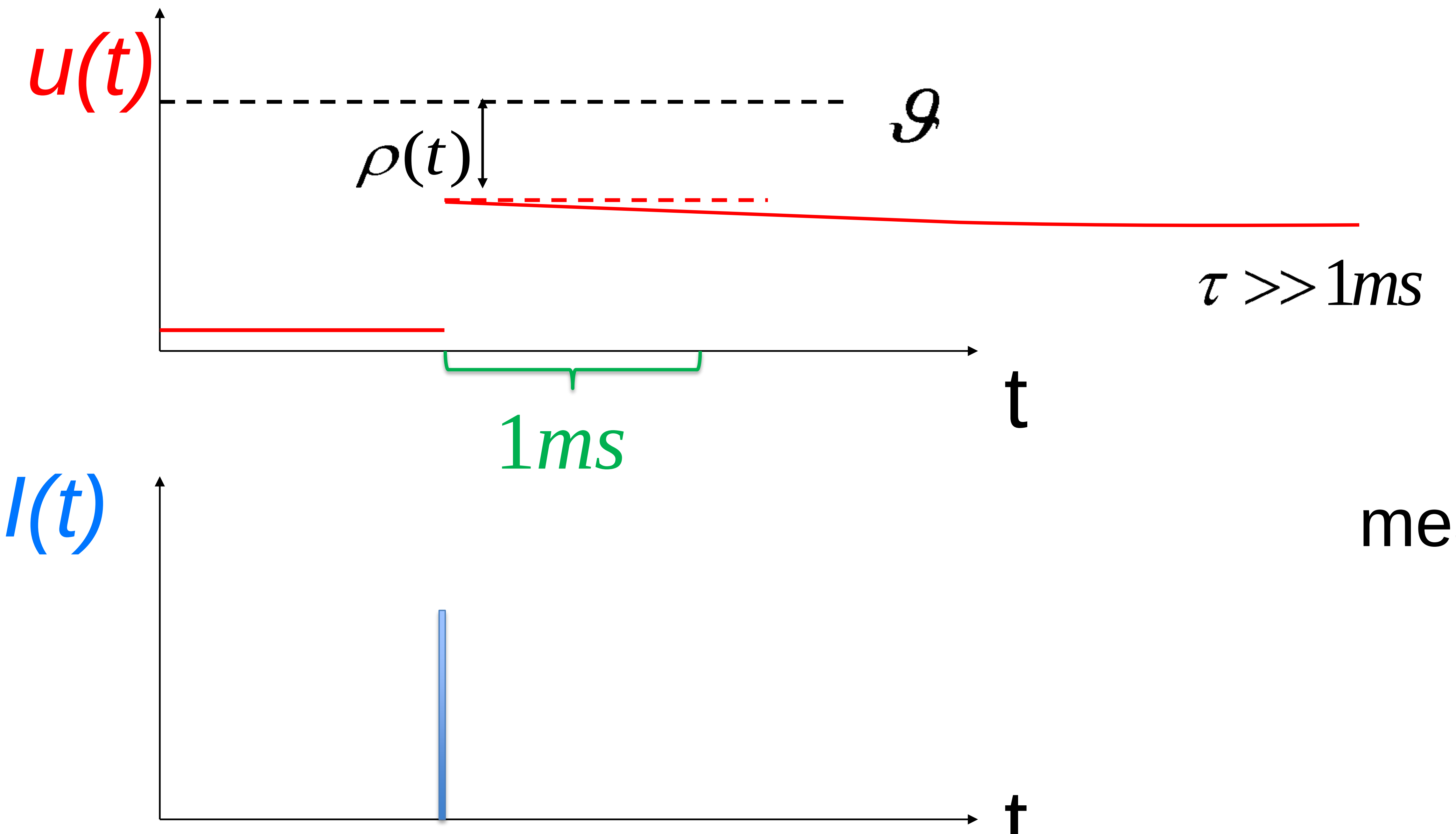
examples

$$\rho(t) = \frac{1}{\Delta} \exp\left(\frac{u(t) - \vartheta}{\Delta}\right)$$

$$\rho(t) =$$

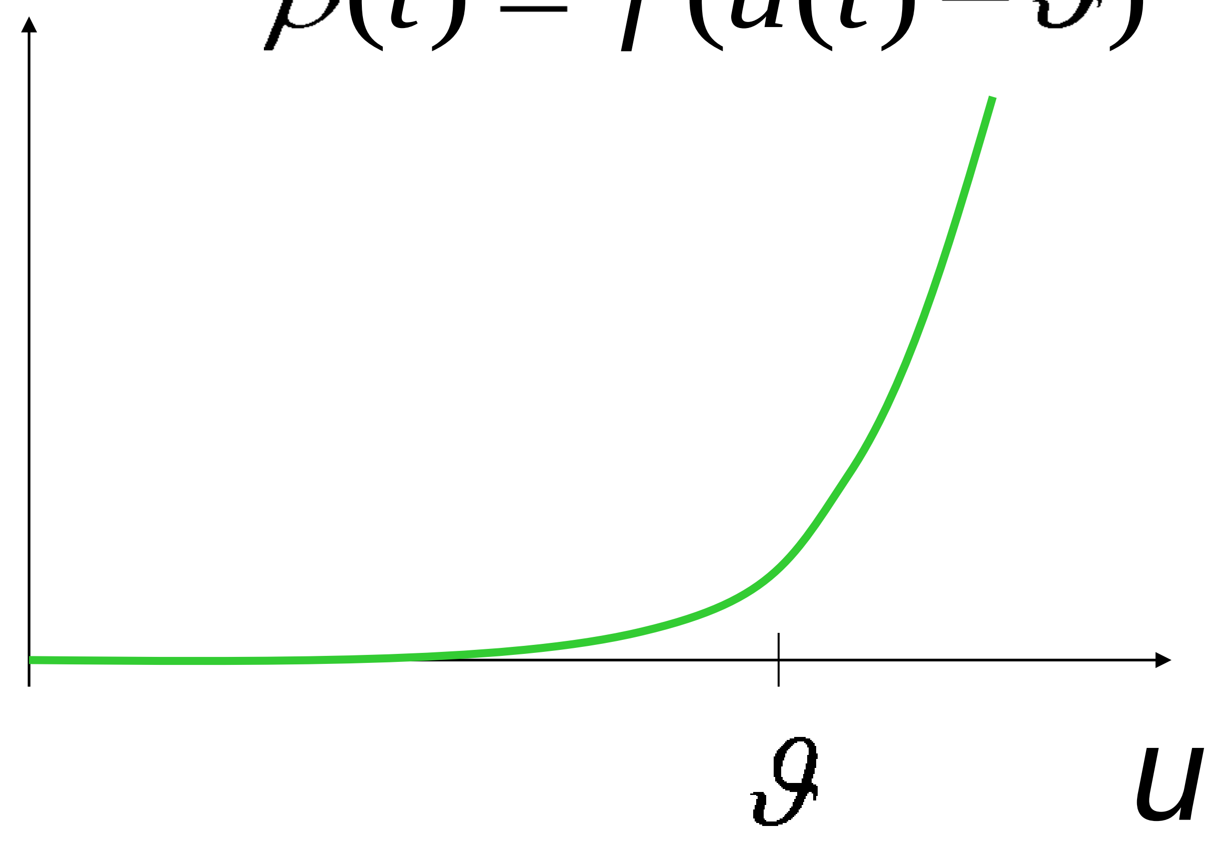
Neuronal Dynamics – 8.4 mean waiting time

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$

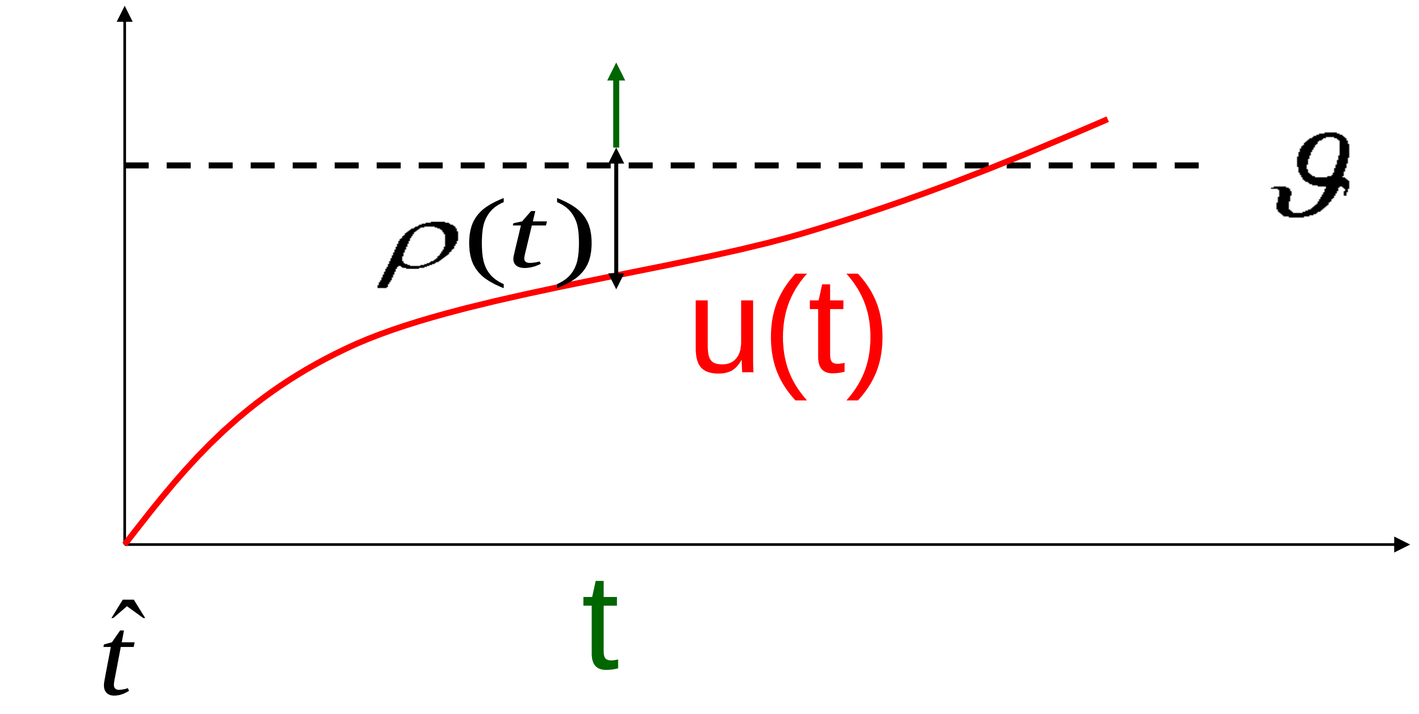


mean waiting time, after switch

Blackboard, Math detour

Escape rate = stochastic intensity of point process

$$\rho(t) = f(u(t))$$



Neuronal Dynamics – Quiz 8.4

Escape rate/stochastic intensity in neuron models

[ ] The escape rate of a neuron model has units one over time

[ ] The stochastic intensity of a point process has units one over time

[ ] For large voltages, the escape rate of a neuron model always saturates at some finite value

[ ] After a step in the membrane potential, the mean waiting time until a spike is fired is proportional to the escape rate

[ ] After a step in the membrane potential, the mean waiting time until a spike is fired is equal to the inverse of the escape rate

[ ] The stochastic intensity of a leaky integrate-and-fire model with reset only depends on the external input current but not on the time of the last reset

[ ] The stochastic intensity of a leaky integrate-and-fire model with reset depends on the external input current AND on the time of the last reset

Week 8 – part 5 : Renewal model

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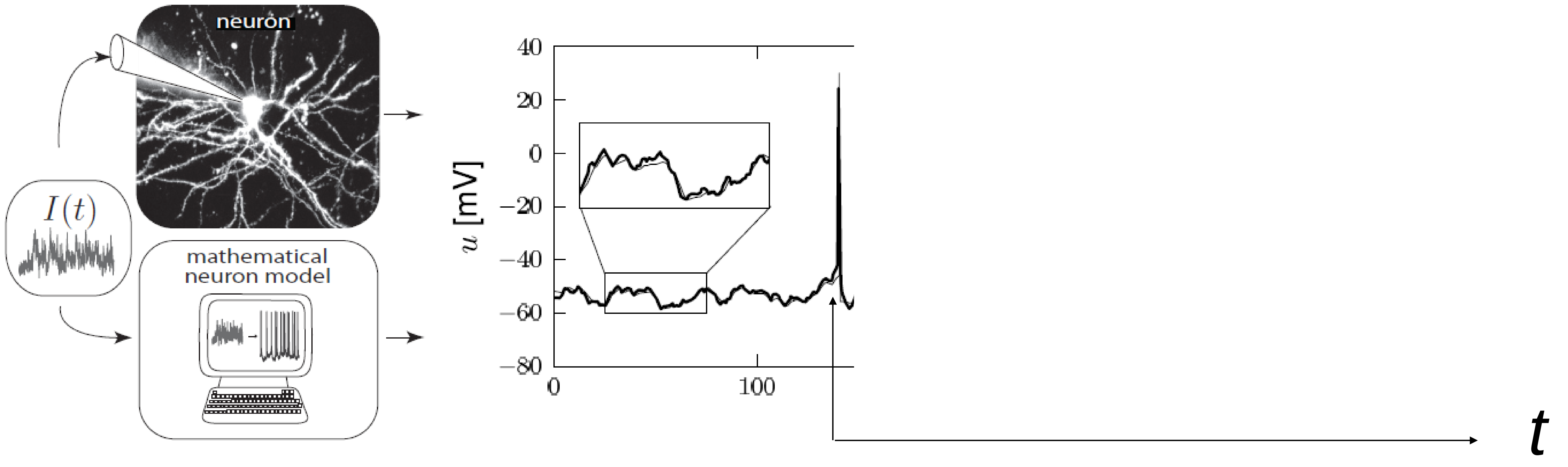
Biological Modeling of Neural Networks

Week 8 – Variability and Noise: Autocorrelation

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Neuronal Dynamics – 8.5. Interspike Intervals



deterministic part of input

$$I(t) \rightarrow u(t)$$

noisy part of input/intrinsic noise

→ *escape rate*

Example:
nonlinear integrate-and-fire model

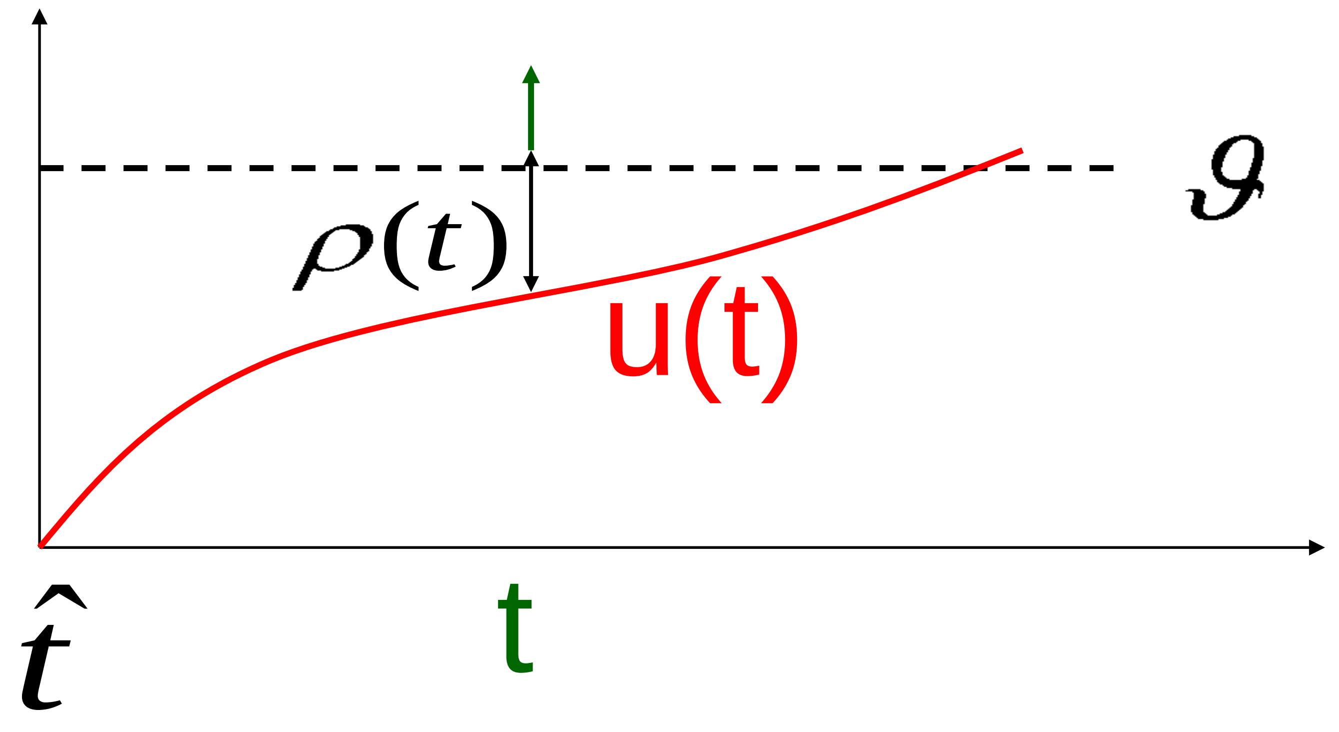
$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

$$\text{if spike at } t^f \Rightarrow u(t^f + \delta) = u_r$$

Example:
exponential stochastic intensity

$$\rho(t) = f(u(t)) = \rho_\vartheta \exp(u(t) - \vartheta)$$

escape process



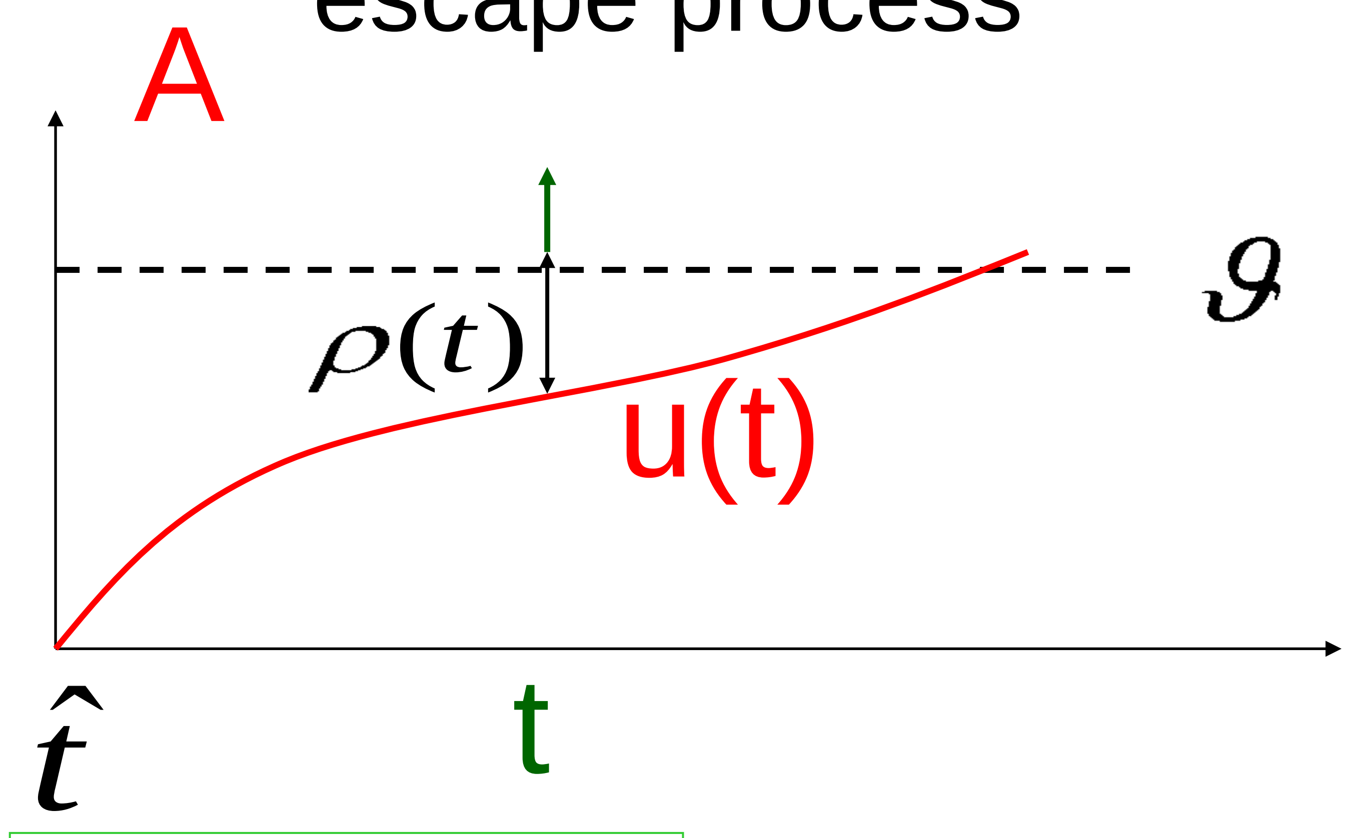
escape rate

$$\rho(t) = f(u(t) - \vartheta)$$

Survivor function

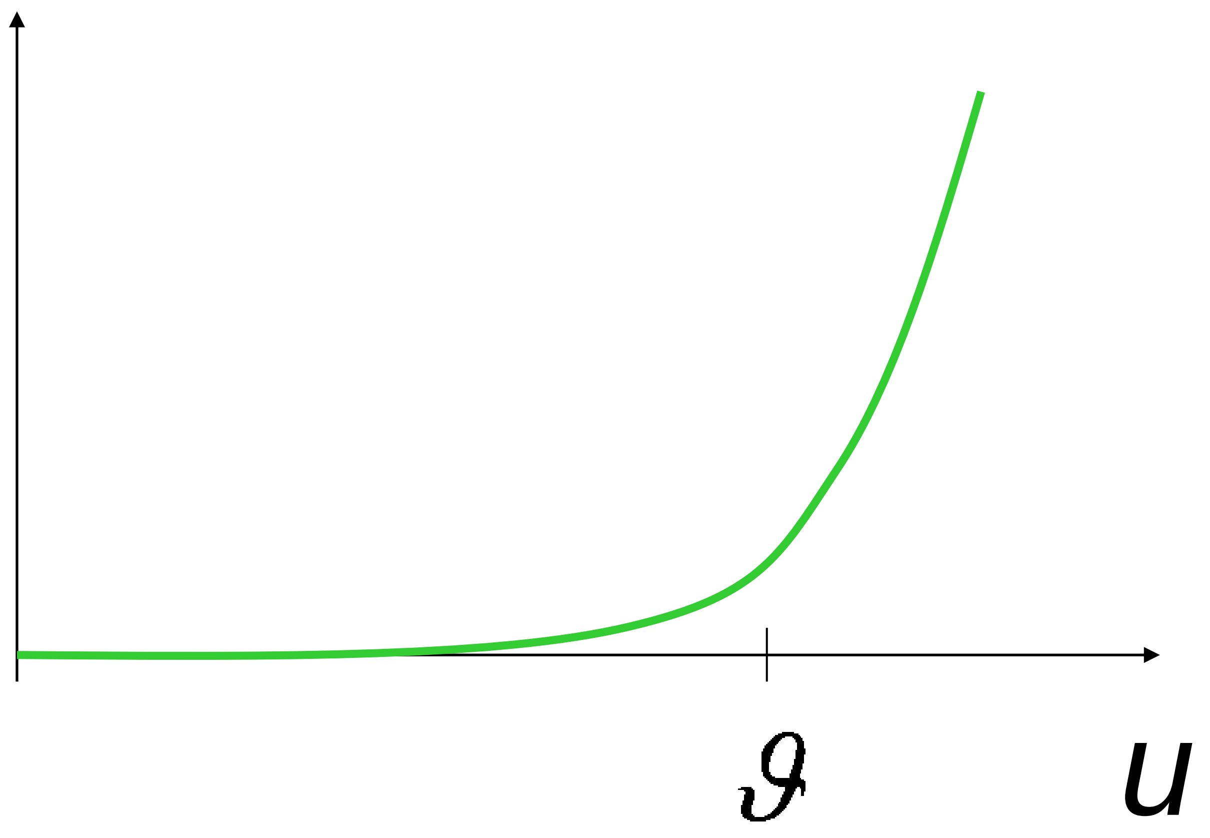
$$\frac{d}{dt} S_I(t|\hat{t}) = -\rho(t)\, S_I(t|\hat{t})$$

Neuronal Dynamics – 8.5. Interspike Intervals



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$



Survivor function

Examples now

$$\frac{d}{dt} S_I(t|\hat{t}) = -\rho(t)\, S_I(t|\hat{t})$$

$$S_I(t|\hat{t}) = \exp\left(-\int_{\hat{t}}^{t} \rho(t')\,dt'\right)$$

Interval distribution

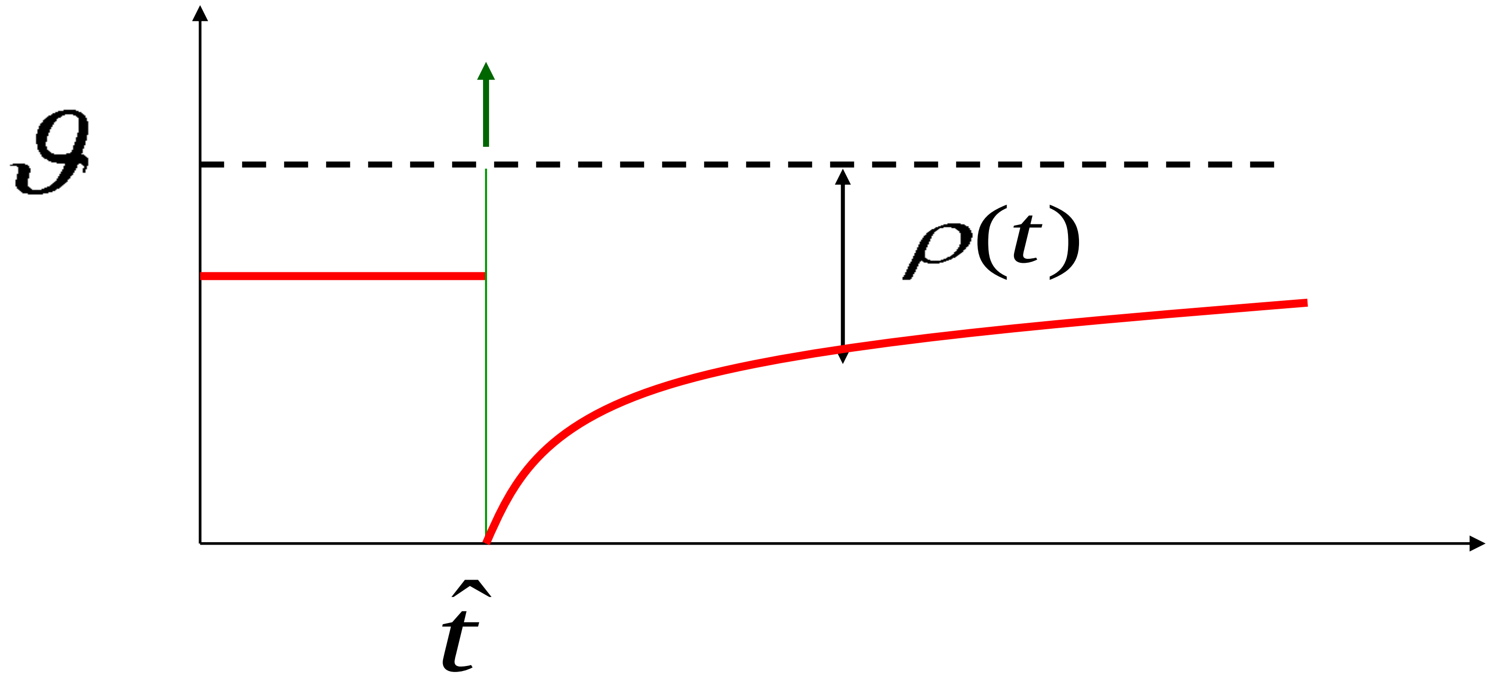
$$P_I(t|\hat{t}) = \rho(t) \cdot \exp\left(-\int_{\hat{t}}^{t} \rho(t')\,dt'\right)$$

escape rate

Survivor function

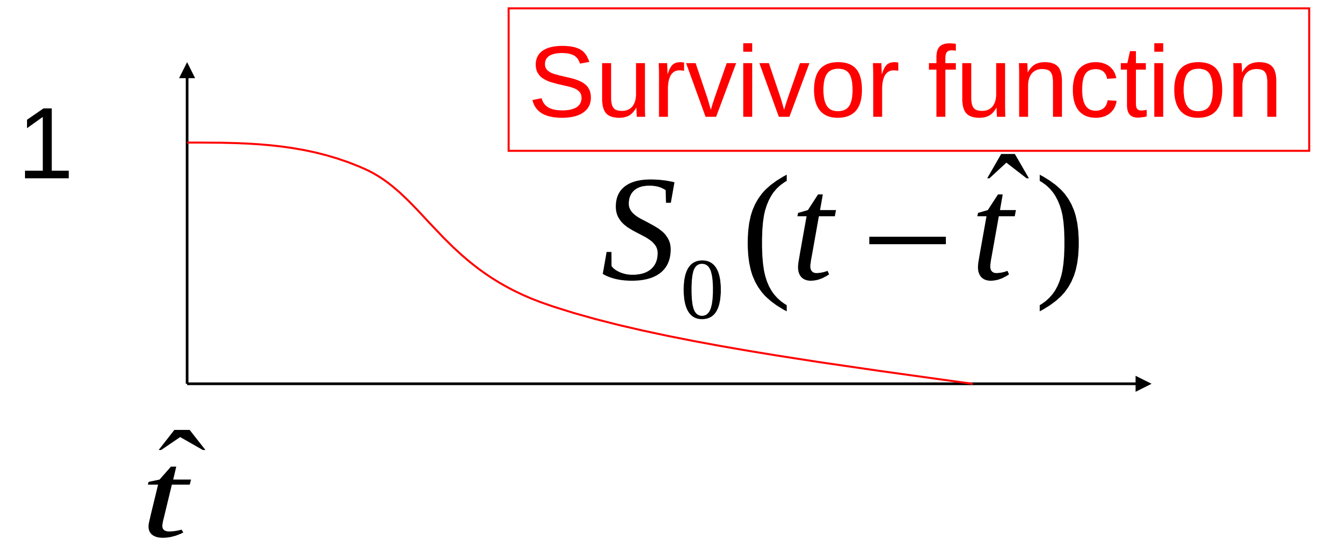
Neuronal Dynamics – 8.5. Renewal theory

Example: I&F with reset, **constant input**

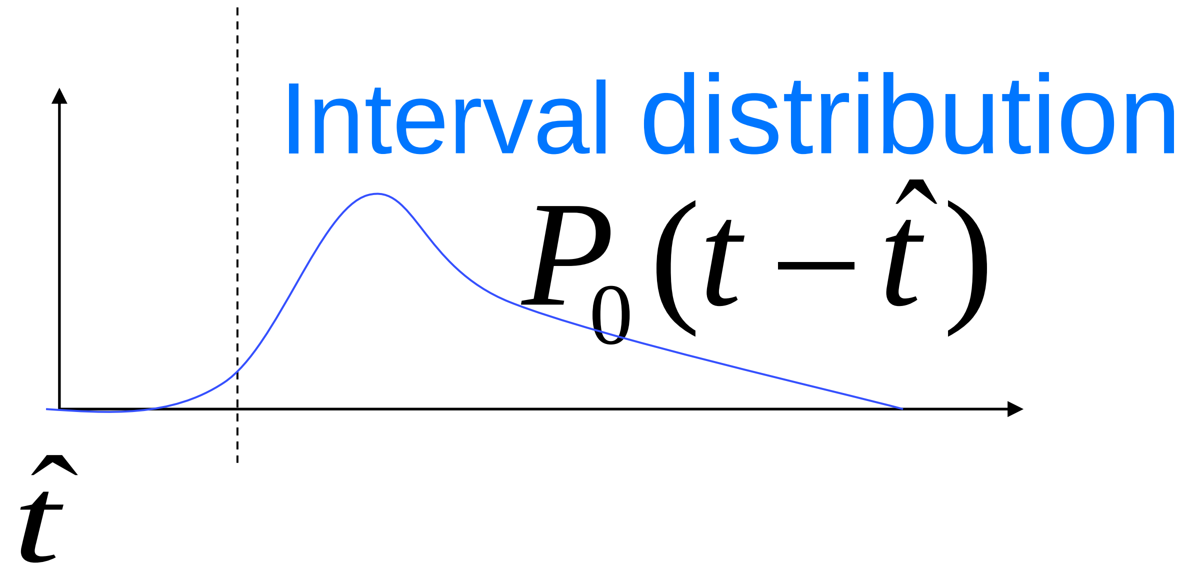


escape rate

$$\rho(t|\hat{t}) = f(u(t|\hat{t})) = \rho_\vartheta \exp(u(t|\hat{t}) - \vartheta)$$



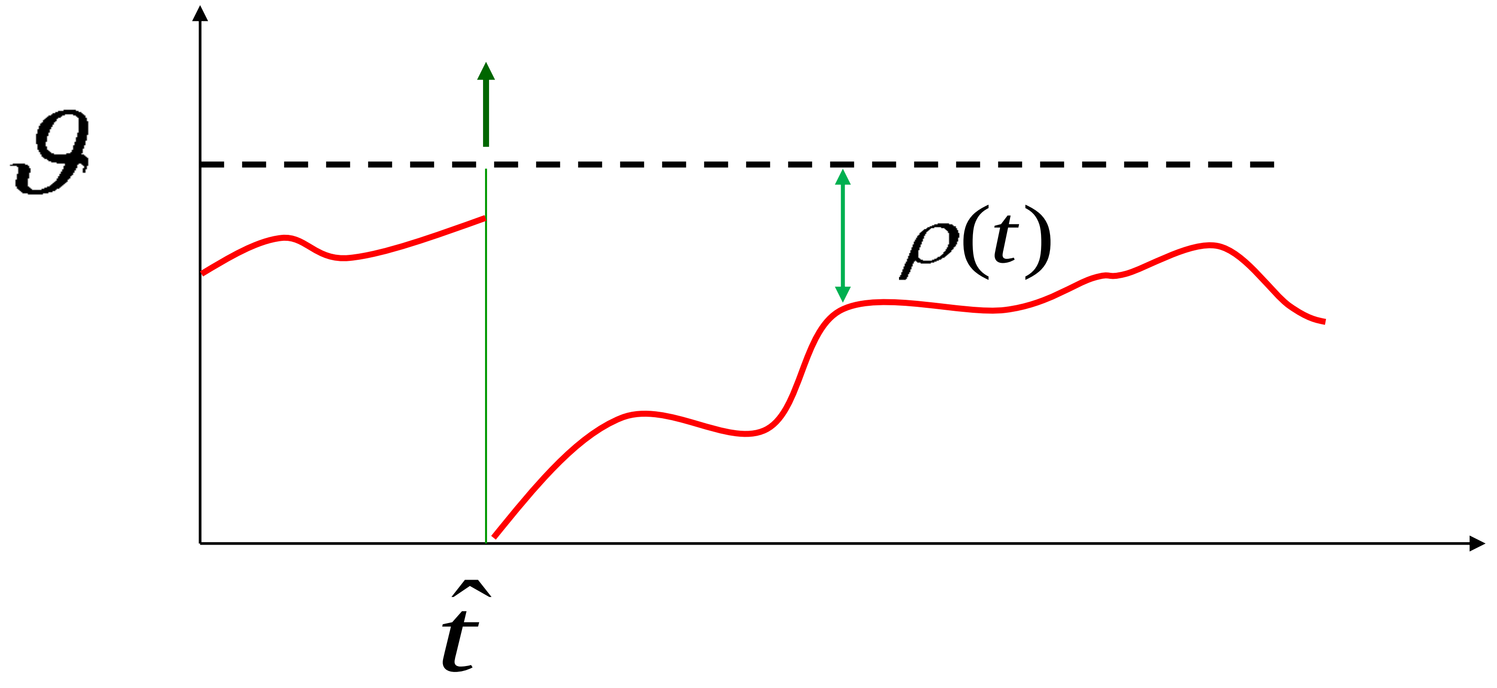
$$S(t|\hat{t}) = \exp(-\int_{\hat{t}}^{t} \rho(t'|\hat{t})dt')$$



$$P(t|\hat{t}) = \rho(t|\hat{t}) \exp(-\int_{\hat{t}}^{t} \rho(t'|\hat{t})dt')$$

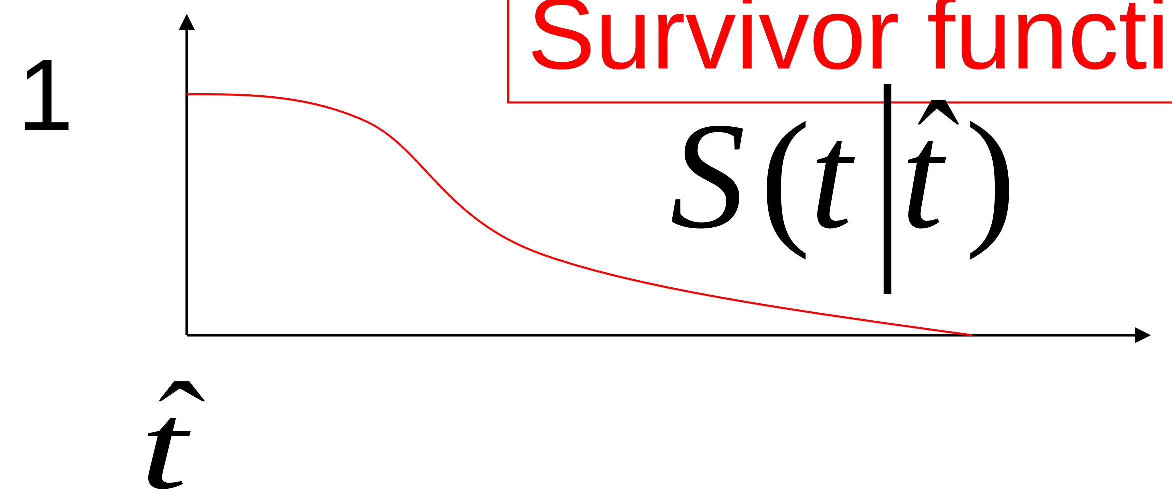
$$= -\tfrac{d}{dt} S(t|\hat{t})$$

Example: I&F with reset, **time-dependent input**,

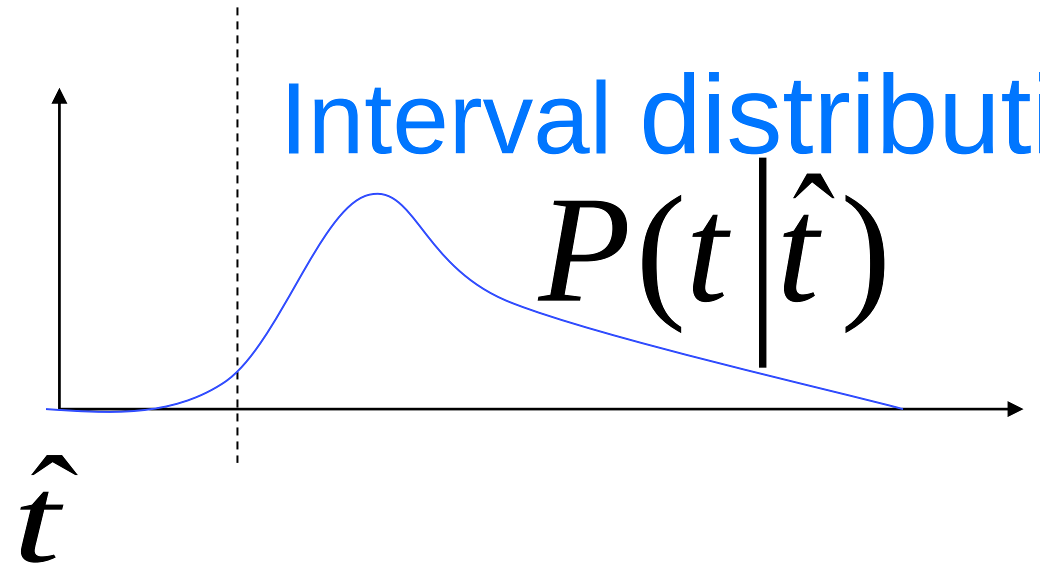


escape rate

$$\rho(t|\hat{t}) = f(u(t|\hat{t})) = \rho_{\vartheta} \exp(u(t|\hat{t}) - \vartheta)$$



$$S(t|\hat{t}) = \exp(-\int_{\hat{t}}^{t} \rho(t'|\hat{t}) dt')$$

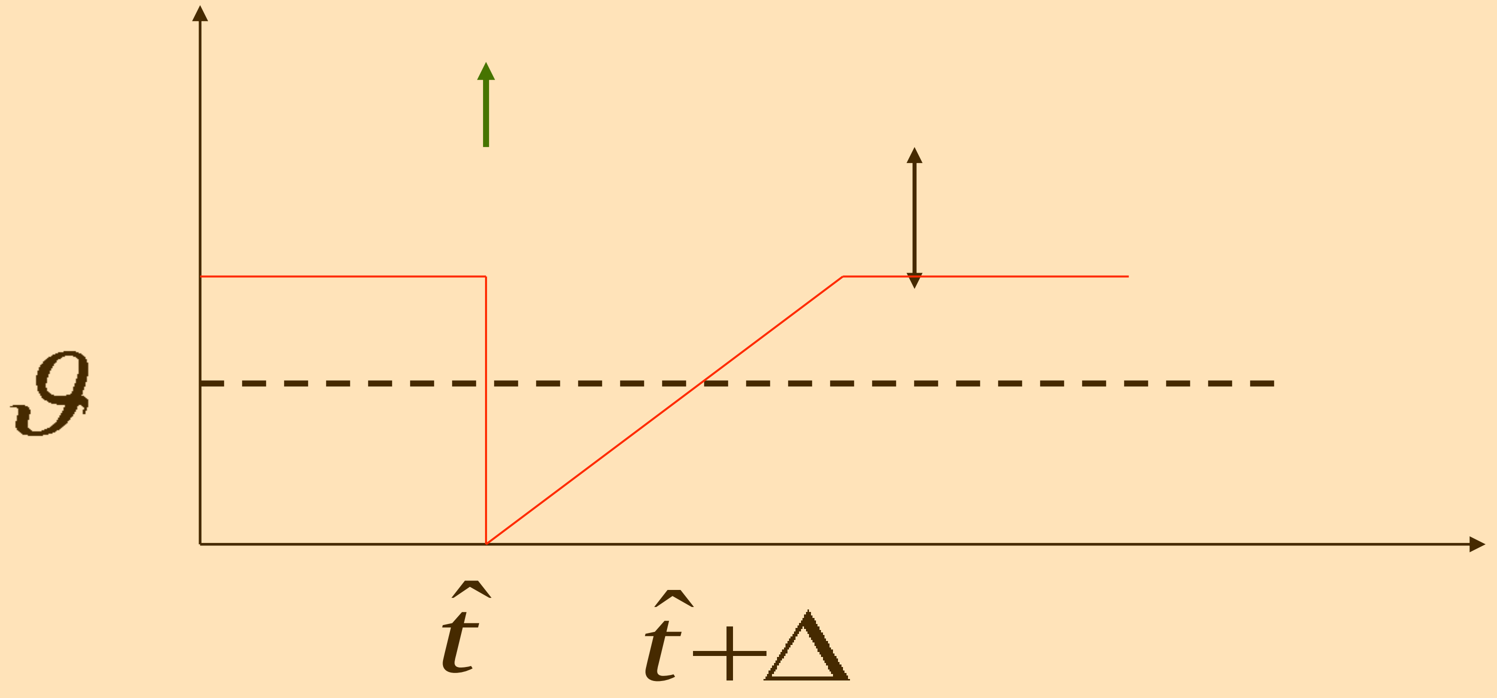


$$P(t|\hat{t}) = \rho(t|\hat{t}) \exp(-\int_{\hat{t}}^{t} \rho(t'|\hat{t}) dt')$$

$$= -\frac{d}{dt} S(t|\hat{t})$$

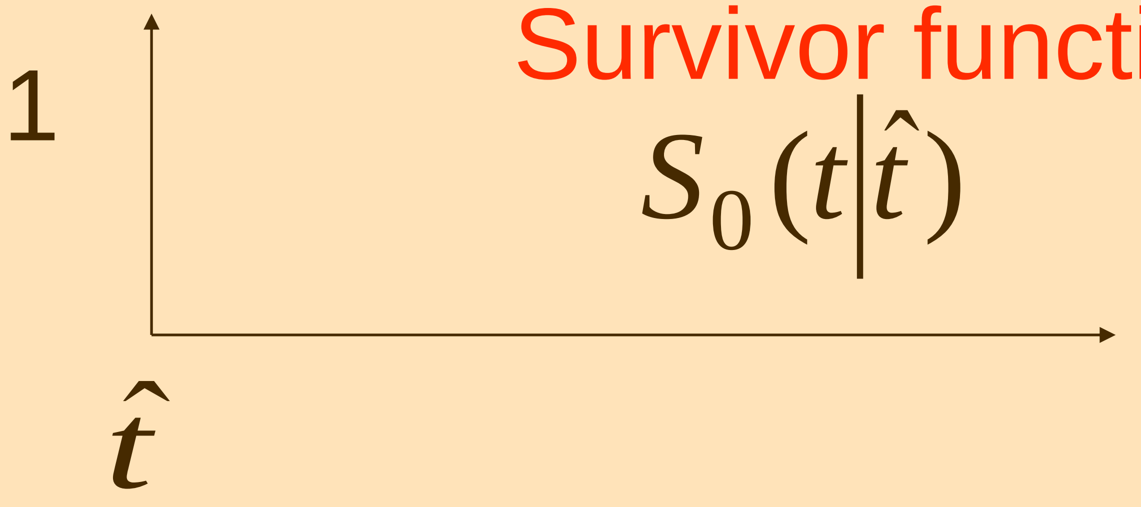
Neuronal Dynamics – Homework assignement

neuron with relative refractoriness, constant input



escape rate

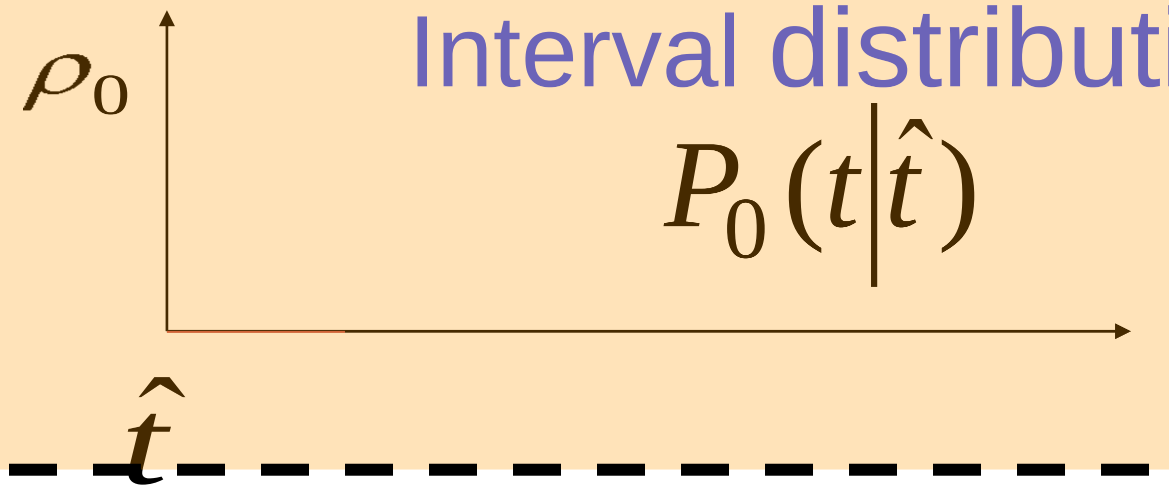
$$\rho(t) = \rho_0 \frac{u}{\vartheta} \quad for \;\; u > \vartheta$$



Survivor function

$S_0(t|\hat{t})$

$$S_0(t|\hat{t}) = \Big\{$$

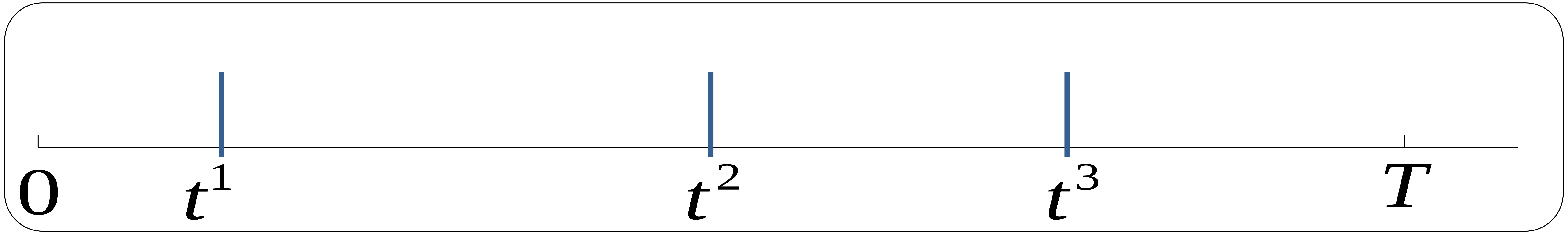


Interval distribution

$P_0(t|\hat{t})$

$$P_0(t|\hat{t}) =$$

Neuronal Dynamics – 8.5. Firing probability in discrete time



Probability to survive 1 time step

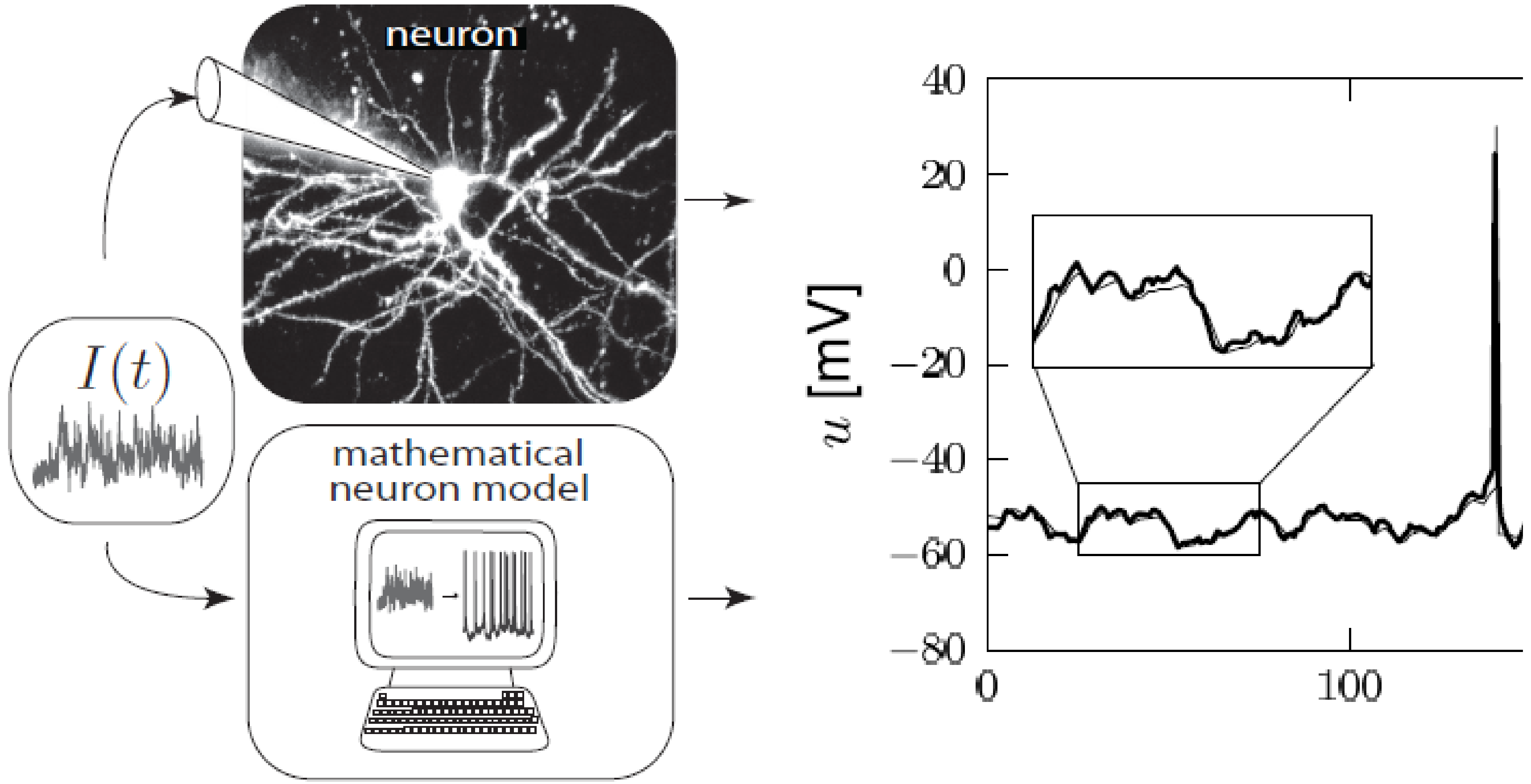
$$S(t_{k+1}|t_k) = \exp[-\int_{t_k}^{t_{k+1}} \rho(t')dt']$$

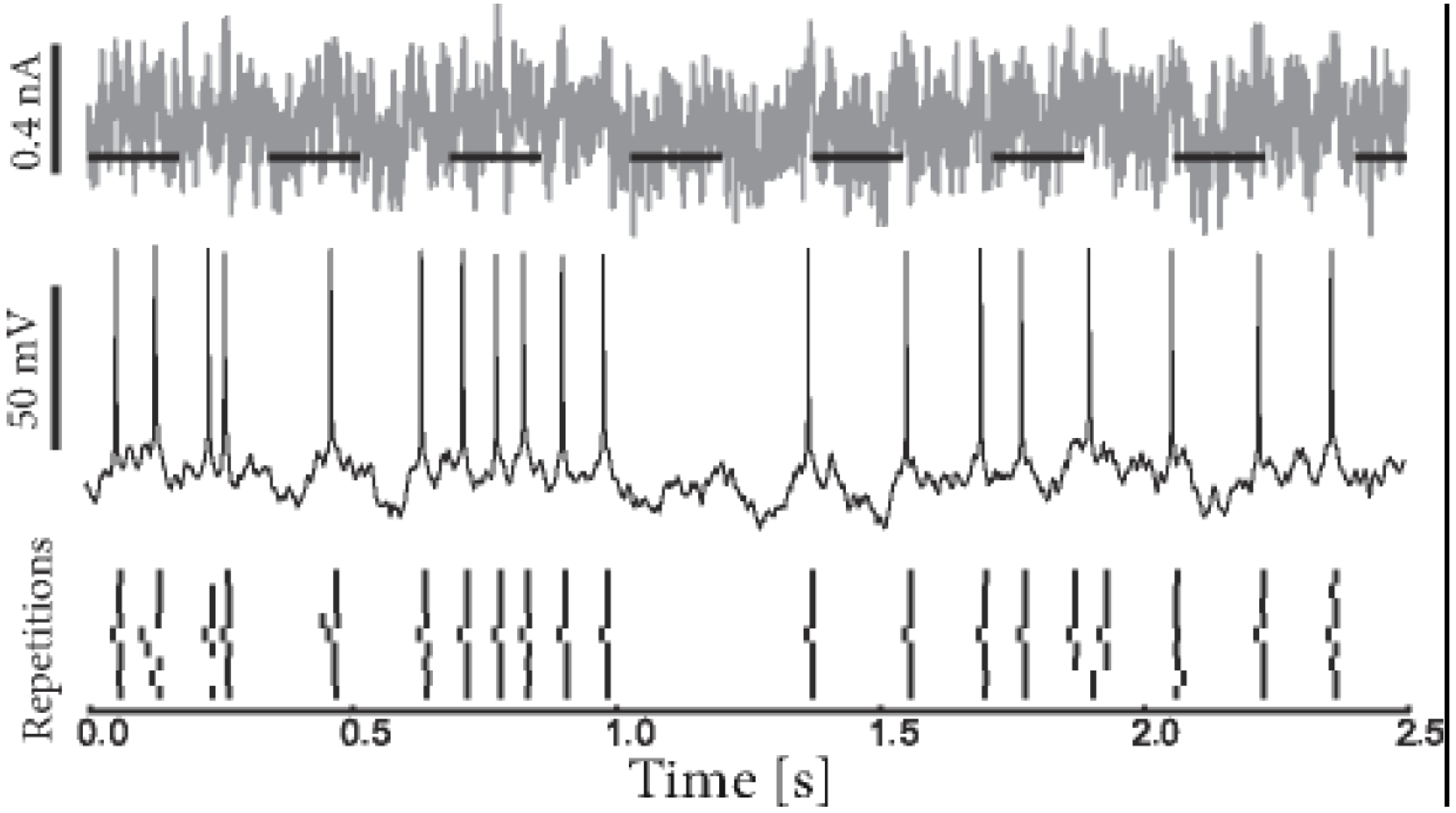
$$S(t_{k+1}|t_k) = \exp[-\rho(t_k)\Delta] = 1 - P_k^F$$

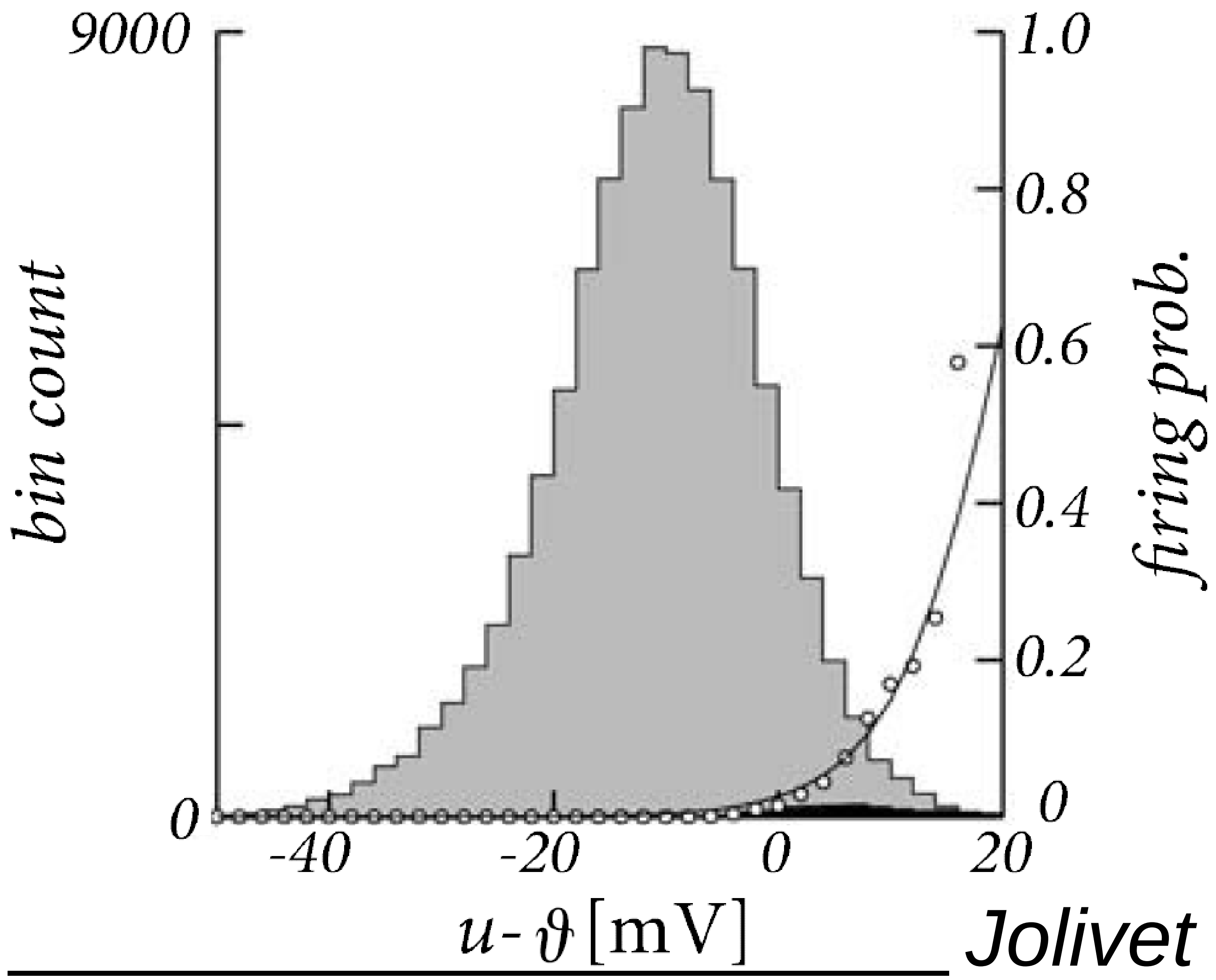
Probability to fire in 1 time step

$$P_k^F =$$

Neuronal Dynamics – 8.5. Escape noise - experiments







Jolivet et al. , J. Comput. Neurosc. 2006

$$P_k^F = 1 - \exp[-\rho(t_k)\Delta]$$

escape rate

$$\rho(t) = \frac{1}{\Delta}\exp\left(\frac{u(t)-\vartheta}{\Delta}\right)$$

Escape noise = stochastic intensity

-Renewal theory
- hazard function
- survivor function
- interval distribution

-time-dependent renewal theory
-discrete-time firing probability
-Link to experiments

→ basis for modern methods of neuron model fitting

Week 8 – part 6 : Comparison of noise models

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Biological Modeling of Neural Networks

Week 8 – Noisy input models: Barrage of spike arrivals

Wulfram Gerstner

EPFL, Lausanne, Switzerland

8.1 Variation of membrane potential
- white noise approximation

8.2 Autocorrelation of Poisson

8.3 Noisy integrate-and-fire
- superthreshold and subthreshold

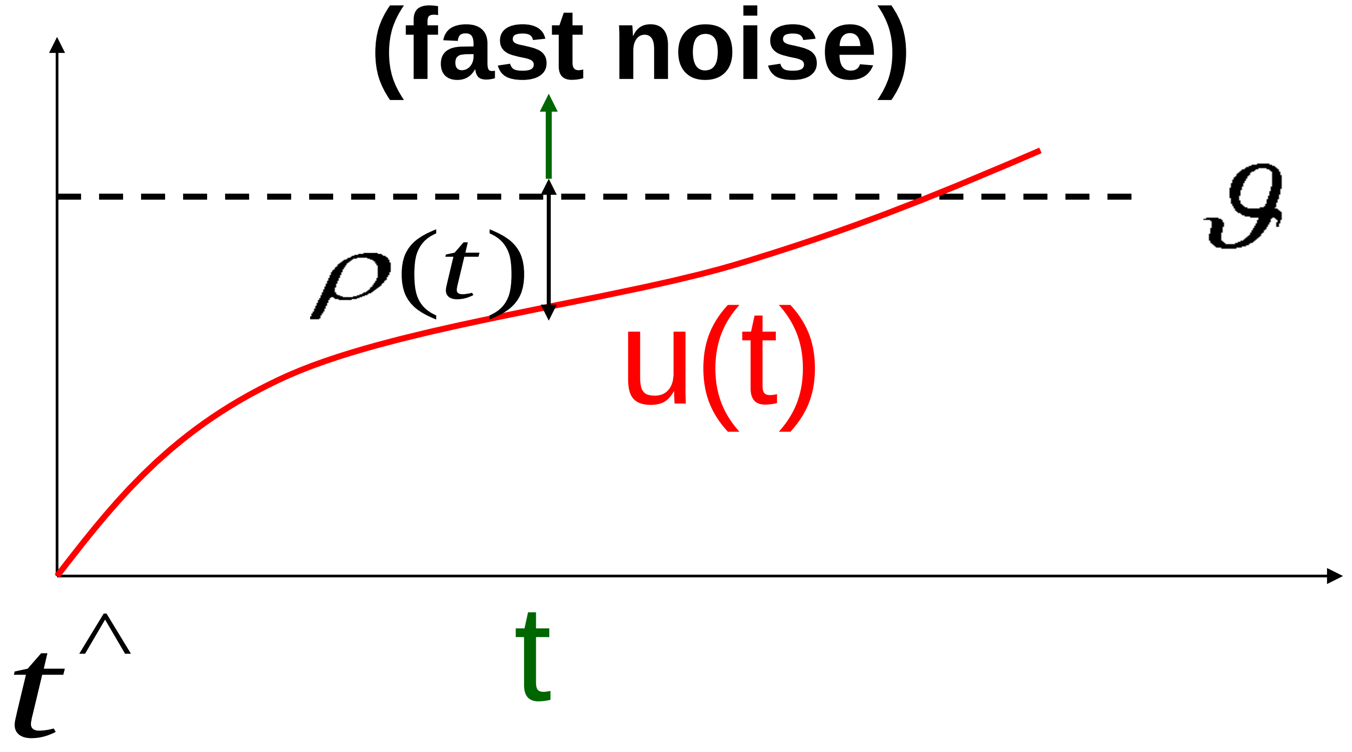
8.4 Escape noise
- stochastic intensity

8.5 Renewal models

8.6 Comparison of noise models

Neuronal Dynamics – 8.6. Comparison of Noise Models

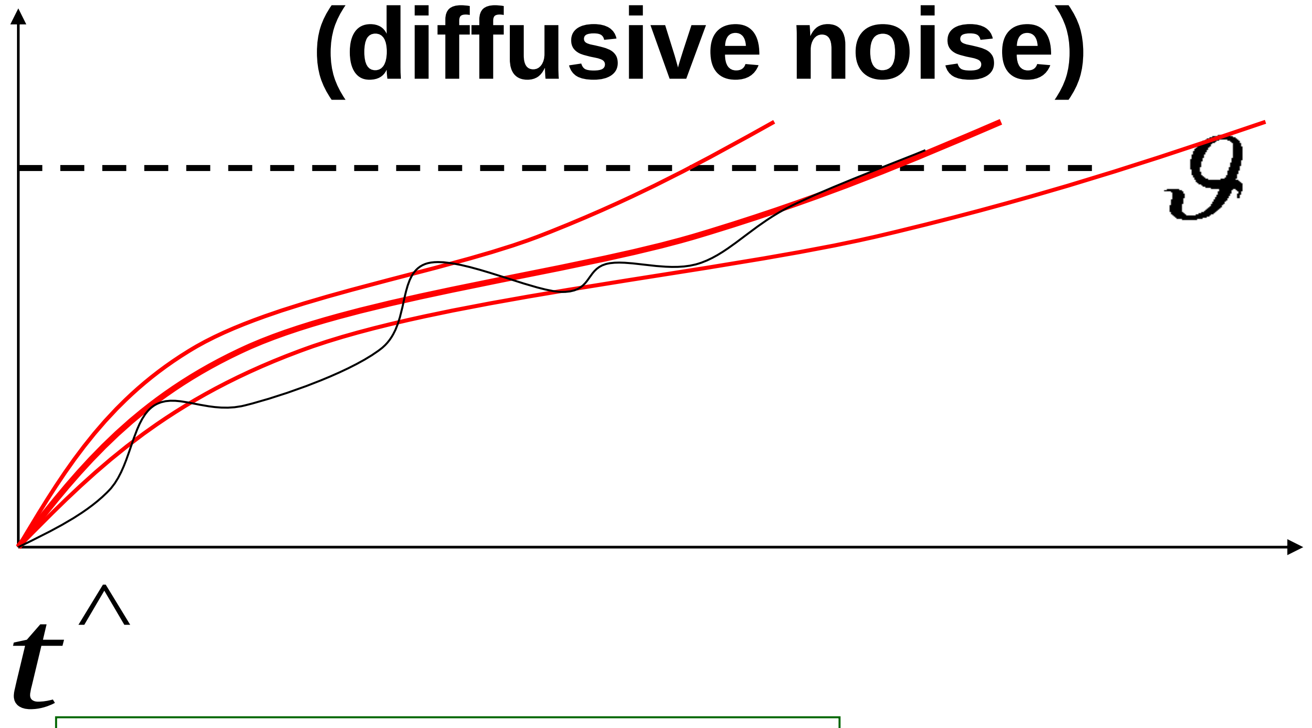
escape process
(fast noise)



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$

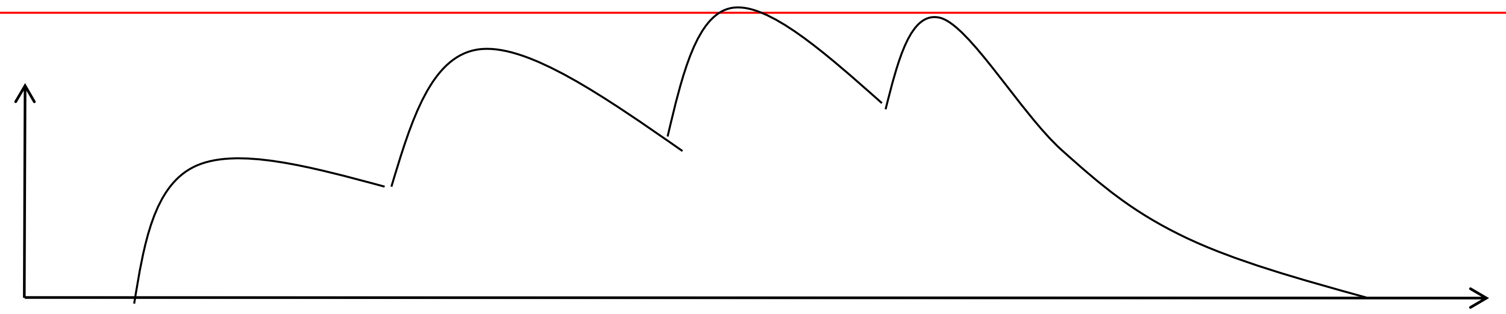
stochastic spike arrival
(diffusive noise)



noisy integration

$$\tau \cdot \frac{du_i}{dt} = -u_i + RI + \xi(t)$$

Poisson spike arrival: **Mean and autocorrelation of filtered signal**



$$S(t) = \sum_f \delta(t - t^f)$$

$$F(s)$$

Filter

$$x(t) = \int F(s) S(t-s) ds$$

Assumption: stochastic spiking

rate $\nu(t)$

$$\langle x(t) \rangle = \int F(s) \langle S(t-s) \rangle ds$$

mean $$\langle x(t) \rangle = \int F(s) \langle \nu(t-s) \rangle ds$$

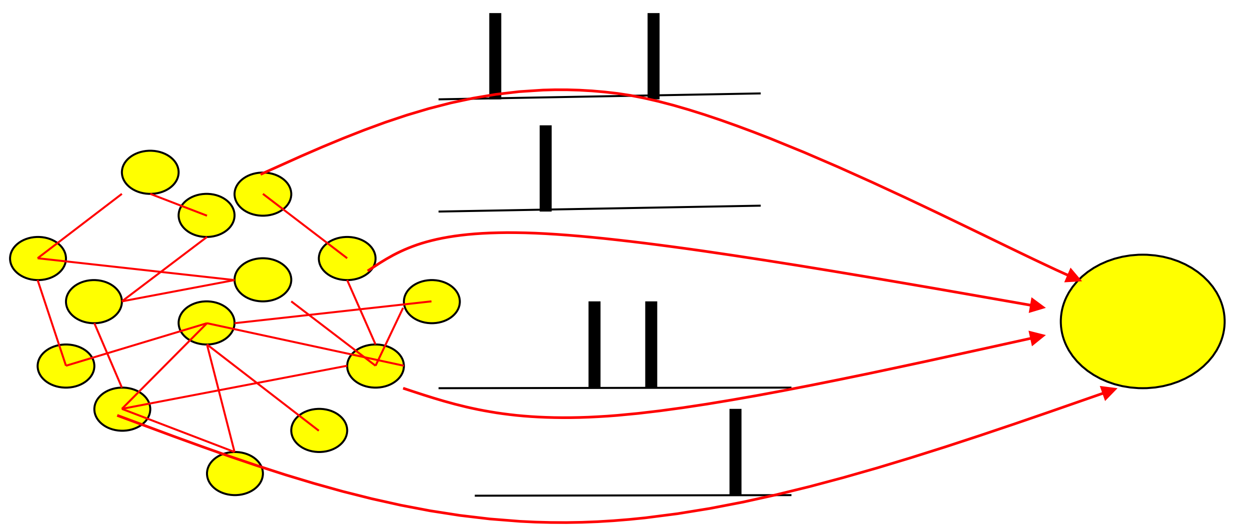
Autocorrelation of output

$$\langle x(t) x(t') \rangle = \left\langle \int F(s) S(t-s) ds \int F(s') S(t'-s') ds' \right\rangle$$

$$\langle x(t) x(t') \rangle = \int F(s) F(s') \underline{\langle S(t-s) S(t'-s') \rangle} \, ds ds'$$

Autocorrelation of input

Diffusive noise (stochastic spike arrival)



Stochastic spike arrival:
excitation, total rate R_e
inhibition, total rate R_i

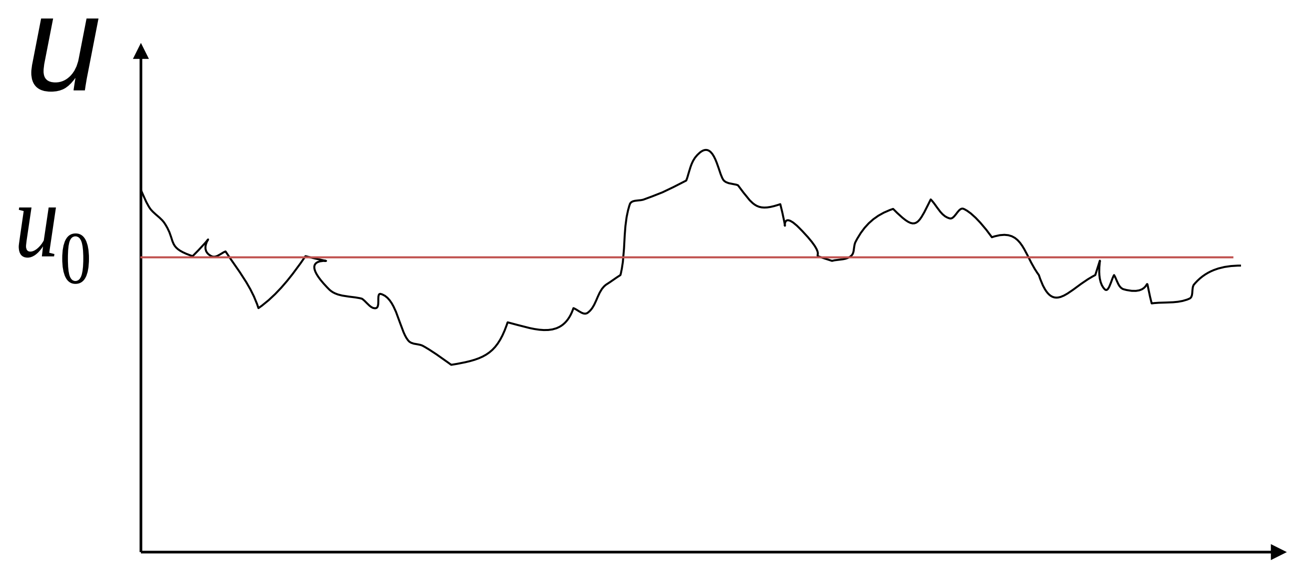
Synaptic current pulses

$$\tau \frac{d}{dt}u = -(u - u_{rest}) + \sum_{k,f} \frac{q_e}{C}\delta(t - t_k^f) - \sum_{k',f'} \frac{q_i}{C}\delta(t - t_{k'}^{f'})$$

EPSC IPSC

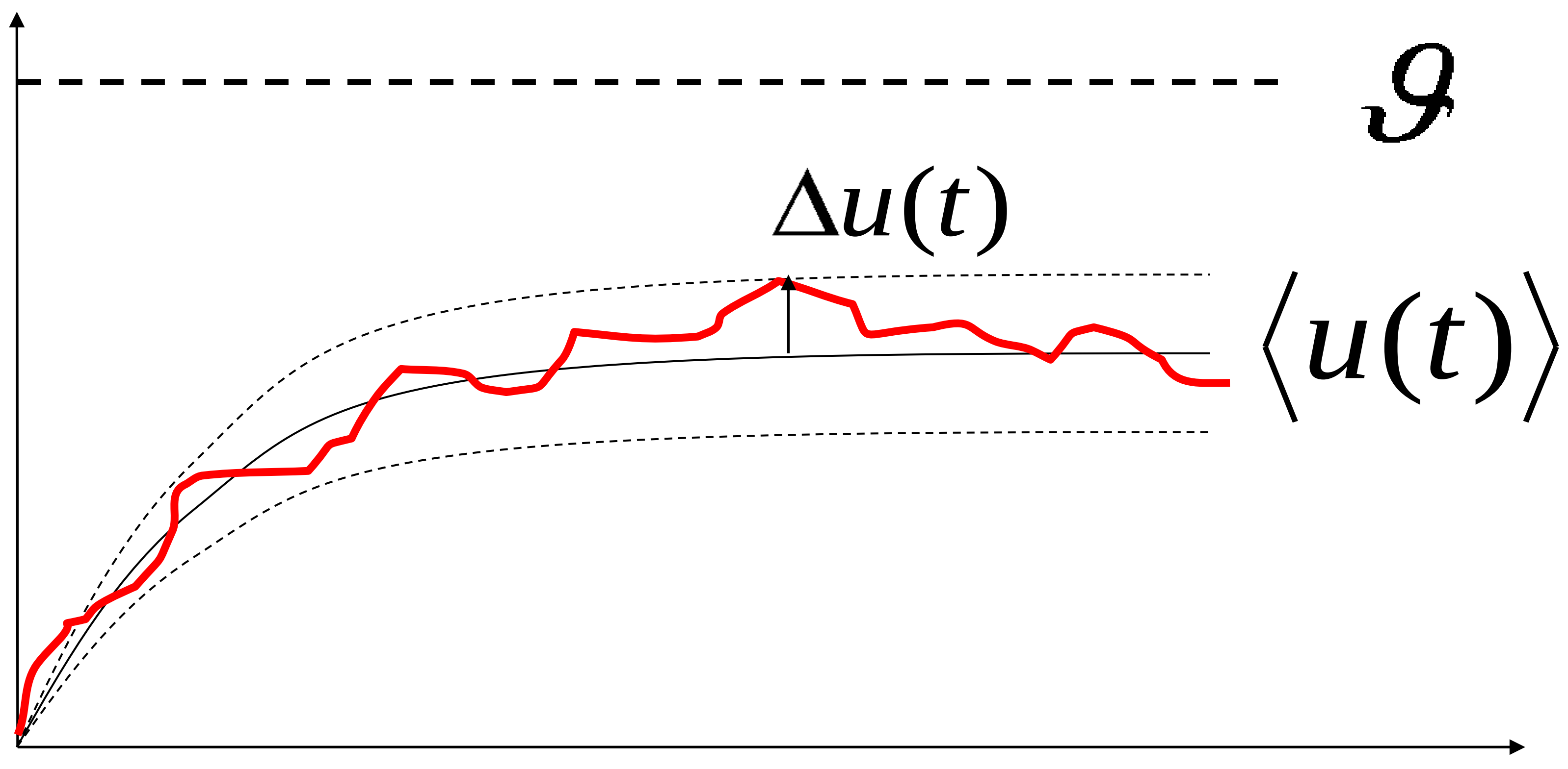
$$\tau \frac{d}{dt}u = -(u - u_{rest}) + RI(t) + \xi(t)$$

Blackboard

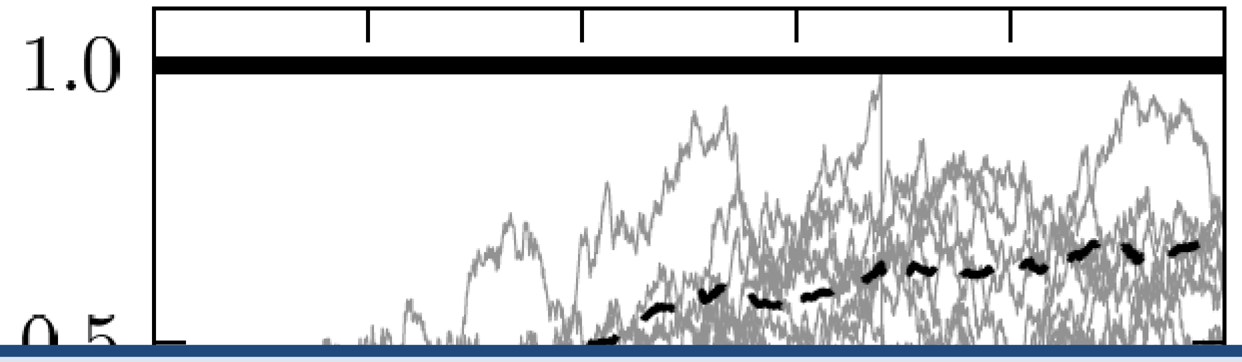


Langevin equation,
Ornstein Uhlenbeck process

Diffusive noise (stochastic spike arrival)



$$\tau \frac{d}{dt}u = -(u - u_{rest}) + RI(t) + \xi(t)$$

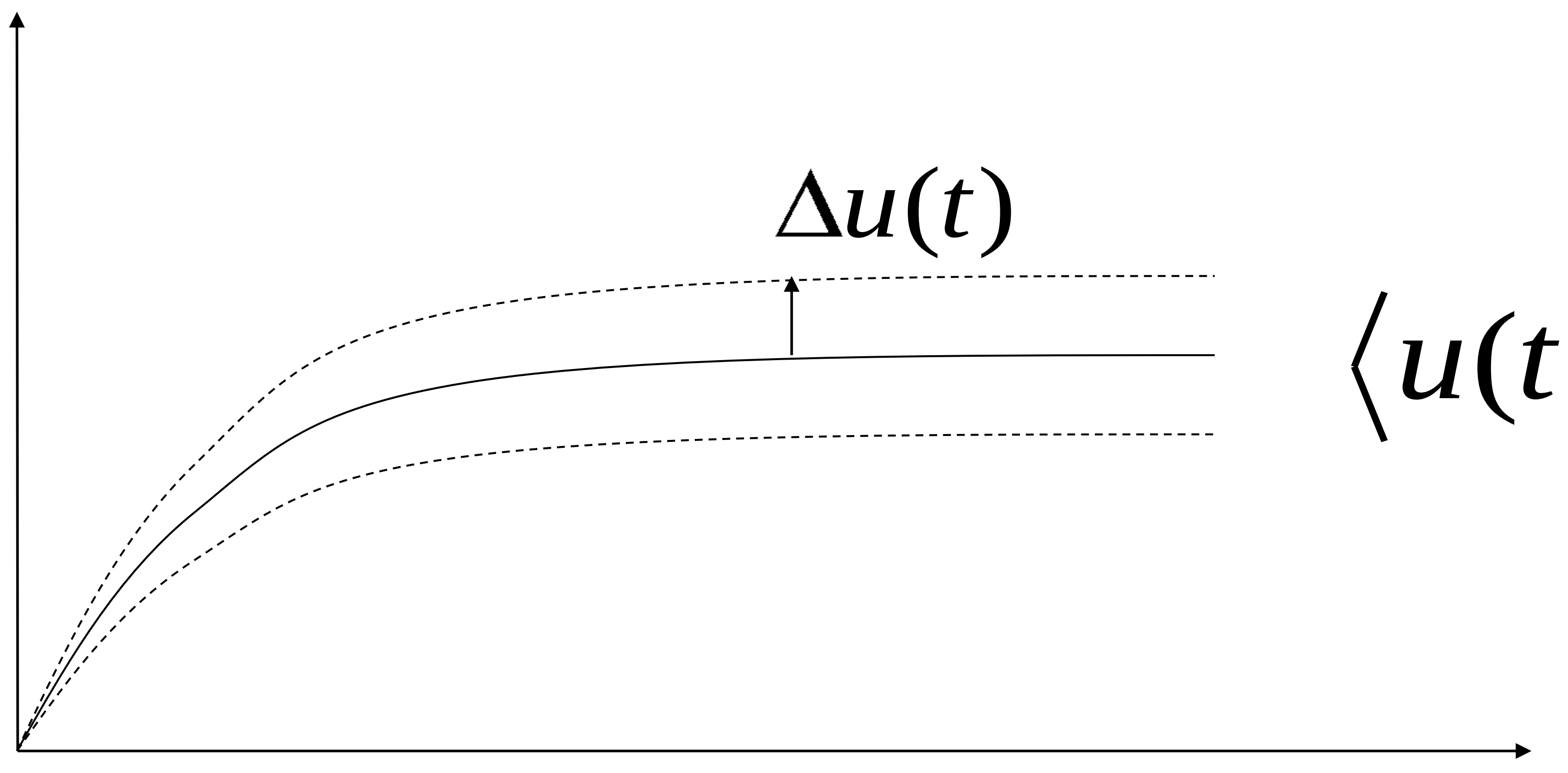


$$\langle \Delta u(t) \Delta u(t) \rangle = \langle u(t) u(t) \rangle - \langle u(t) \rangle^2 =$$

$$\langle \Delta u(t') \Delta u(t) \rangle = \langle u(t) u(t') \rangle - \langle u(t) \rangle \langle u(t') \rangle =$$

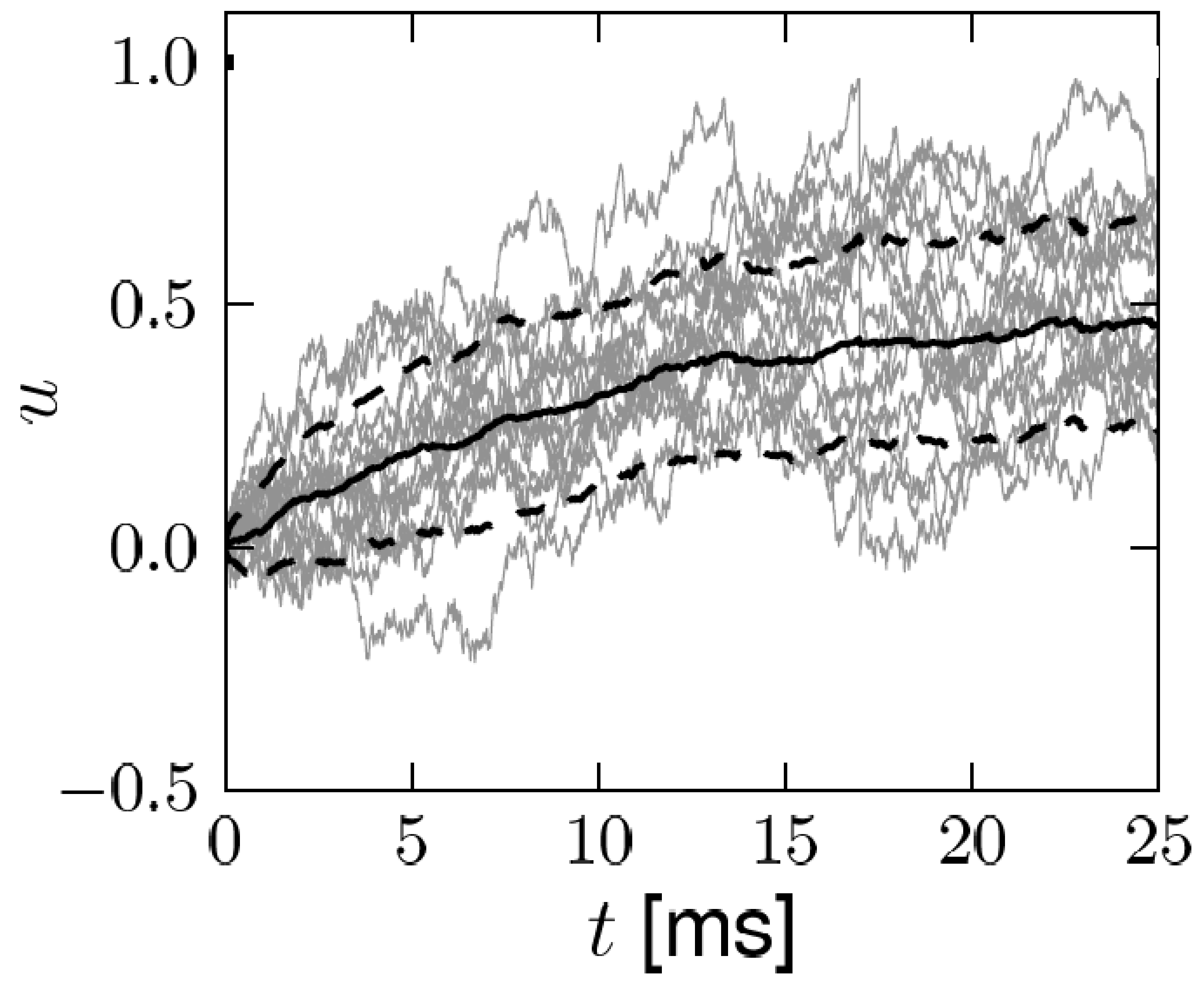
Math argument:
- *no threshold*
- *trajectory starts at known value*

Diffusive noise (stochastic spike arrival)



$$\langle u(t) \rangle = u_0(t)$$

$$\tau \frac{d}{dt} u = -(u - u_{rest}) \quad + R\, I(t) + \quad \xi(t)$$



$$\langle \Delta u(t) \Delta u(t) \rangle = \langle u(t) u(t) \rangle - \langle u(t) \rangle^2 =$$

$$\langle \Delta u(t') \Delta u(t) \rangle = \langle u(t) u(t') \rangle - \langle u(t) \rangle \langle u(t') \rangle =$$

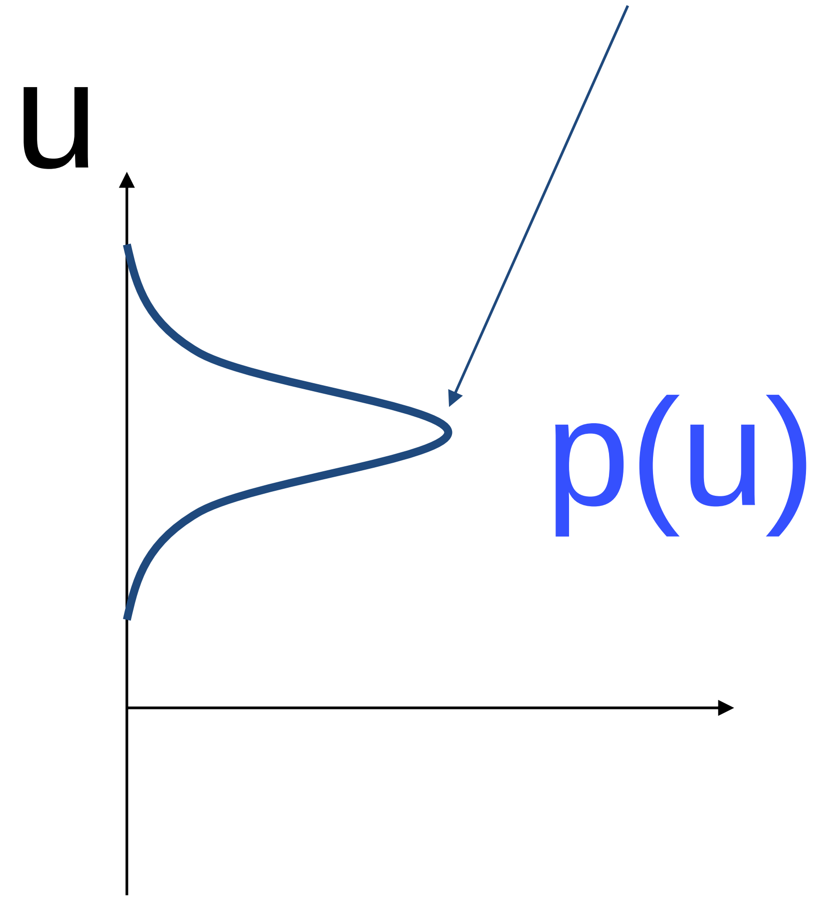
Math argument

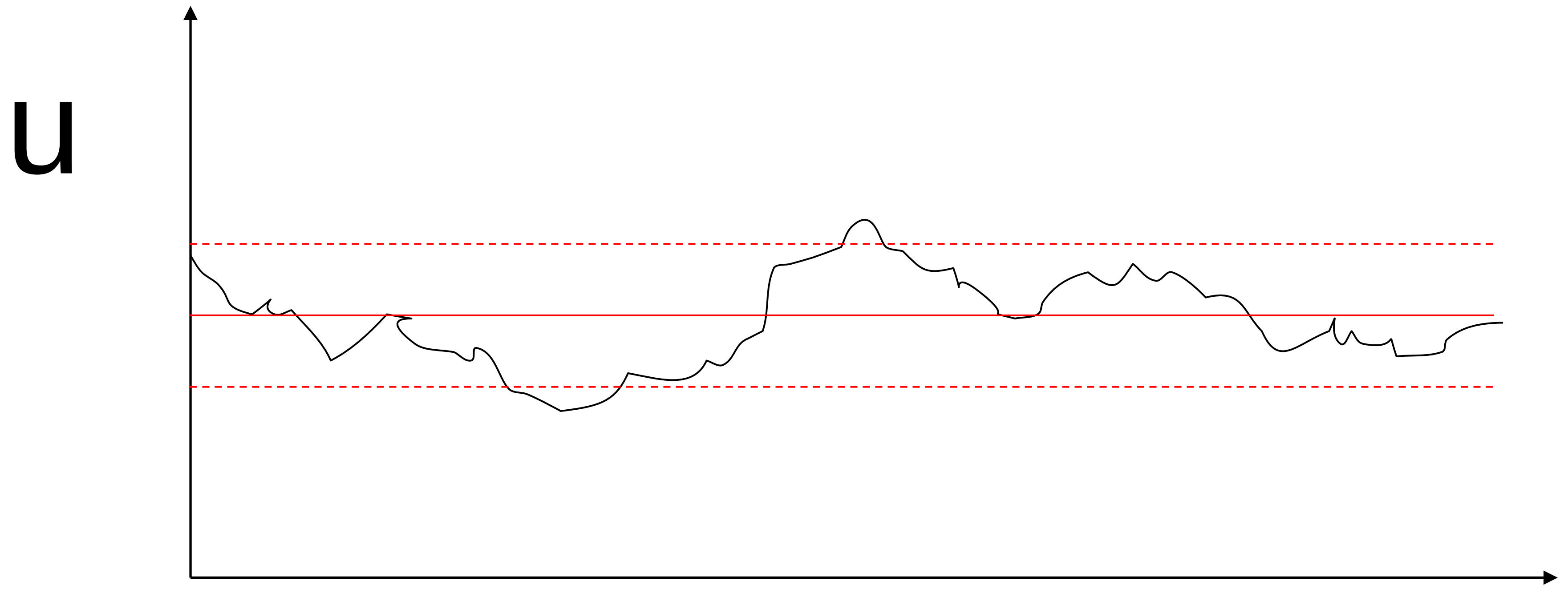
$$p(u,t) = \frac{1}{\sqrt{2\pi \langle \Delta u^2(t) \rangle}} \exp\left\{ -\frac{[u(t|\hat{t}) - u_0(t)]^2}{2 \langle \Delta u^2(t) \rangle} \right\}$$

$$\langle [\Delta u(t)]^2 \rangle = \sigma_u^2 [1 - \exp(-2t/\tau)]$$

A) No threshold, stationary input

Membrane potential density: Gaussian





constant input rates
no threshold

noisy integration

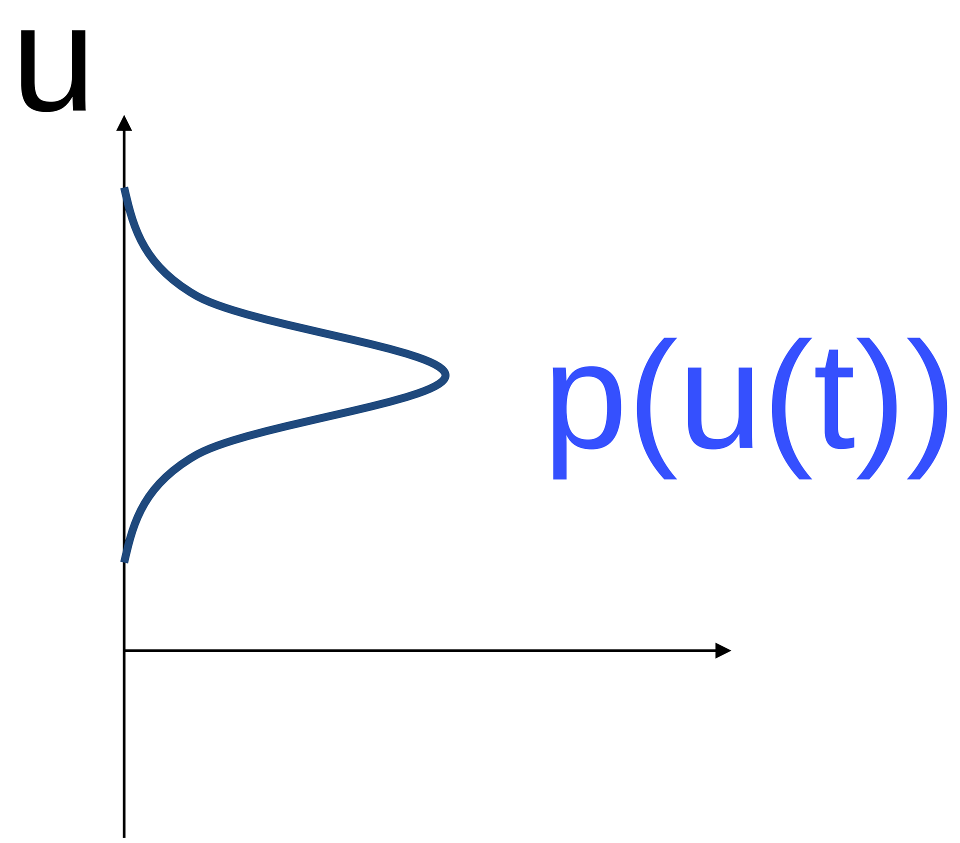
$$\tau \cdot \frac{du_i}{dt} = -u_i + RI + \xi(t)$$

B) No threshold, oscillatory input

Membrane potential density:
Gaussian at time t

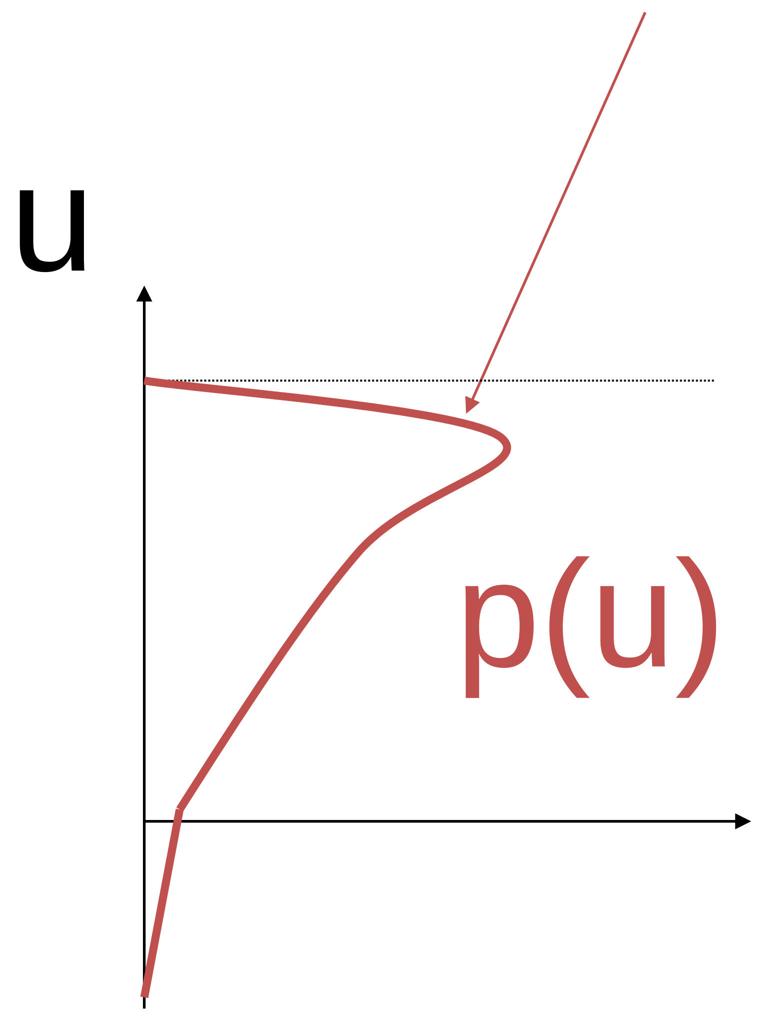
noisy integration

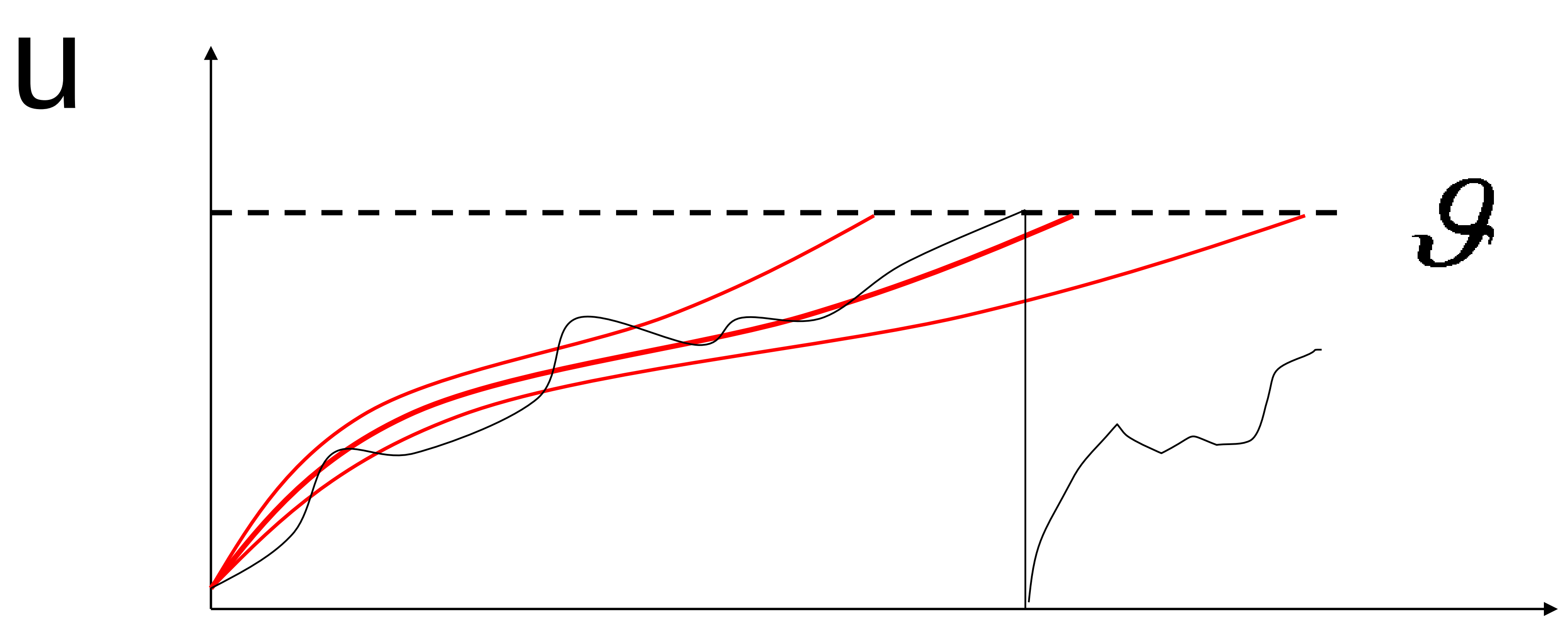
$$\tau \cdot \frac{du_i}{dt} = -u_i + RI + \xi(t)$$



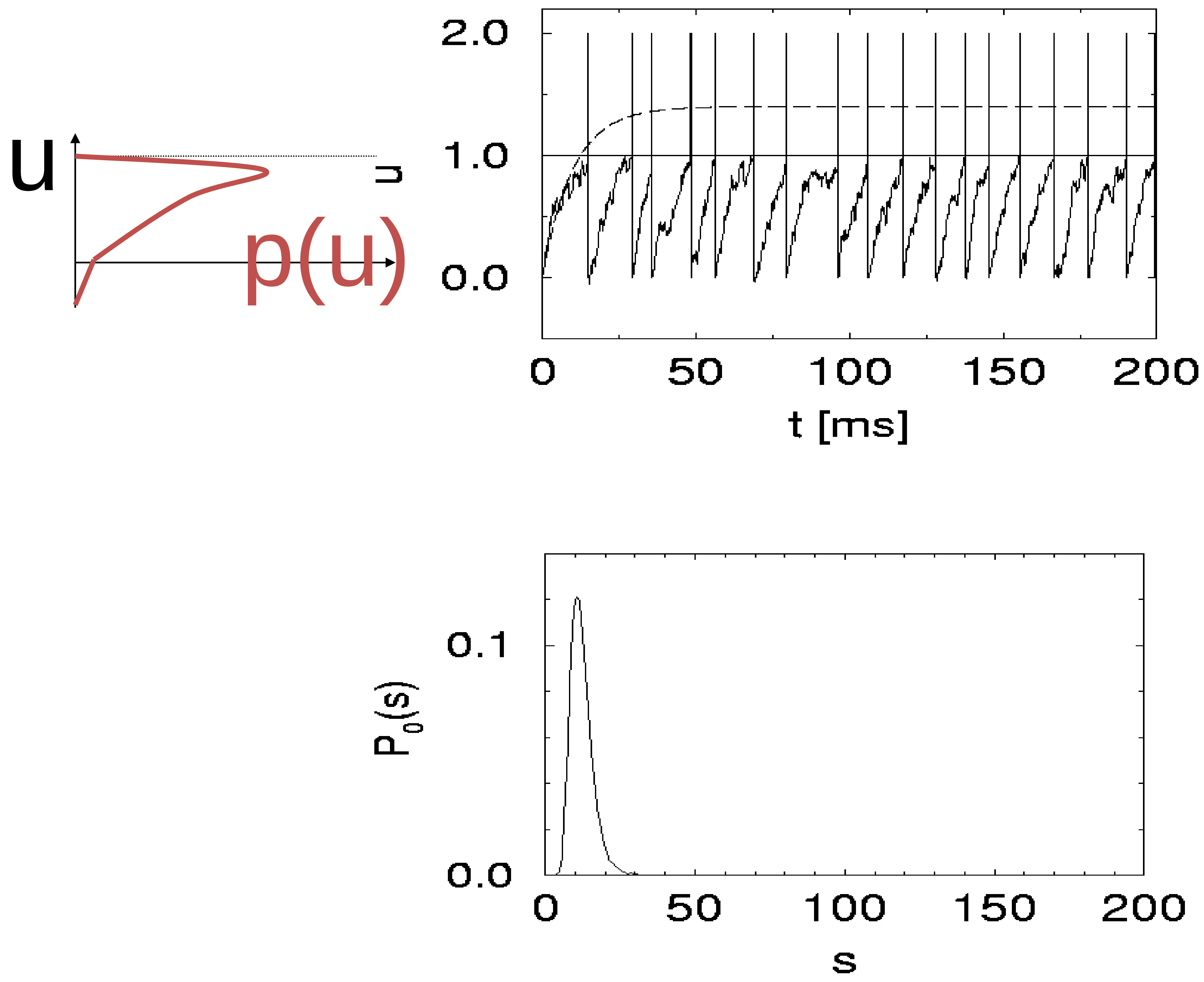
C) With threshold, reset/ stationary input

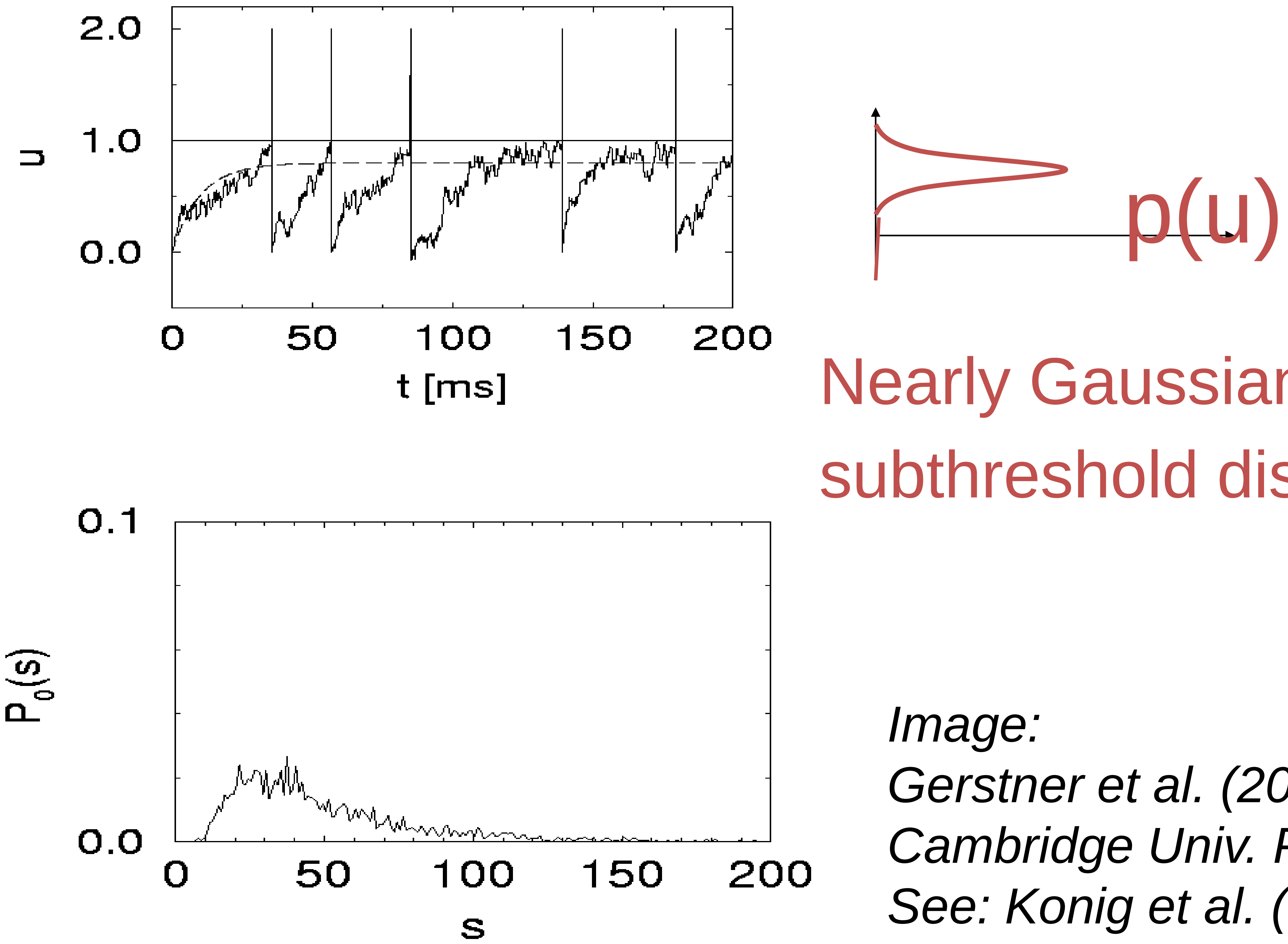
Membrane potential density





Superthreshold vs. Subthreshold regime

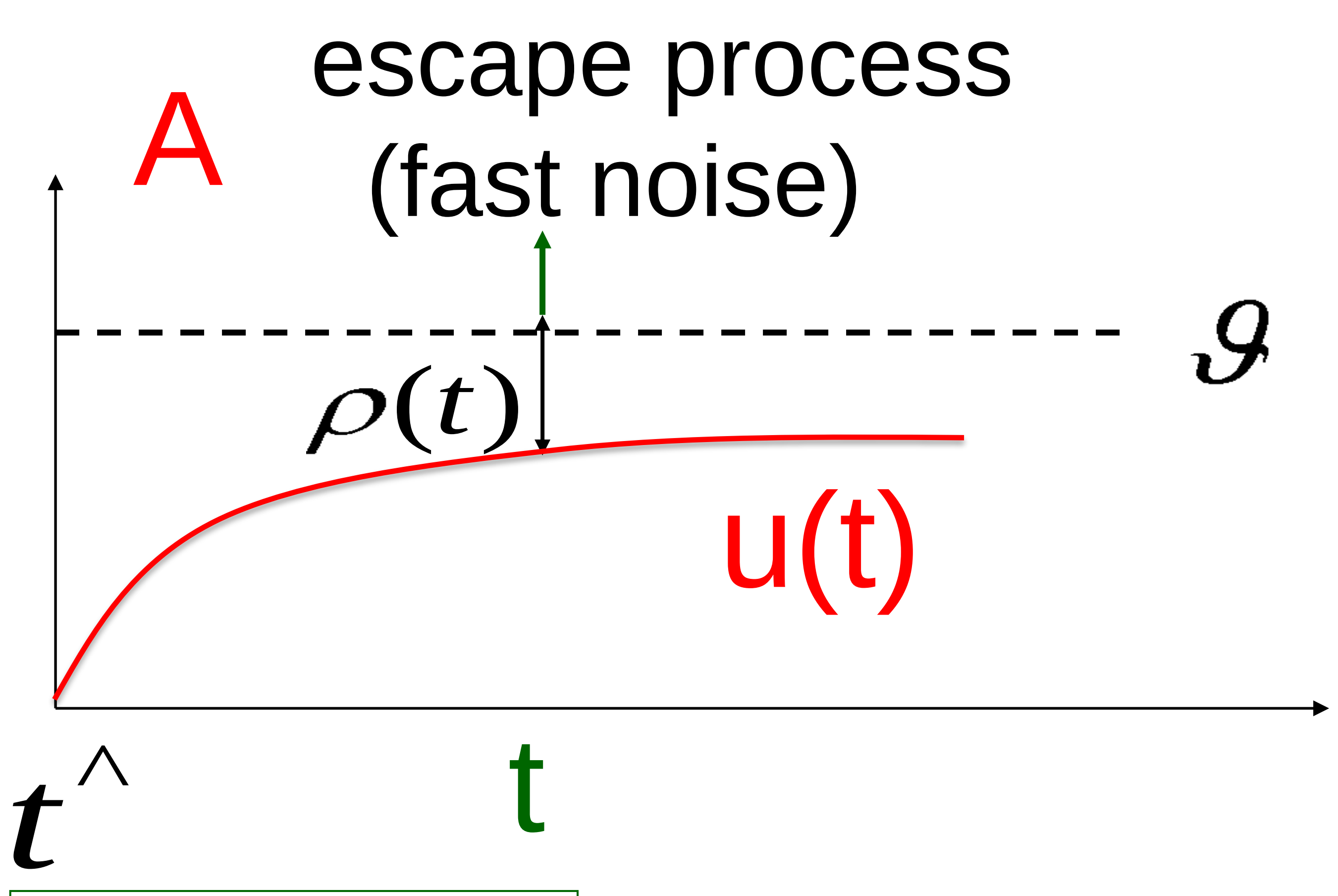




Nearly Gaussian subthreshold distr.

Image:
Gerstner et al. (2013)
Cambridge Univ. Press;
See: Konig et al. (1996)

Neuronal Dynamics – 8.6. Comparison of Noise Models



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$

Interval distribution

$$P_I(t|\hat{t}) =$$

$$= \rho(t) \cdot \exp\left(-\int_{\hat{t}}^{t} \rho(t')dt'\right)$$

escape rate

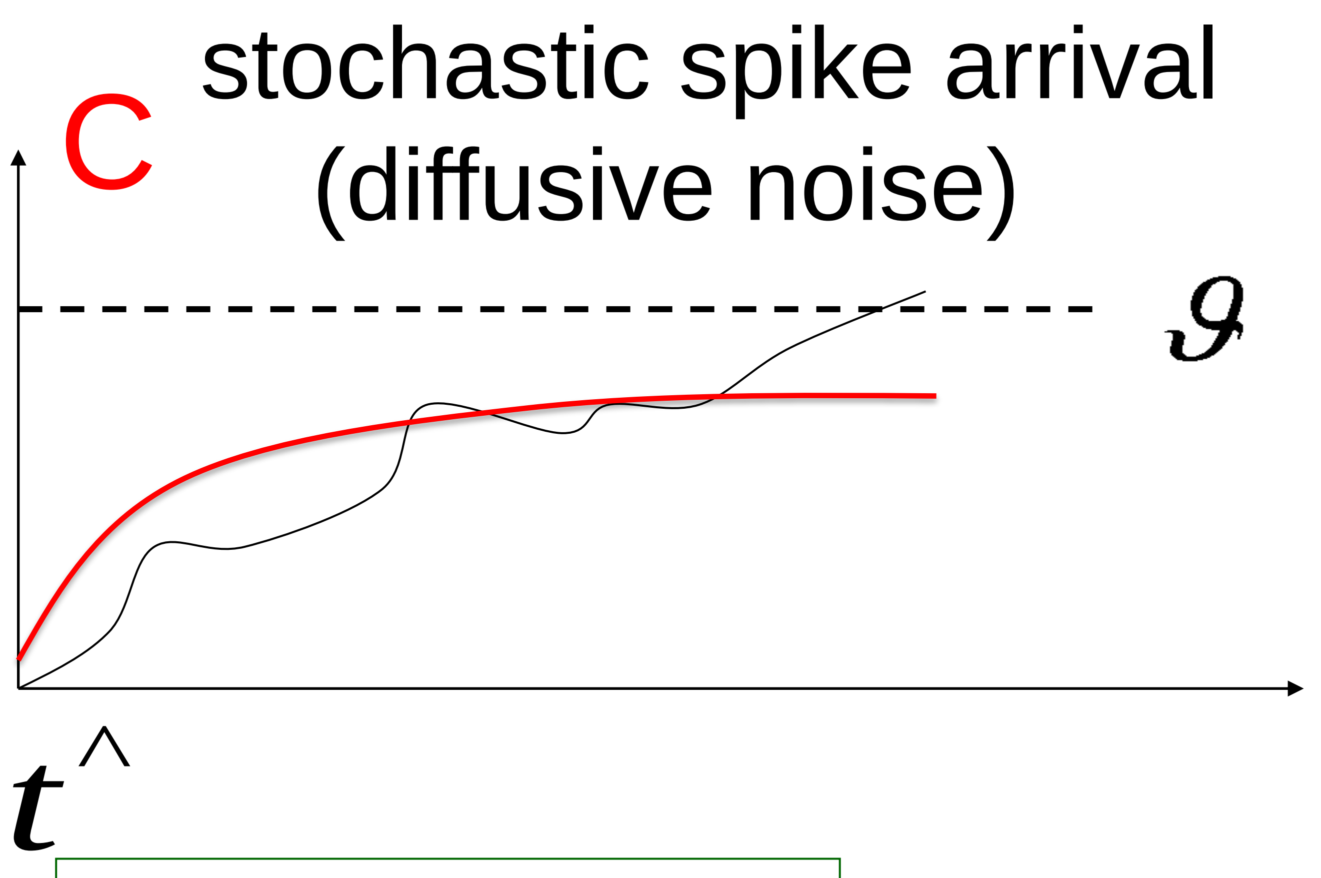
Survivor function

Stationary input:
-Mean ISI

$$\langle s \rangle = \tau_m \sqrt{\pi} \int_{\frac{u_r - h_0}{\sigma}}^{\frac{\vartheta - h_0}{\sigma}} \mathrm{d}u \, \exp\left(u^2\right) \, [1 + \mathrm{erf}(u)]$$

Siegert 1951

-Mean firing rate



noisy integration

$$\tau \cdot \frac{du_i}{dt} = -u_i + RI + \xi(t)$$

noise

white (fast noise)

synapse (slow noise)

$P_I(t|\hat{t})$: first passage time problem

(Brunel et al., 2001)

Diffusive noise

- distribution of potential
- mean interspike interval

FOR CONSTANT INPUT

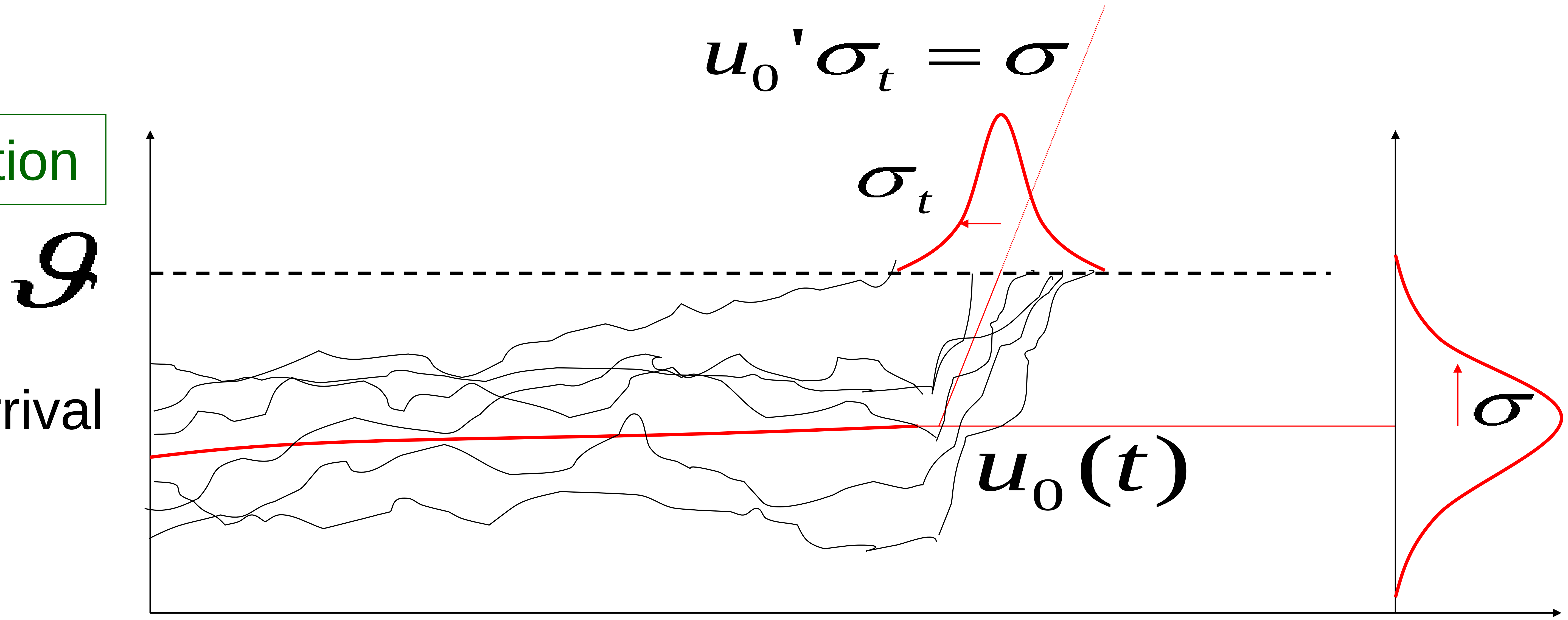
- time dependent-case difficult

Escape noise

- time-dependent interval distribution

Noise models: from diffusive noise to escape rates

noisy integration



escape rate

$$\rho(t) = f(u_0(t), u'_0(t)) \propto \frac{\exp\left(-\frac{(u_0(t)-\vartheta)^2}{2\sigma^2}\right)}{erf((u_0(t)-\vartheta)/\sigma)} [1 + u'_0(t)]$$

Comparison: diffusive noise vs. escape rates

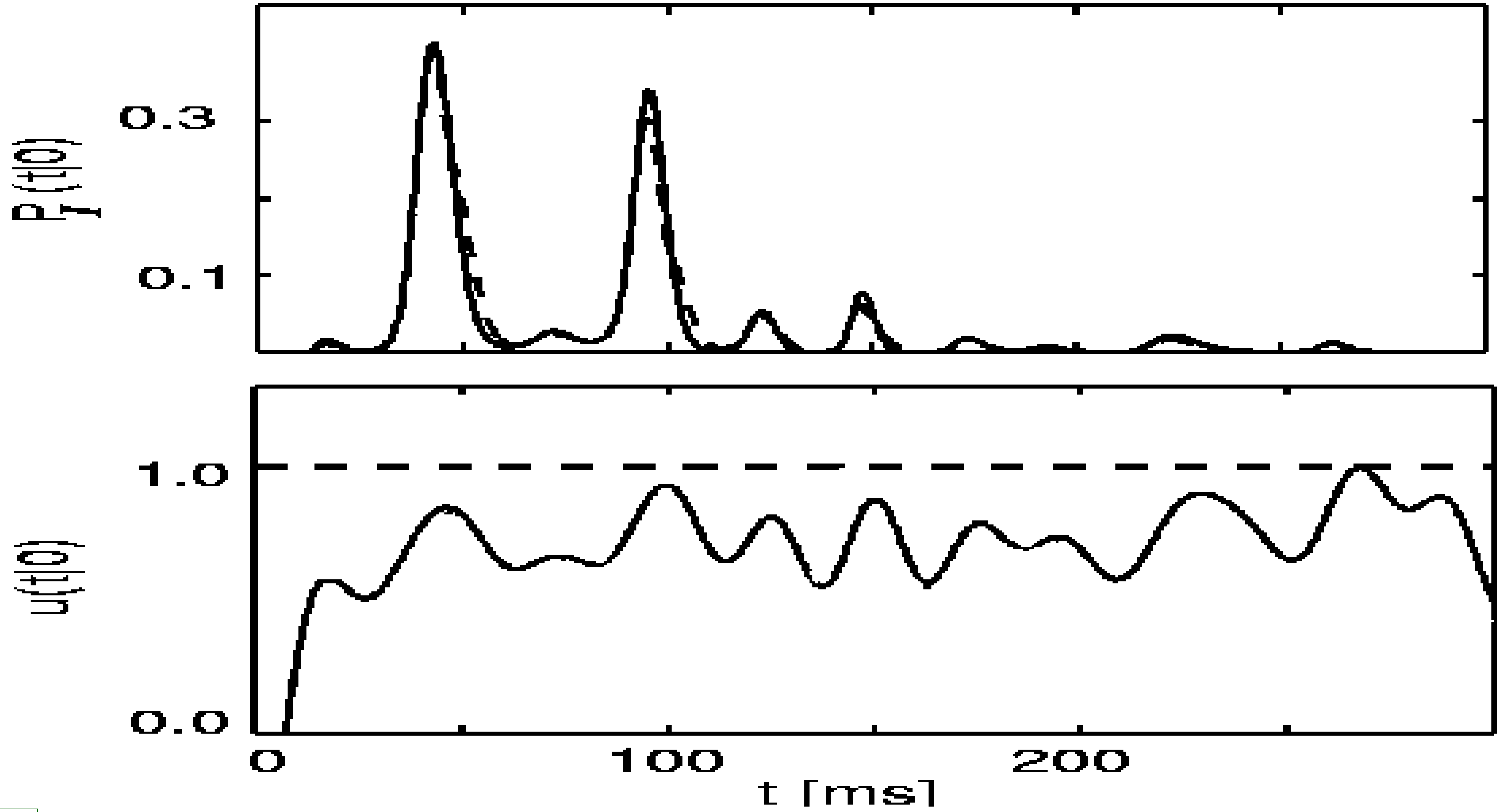
Plesser and Gerstner (2000)

Probability of first spike

diffusive

escape

subthreshold potential



escape rate

$$\rho(t) = f(u_0(t), u'_0(t)) \propto \exp\left(-\frac{(u_0(t)-\vartheta)^2}{2\sigma^2}\right) [1 + u'_0(t)]$$

Diffusive noise

- represents stochastic spike arrival
- easy to simulate
- hard to calculate

Escape noise

- represents internal noise
- easy to simulate
- easy to calculate
- approximates diffusive noise
- basis of modern model fitting methods

Neuronal Dynamics – Quiz 8.4.

A. Consider a leaky integrate-and-fire model with diffusive noise:

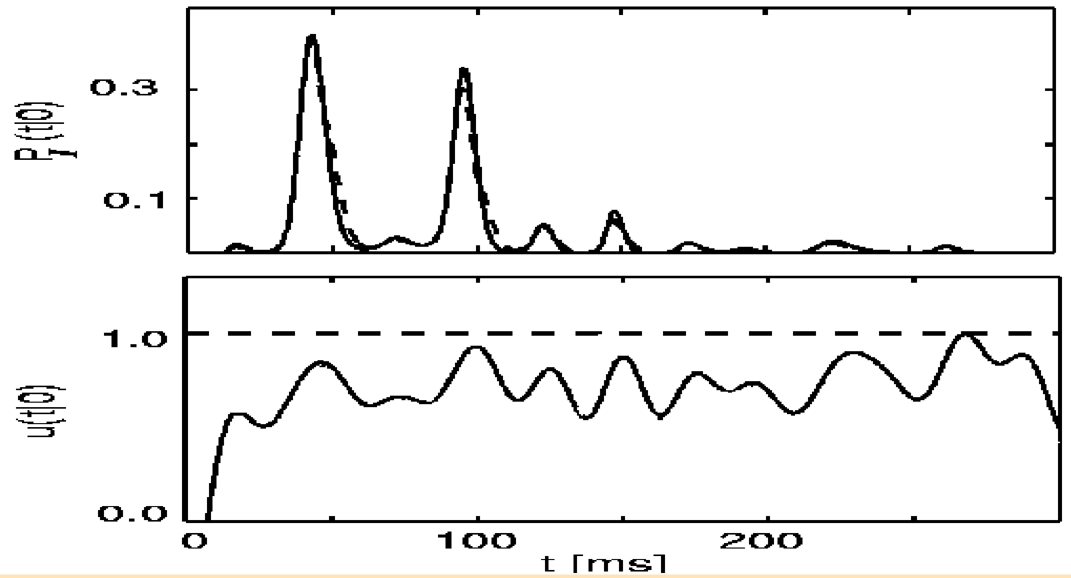
[ ] The membrane potential distribution is always Gaussian.

[ ] The membrane potential distribution is Gaussian for any time-dependent input.

[ ] The membrane potential distribution is approximately Gaussian for any time-dependent input, as long as the mean trajectory stays ‘far’ away from the firing threshold.

[ ] The membrane potential distribution is Gaussian for stationary input in the absence of a threshold.

[ ] The membrane potential distribution is always Gaussian for constant input and fixed noise level.



B. Consider a leaky integrate-and-fire model with diffusive noise for time-dependent input. The above figure (taken from an earlier slide) shows that

[ ] The interspike interval distribution is maximal where the determinstic reference trajectory is **closest** to the threshold.

[ ] The interspike interval vanishes for very long intervals if the determinstic reference trajectory has stayed close to the threshold before - even if for long intervals it is very close to the threshold

[ ] If there are several peaks in the interspike interval distribution, peak n is always of smaller amplitude than peak n-1.

[ ] I would have ticked the same boxes (in the list of three options above) for a leaky integrate-and-fire model with escape noise.