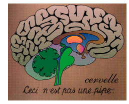
Lecture 5 – Network of neurons and associative memory

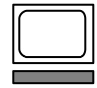
-Introduction
-Classification by similarity
-Detour: magnetic materials
-Associative Memory
-Dense networks (mean-field)

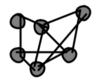
Systems for computing and information processing

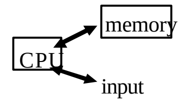


Brain

Computer







Distributed architecture

(10^{10} proc. Elements/neurons)

No separation of processing and memory

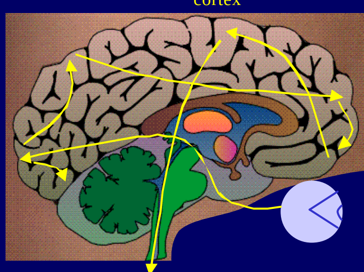
Von Neumann architecture

1 CPU

(10^{10} transistors)

motor cortex

visual cortex



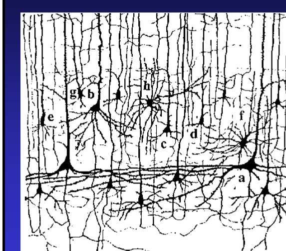
association cortex

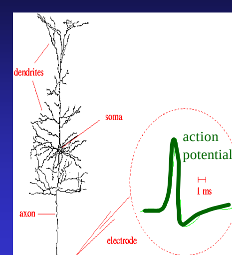
to motor output

1mm

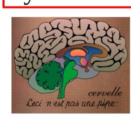
10 000 neurons
3 km wires

Signal:
action potential (spike)

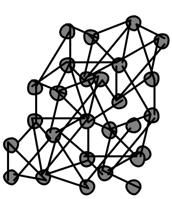




Systems for computing and information processing



Brain



Distributed architecture

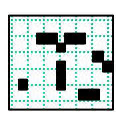
10^{10} neurons

10^{4} connections/neurons

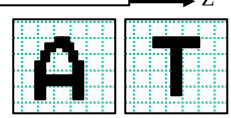
No separation of processing and memory

- Classification by similarity: **pattern recognition**

image → Classification: comparison with prototypes → A, B, T, Z



Noisy image



Prototypes

blackboard

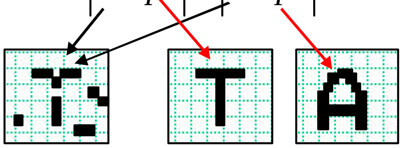
- recognize/understand images:

pattern recognition

Blackboard :

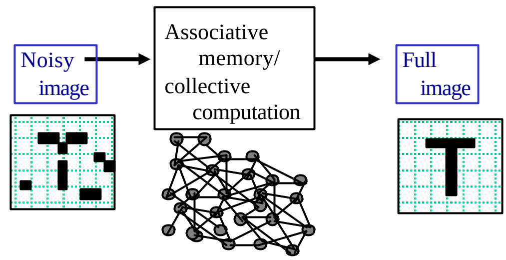
Classification by closest prototype

$$\left|x - p^{T}\right| \le \left|x - p^{A}\right|$$



- recognize/understand images:

pattern recognition



Brain-style computation

Lecture 5 – Network of neurons and associative memory

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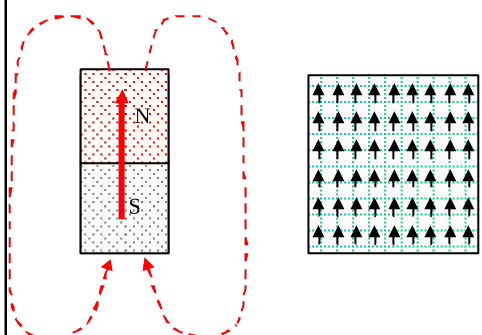
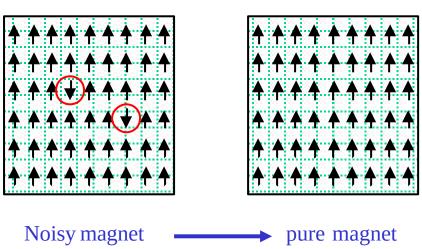
-Detour: magnetic materials
-Associative Memory
-Dense networks (mean-field)

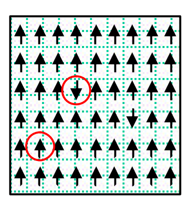
Detour: magnetism



Detour: magnetism



Detour: magnetism



Elementary magnet

↑ $S_i = +1$

↓ $S_i = -1$

Blackboard : example

dynamics

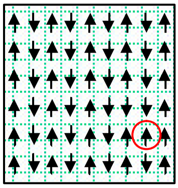
$$S_i\,(t+1) = \mathrm{sgn}\left[\sum S_j(t)\right]$$

Sum over all interactions with i

blackboard

Detour: magnetism

Anti-ferromagnet



Elementary magnet

↑ $S_i = +1$

↓ $S_i = -1$

$w_{ij} = +1$

$w_{ij} = -1$

dynamics

$$S_i\,(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j(t)\right]$$

Sum over all interactions with i

blackboard

Lecture 5 – Network of neurons and associative memory

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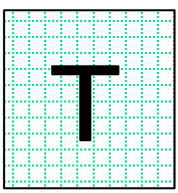
- -Dense networks (mean-field)

Associative memory



Elementary pixel

■ $S_i = +1$

□ $S_i = -1$

$w_{ij} = +1$

$w_{ij} = +1$

$w_{ij} = -1$

dynamics

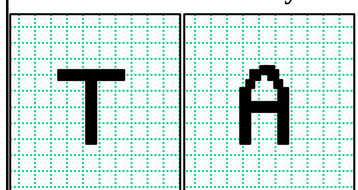
$$S_i\,(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j(t)\right]$$

Sum over all interactions with i

blackboard

Hopfield model

Associative memory



Prototype $\vec{p}^1$

Prototype $\vec{p}^2$

interactions

$$w_{ij} = \sum_m p_i^m p_j^m$$

Sum over all prototypes

dynamics

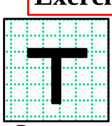
$$S_i\,(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j(t)\right]$$

Sum over all interactions with i

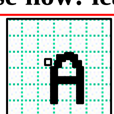
blackboard

Hopfield model

Exercise now: learning of prototypes



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

interactions

$$(1)\quad w_{ij} = \sum_m p_i^m p_j^m$$

Sum over all prototypes

a) Show that (1) corresponds to a rate learning rule

$$(2)\quad \frac{d}{dt} w_{ij} = a_2^{corr}(n_j^{pre} - J)(n_i^{post} - J)$$

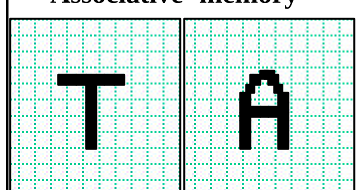
Assume that weights are zero at the beginning;

Each pattern is presented (enforced) during 0.5 sec (One after the other).

note that $p_j^m = \pm 1$ *but* $n_j \geq 0$

b) Compare with: $\frac{d}{dt} w_{ij} = a_0 + a_1^{pre} n_j^{pre} + a_1^{post} n_i^{post} + a_2^{corr} n_j^{pre} n_i^{post} + \ldots$

Associative memory



Prototype $\vec{p}^1$

Prototype $\vec{p}^2$

interactions

$$w_{ij} = \sum_m p_i^m p_j^m$$

Sum over all prototypes

dynamics

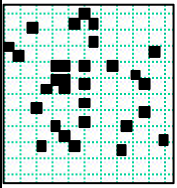
$$S_i\,(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j(t)\right]$$

Sum over all interactions with i

blackboard

Hopfield model

Associative memory



Prototype

$\vec{p}^1$

interactions

$$w_{ij} = \sum_m p_i^m p_j^m$$

Sum over all prototypes

This rule is optimal for **random** patterns

It does not work well for correlated patters

DEMO

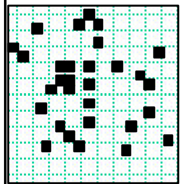
dynamics

$$S_i\ (t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j(t)\right]$$

Sum over all interactions with i

Hopfield model

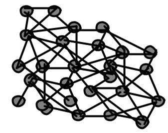
Associative memory



Prototype

$\vec{p}^1$

Interacting neurons



Finds the prototype with maximal overlap

$$m^m = \sum_j p_j^m S_j$$

Hopfield model

Computation
- without CPU,
- without explicit memory unit

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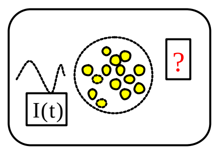
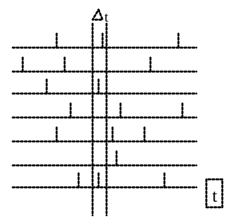
Populations of spiking neurons



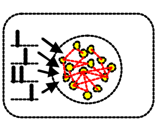
population dynamics?



population activity

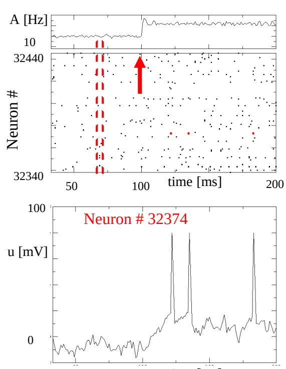
$$A(t) = \frac{n(t; t+\Delta t)}{N\Delta t}$$

Activity in a populations of neurons



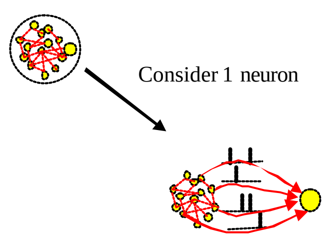
input {-low rate, -high rate}

Population
- 50 000 neurons
- 20 percent inhibitory
- randomly connected



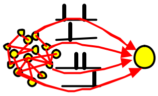
Homogeneous network (I&F)

Consider 1 neuron



Homogeneous network (I&F)

Assumption of Stochastic spike arrival:
network of exc. neurons,
total spike arrival rate *A(t)*



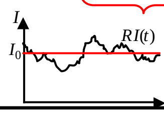
$$\tau \frac{d}{dt}u = -(u-u_{rest}) \quad + RI(t)$$ ← *Passive membrane*

Synaptic current pulses of shape α

$$\tau \frac{d}{dt}u = -(u-u_{rest}) - \sum_k \frac{J_0}{N} \sum_f \alpha(t-t_k^f)$$

EPSC

Blackboard : sum of EPSCs



Homogeneous network (I&F)

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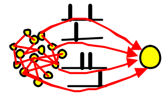
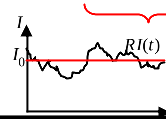


$$\tau \frac{d}{dt}u = -(u-u_{rest}) \quad + RI(t)$$ ← *Passive membrane*

$$RI(t) = R[I_o + I_{noise}]$$

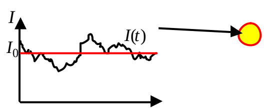


$I_0 = g\ A_0$

Population activity

Homogeneous network (I&F)

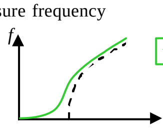
Step 1: Inject noise current



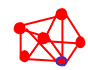
$$\tau \frac{d}{dt}u = -(u-u_{rest}) \quad + RI(t)$$

$$I(t) = [I_o + I_{noise}]$$

Measure frequency



Step 2 : consider 1 neuron in the network



fully connected

$$\tau \frac{d}{dt}u = -(u-u_{rest}) \quad + RI(t)$$ ← *integrate&fire*

Synaptic current pulses of shape α

$$\tau \frac{d}{dt}u_i = -(u_i-u_{rest}) - \sum_k w_{ij} \sum_f \alpha(t-t_k^f) \qquad w_{ij} = \frac{J_0}{N}$$

EPSC

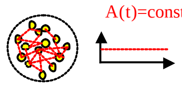
$$\sum_k \frac{J_0}{N} \sum_f \alpha(t-t_k^f) = R[I_0 + I_{noise}]$$

All neurons receive the same input ('mean field')

Step 3: assume Stationary State/Asynchronous State

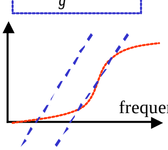
$$\sum_k \frac{J_0}{N} \sum_f \alpha(t-t_k^f) = R[I_0 + I_{noise}]$$

$$I_0 = g\ A_0 + I^{ext}$$



fully connected coupling J_0/N

$$A_0 = \frac{1}{g}\left[I_0 - I^{ext}\right]$$



typical mean field (Curie Weiss)

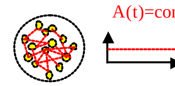
Blackboard

Step 4: close equation– calculate A_0

Step 3: assume Stationary State/Asynchronous State

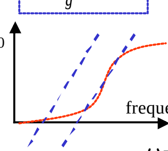
$$\sum_k \frac{J_0}{N} \sum_f \alpha(t-t_k^f) = R[I_0 + I_{noise}]$$

$$I_0 = g\ A_0 + I^{ext}$$



fully connected coupling J_0/N

$$A_0 = \frac{1}{g}\left[I_0 - I^{ext}\right]$$



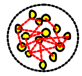
typical mean field (Curie Weiss)

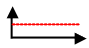
$$f = \langle s \rangle^{-1} = \left[\int_0^\infty P_I(\hat{t}+s\,|\,\hat{t})\, ds\right]^{-1}$$

Step 4: close equation– calculate A_0

Random Connectivity/Asynchronous State

C inputs per neuron $w_{ij} = \begin{cases} \frac{J_0}{C} \\ 0 \end{cases}$





randomly connected

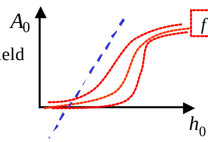
mean drive

$I_0 = g\, A_0 + I^{ext}$

variance

$s^2 = b\, A_0$

improved mean field



$f = g(I_0, s)$

Analogous for column of 1 exc. + 1 inhib. Pop.

frequency (single neuron) $f = \langle s \rangle^{-1} = \left[\int_0^\infty P_{I,s}(\hat{t} + s \mid \hat{t})\, ds \right]^{-1} = g(h_0, s)$

(Amit&Brunel 1997, Brunel 2000)

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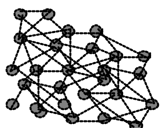
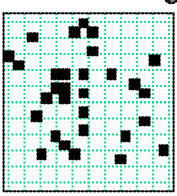
- -Associative Memory
- -Dense networks (mean-field)
- → -Associative memory in neural populations

Back to Associative memory

Interacting neurons

- -Possible with spiking neurons
- -Calculation: mean-field
- -Prototypes = random patterns



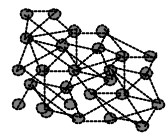


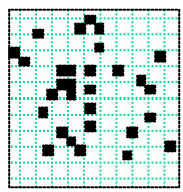
Computation
- without CPU,
- without explicit memory unit

Associative memory - simple model

Interacting neurons

- -Rate model
- -Calculation: mean-field
- -Prototypes = random patterns





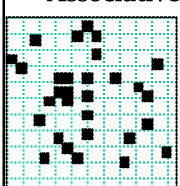
rate model

$f = g(I_0)$

$f_i = g(\sum_j w_{ij} f_j)$

$w_{ij} = \sum_m p_i^m p_j^m$

Associative memory



Prototype $\vec{p}^1$

For comparison with Spin S_j=+/-1

$m^m = \sum_j p_j^m S_j$

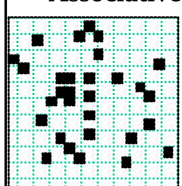
Task: Find the prototype with maximal overlap

$m^m = \sum_j p_j^m (f_j - J)$

+/-1

Blackboard

Associative memory



Prototype $\vec{p}^1$

$f = g(I_0)$

$f_i = g(\sum_j w_{ij} f_j)$

$w_{ij} = \sum_m p_i^m p_j^m$

$f_i = g(\sum_j \sum_m p_i^m p_j^m f_j)$

$f_i = g(\sum_m p_i^m \sum_j p_j^m f_j)$

$f_i = g(\sum_m p_i^m m^m)$

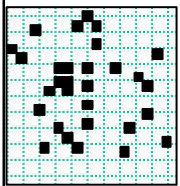
Blackboard

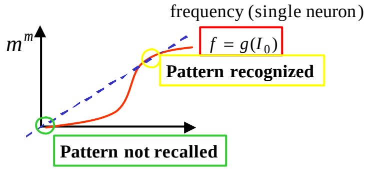
overlap

$m^m = \sum_j p_j^m (f_j - J) = \sum_j p_j^m f_j - \sum_j p_j^m J$

+/-1 with prob. 0.5 +/-1

Associative memory





Prototype

$\vec{p}^1$

Task: Find the prototype with maximal overlap

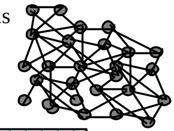
$$m^m = \sum_j p_j^m (f_j - J)$$

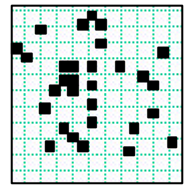
+/-1

Conclusion - Associative memory

Interacting neurons

- -Possible with spiking neurons
- -Calculation: mean-field
- -Prototypes = random patterns





Computation

- without CPU,
- without explicit memory unit

The end