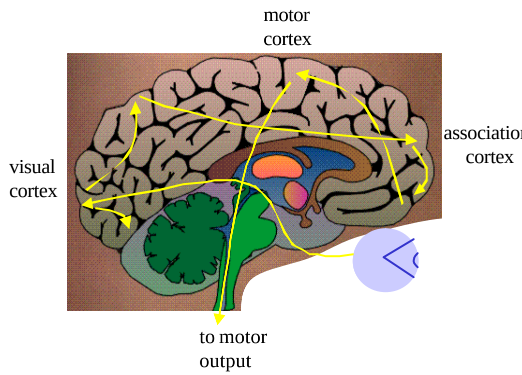
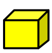
Introduction to computational neuroscience.
Doctoral school Oct-Nov 2006

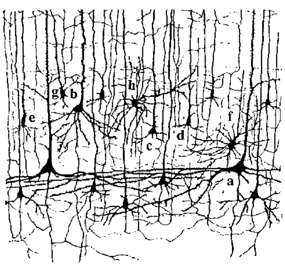
Wulfram Gerstner
http://diwww.epfl.ch/w3mantra/

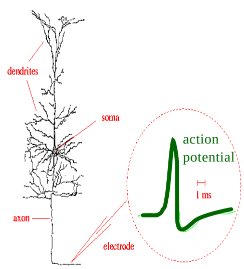




1mm

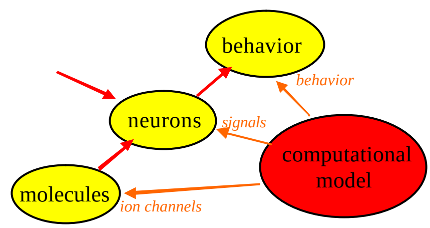
10 000 neurons
3 km wires

Signal:
action potential (spike)




## Computational Neuroscience

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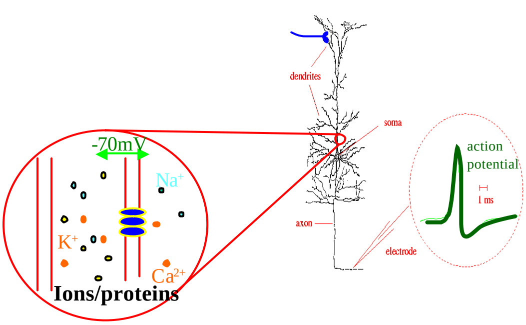
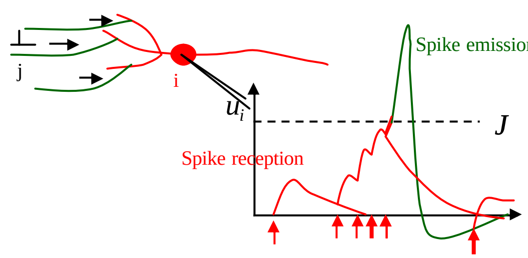
## Hodgkin-Huxley type models

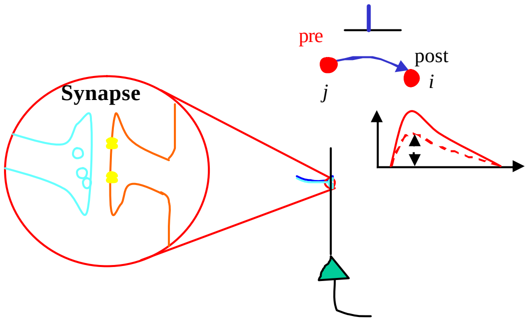



## Integrate-and-fire type models

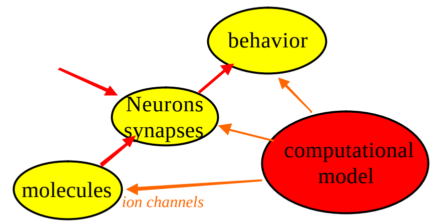



-spikes are events
-threshold
-spike/reset/refractoriness

## Models of synaptic Plasticity




## Computational Neuroscience

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## Introduction to computational neuroscience

Lecture 1: Passive membrane and Integrate-and-Fire model

Lecture 2: Hodgkin-Huxley models (detailed models)

Lecture 3: Two-dimensional models (FitzHugh Nagumo)

Lecture 4: synaptic plasticity

Lecture 5: noise, network dynamics, associative memory

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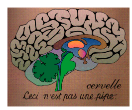
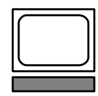
## *Background: What is brain-style computation?*

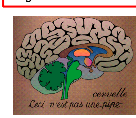


Brain

Computer

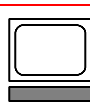


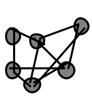
## Systems for computing and information processing



Brain

Computer

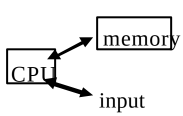




Distributed architecture

($10^{10}$ proc. Elements/neurons)

No separation of processing and memory

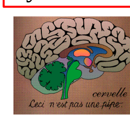



Von Neumann architecture

1 CPU

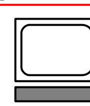
($10^{10}$ transistors)

## Systems for computing and information processing



Brain

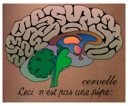
Computer



Tasks:

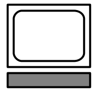
| Brain | Tasks | Computer |
|---|---|---|
| slow | Mathematical $\sqrt{5}\cos\left(\frac{7p}{5}\right)$ | fast |
| fast | Real world<br>E.g. complex scenes | slow |

## Systems for computing and information processing




Brain

Computer



Where is the program?

Clear separation:
software (pogram)/hardware

Where is the memory?

Clear separation:
memory/processing

In the synaptic connections

## Introduction to computational neuroscience

Lecture 1: Passive membrane and Integrate-and-Fire model

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Lecture 3: Two-dimensional models (FitzHugh Nagumo)

Lecture 4: synaptic plasticity

Lecture 5: noise, network dynamics, associative memory


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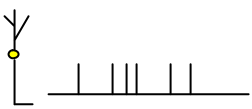
## Why spiking neuron models?

Spikes versus rates

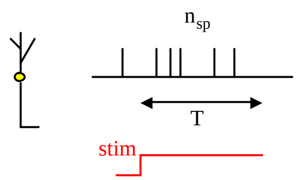


## The Problem of Neuronal Coding

Book: Spiking Neuron Models
Chapter 1.4



## The Problem of Neuronal Coding




Rate $n = \frac{n_{sp}(t;t+T)}{T}$

Rate defined as temporal average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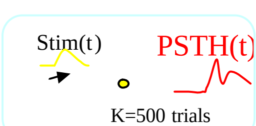
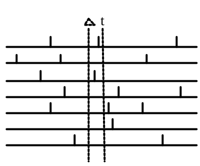
## The Problem of Neuronal Coding

Rate defined as average over stimulus repetitions

Peri-Stimulus Time Histogram




$$PSTH(t) = \frac{n(t;t+\Delta t)}{K\,\Delta t}$$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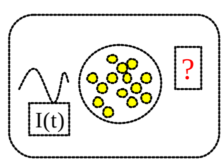
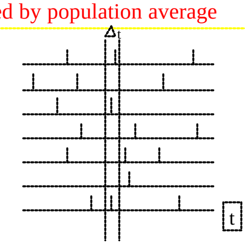
## The problem of neural coding:

population activity - rate defined by population average



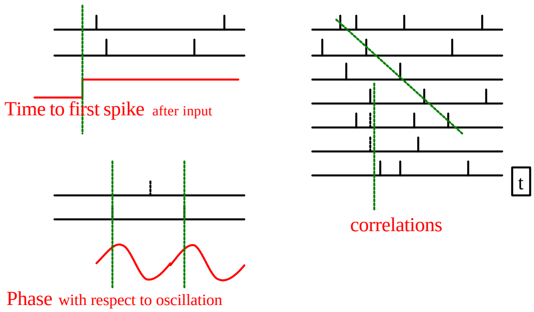

population dynamics?




population activity

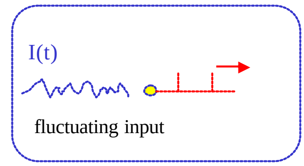
$$A(t) = \frac{n(t;t+\Delta t)}{N\Delta t}$$

## The problem of neural coding:
temporal codes




## Relevance of temporal aspects -1

### Reverse Correlations





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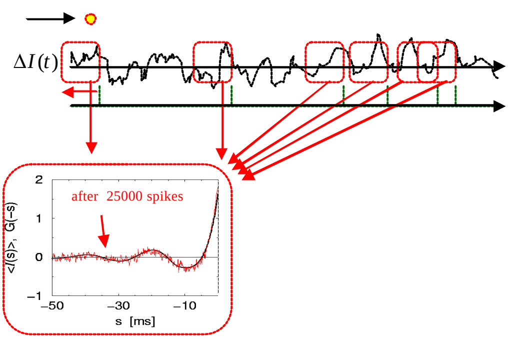
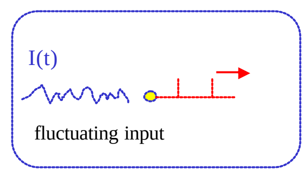
## Reverse-Correlation Experiments (simulations)




## Relevance of temporal aspects -2

### Stimulus Reconstruction





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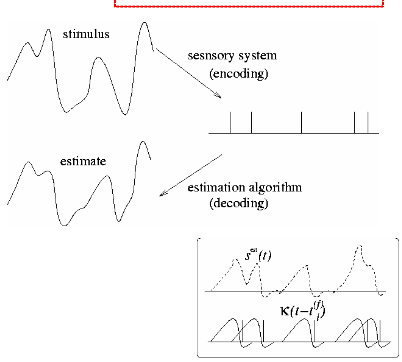
## Stimulus Reconstruction

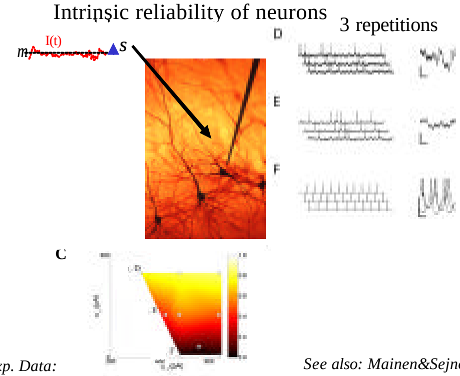



*Bialek et al*

## Relevance of temporal aspects -3

Intrinsic reliability of neurons

3 repetitions




*Exp. Data:*
*A. Rauch, H.Lüscher,* U. Berne

*See also: Mainen&Sejnowski*

If we want to avoid prior
assumptions about neural coding,
we need to model neurons
on the level of action potentials:

spiking neuron models

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## Introduction to computational neuroscience

**Lecture 1:**

**Passive membrane and Integrate -and-Fire model**

-The problem of neural coding
-The passive membrae
-Leaky ingreate-and-fire model
-Generalized integrate-and-fire model
-Quality of integrate-and-fire models
-Coding revisited

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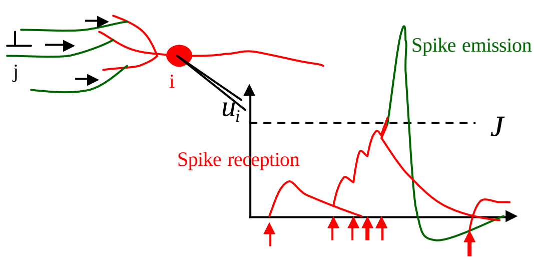
## Integrate-and-fire type models




-spikes are events
-threshold
-spike/reset/refractoriness

**Subthreshold regime**
- linear
- passive membrane

---

## Introduction to computational neuroscience

**Lecture 1:**

**Passive membrane and Integrate -and-Fire model**

-The problem of neural coding
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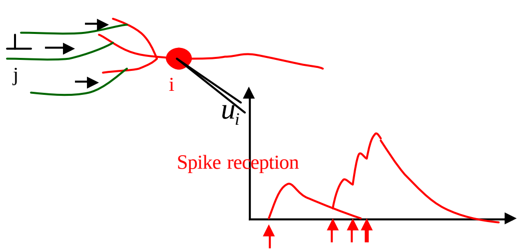
## Subthreshold regime – passive membrane




**Subthreshold regime**
- linear
- passive membrane

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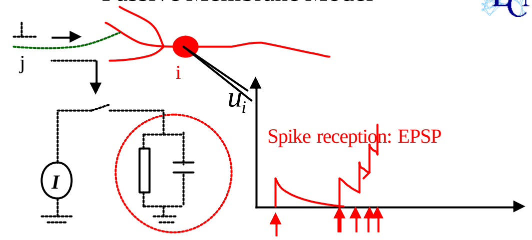
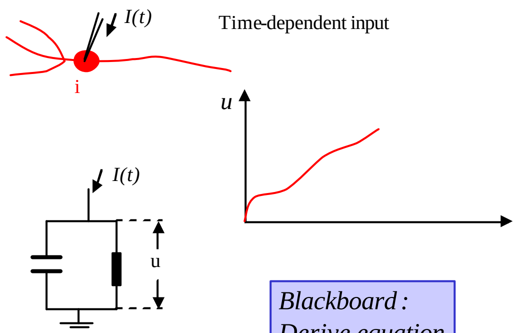
## Passive Membrane Model

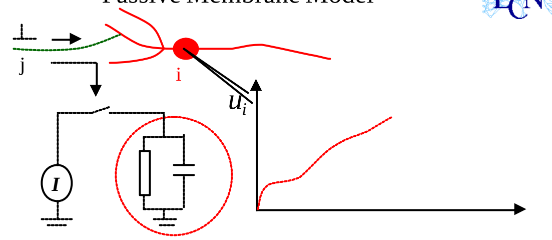


## Passive Membrane Model




*Blackboard: Derive equation*

## Passive Membrane Model

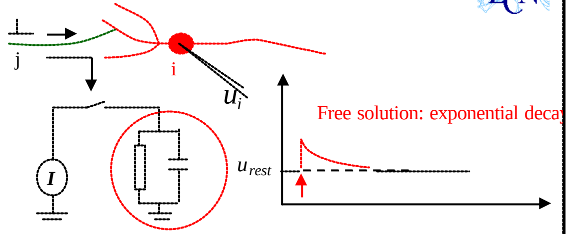



$$t \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

$$t \cdot \frac{d}{dt} V = -V + RI(t); \quad V = (u - u_{rest})$$

*Blackboard: Voltage scale*

## Passive Membrane



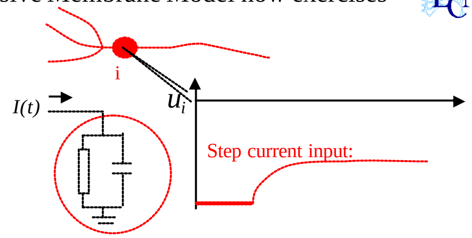

$$t \cdot \frac{d}{dt} u = -(u - u_{rest});$$

Start at $t_0$ with $u_0$

$$u(t) = u_{rest} + (u_0 - u_{rest}) \exp\left(-\frac{t - t_0}{t}\right)$$

*Blackboard: free solution*

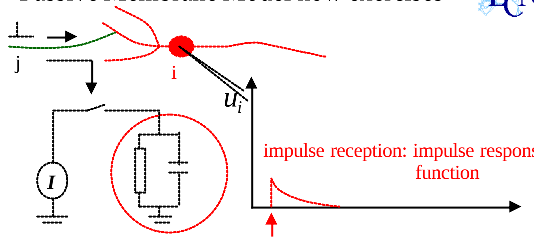
## Passive Membrane Model-now exercises




$$t \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

linear

## Passive Membrane Model-now exercises




$$t \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

linear

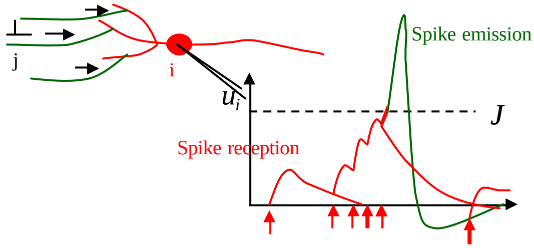
# Chapter 4: Formal Spiking models

## Integrate-and-Fire model

BOOK: Spiking Neuron Models,
W. Gerstner and W. Kistler
Cambridge University Press, 2002

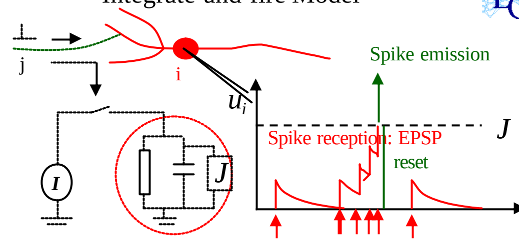
**Chapter 4**

## Integrate-and-fire type models




-spikes are events
-threshold
-spike/reset/refractoriness

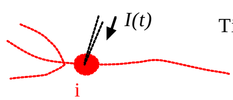
## Integrate-and-fire Model



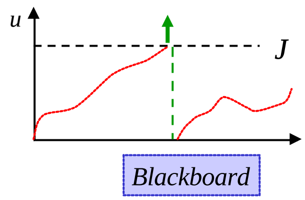

$$t \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$ linear

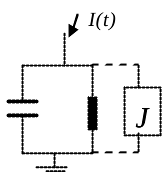
$u_i(t) = J \Rightarrow$ Fire+reset threshold

## Integrate-and-fire type models




Time-dependent input



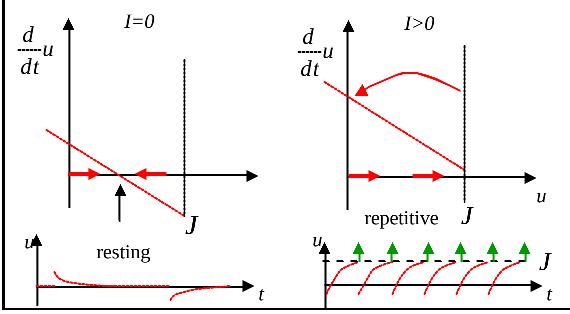




-spikes are events
-threshold
-spike/reset/refractoriness

## Integrate-and-fire Models

$$t \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$ **LIF**

If firing: $u \rightarrow u_{reset}$



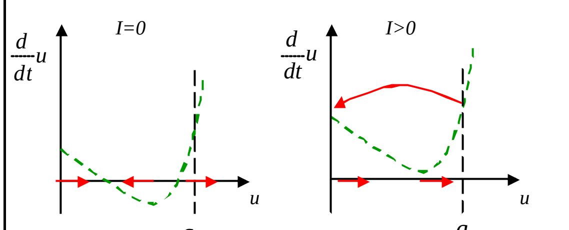

## Integrate-and-fire Models

$$t \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$ **LIF**

$$t \cdot \frac{d}{dt} u = F(u) + RI(t)$$ **NLIF**

If firing: $u \rightarrow u_{reset}$

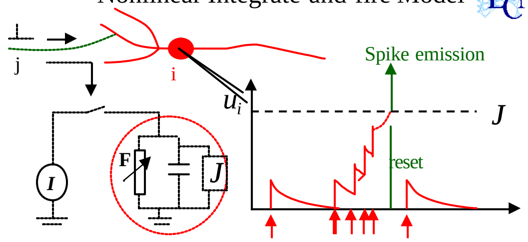
## Nonlinear Integrate-and-fire Model




$$t \cdot \frac{d}{dt} u = F(u) + RI(t)$$ NONlinear

$u(t) = q \Rightarrow$ Fire+reset threshold

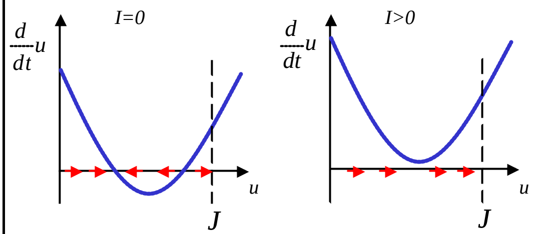
## Nonlinear Integrate-and-fire Model




$$t \cdot \frac{d}{dt} u = F(u) + RI(t)$$ NONlinear

$u_i(t) = J \Rightarrow$ Fire+reset threshold

## Nonlinear Integrate-and-fire Model



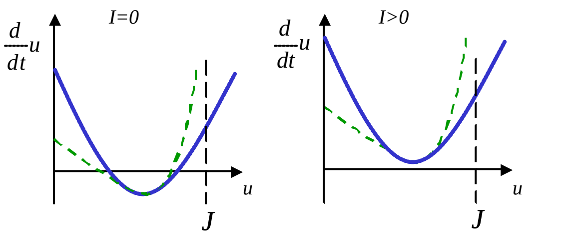

$$t \cdot \frac{d}{dt} u = F(u) + RI(t)$$ NONlinear

$u(t) = J \Rightarrow$ Fire+reset threshold

Quadratic I&F:

$F(u) = c_2 (u - c_1)^2 + c_0$

## Nonlinear Integrate-and-fire Model




$$t \cdot \frac{d}{dt} u = F(u) + RI(t)$$

$u(t) = J \Rightarrow$ Fire+reset

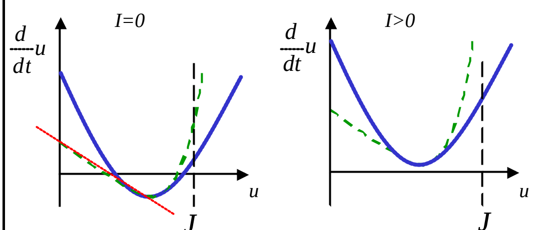
Quadratic I&F:

$F(u) = c_2 (u - c_1)^2 + c_0$

exponential I&F:

$F(u) = -(u - u_{rest}) + c_0 \exp(u - J)$

## Nonlinear Integrate-and-fire Model




$$t \cdot \frac{d}{dt} \mathbf{u} = F(u) - u_{rest} + RI(t)$$ NONlinear

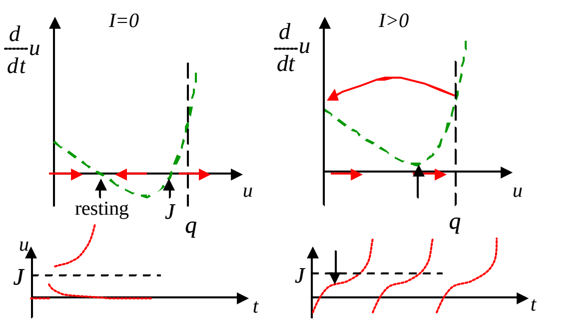
$u(t) = J \Rightarrow$ Fire+reset threshold

exponential I&F:

$F(u) = -(u - u_{rest}) + c_0 \exp(u - J)$

## Nonlinear Integrate-and-fire Model

**Where is the firing threshold?**




$$t \cdot \frac{d}{dt} u = F(u) + RI(t)$$

## Integrate-and Fire type models

**Where is the firing threshold?**

**Leaky integrate-and-fire (LIF)**

**Strict voltage threshold**

- by construction
- spike threshold = reset condition

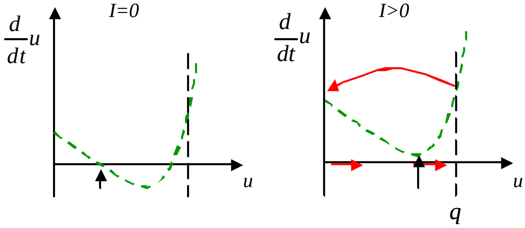
**Nonlinear integrate-and-fire (eIF)**

**There is no strict firing threshold**

- firing depends on input
- exact reset condition of minor relevance

## Nonlinear Integrate-and-fire Model

**Where is the firing threshold?**




**Exercises 2+3 now!**

$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

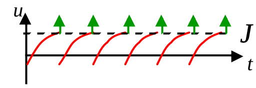
## Leaky Integrate-and-fire Model

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI_0$$

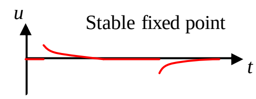
**LIF**

If firing: $u \rightarrow u_{reset}$

repetitive




Stable fixed point




What is the firing rate?
f=g(I)
**Exercise 2+3 now!**

## Integrate-and Fire type models

**Leaky integrate-and-fire (LIF)**

**Strict voltage threshold**

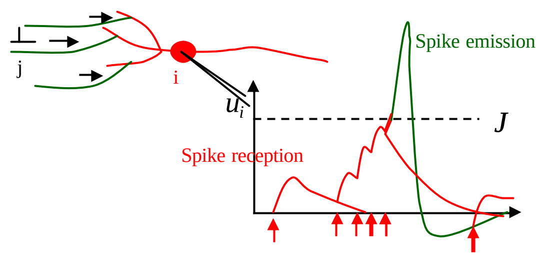
**Nonlinear integrate-and-fire**

**no strict firing threshold**

**Spike Response Model (SRM)**

**Includes refractoriness**

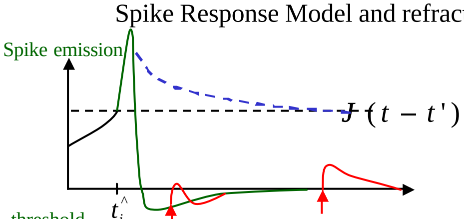
## Integrate-and-fire type models




-spikes are events
-threshold
-spike/reset/refractoriness

**Subthreshold regime**
- linear
- passive membrane

## Spike Response Model and refractoriness




$u(t) = J$

$\frac{du(t)}{dt} \geq 0$ → Firing: $t' = t$

## Integrate&Fire:
## Where is the firing threshold?

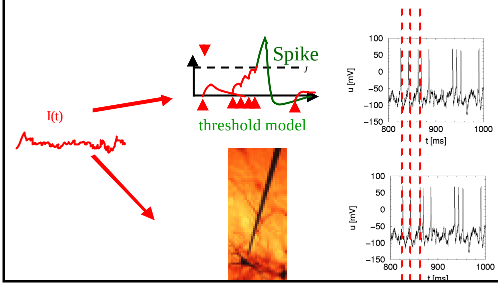
1: Introduction

2: Integrate-and-Fire (and generalisations)

3. Quality of Integrate-and-Fire models


Wulfram Gerstner
http://diwww.epfl.ch/w3mantra/

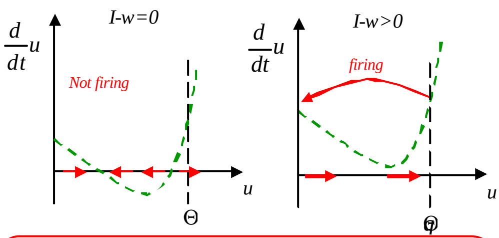
## Validation of neuron models




## Real neuron vs. Nonlinear I&F

*Cortical layer V pyramidal neuron*

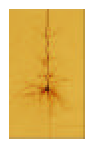
## exponential Integrate-and-fire Model

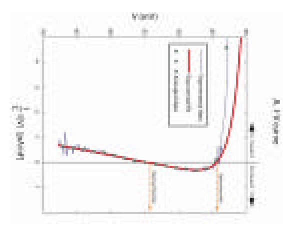



$$\tau \cdot \frac{d}{dt}u = u - u_{rest} + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) + R[I(t)]$$

$u(t)=\Theta \Rightarrow$ Fire --- reset of $u$

## Measurement of the dynamic *IV* curve



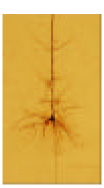


voltage

Experiments:
*Sandrine Lefort*
*Carl Petersen*

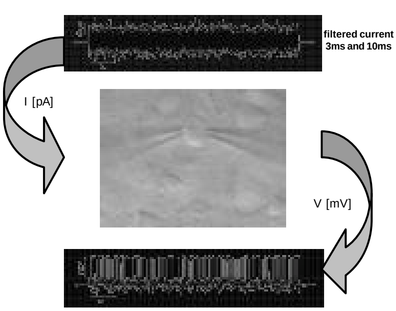
Theory:
*Laurent Badel*
*Magnus Richardson*

## Experimental method

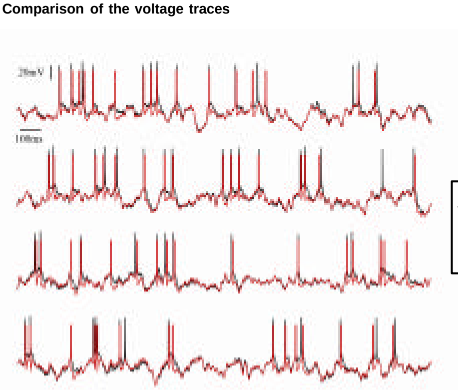


Layer 5 pyramidal

somatosensory cortex



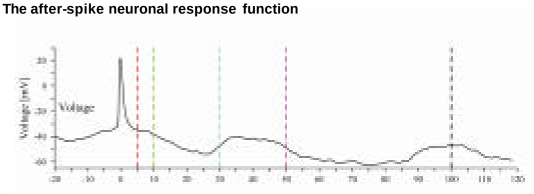

## Comparison of the voltage traces



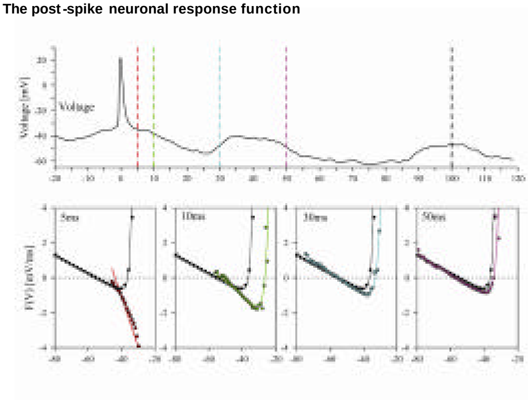
Spikes predicted with 5ms accuracy

model
77% of spikes

## The after-spike neuronal response function

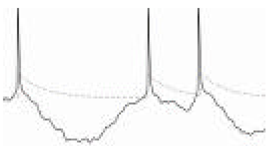



## The post-spike neuronal response function




## EIF with a dynamic threshold, resting potential and time constant

$$\frac{dV}{dt}=\frac{(E_L-V)}{t_L}+\frac{\Delta_T}{t_L}\exp\left(\frac{V-V_T}{\Delta_T}\right)$$



$$\frac{dV_T}{dt}=\frac{(V_{T\infty}-V_T)}{t_{V_T}}$$

*Fuortes and Mantegazzini (1962)*

$$\frac{dE_L}{dt}=\frac{(E_{L\infty}-E_L)}{t_{E_L}}$$

*Jolivet et al (2003)*
*Rauch et al (2003)*
*Brette and Gerstner (2005)*

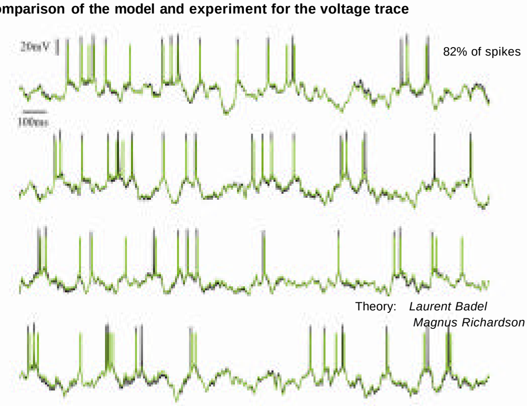
With a high reset:

$$V_{re}=-39\text{mV} \quad > \quad V_{T\infty}=-41\text{mV}$$

$$\frac{dt_L}{dt}=\frac{(t_{L\infty}-t_L)}{t_{t_L}}$$

*Wehmeier et al (1989)*

## Comparison of the model and experiment for the voltage trace

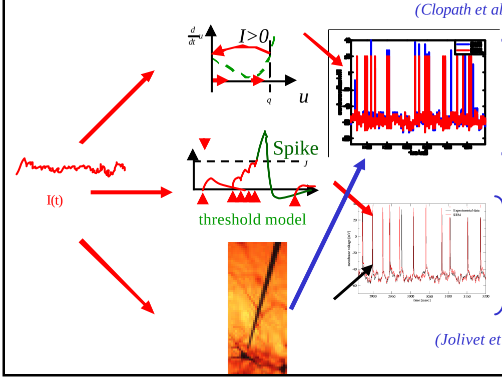



82% of spikes

Theory: *Laurent Badel*
*Magnus Richardson*

# Validation of neuron models

*(Clopath et al. 2006)*




75 %
of spikes
correct
(+/-2ms)

75 %
of spikes
correct
(+/-2ms)

*(Jolivet et al. 2004)*

## Integrate&Fire:
## Where is the firing threshold?

√ 1: Introduction

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