Detailed neuron models:
Hodgkin-Huxeley model

1: Introduction to Hodgkin-Huxley models

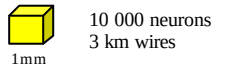
2: threshold in the Hodgkin-Huxeley model

3. Comparison to Integrate-and-Fire model

4. Synaptic input (conductance input)

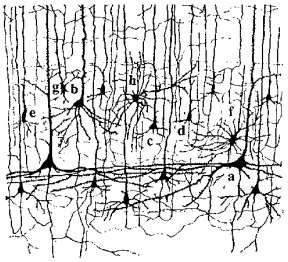
5. Dendrite model and cable equation

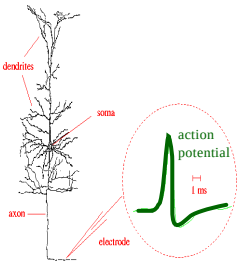
Wulfram Gerstner
http://diwww.epfl.ch/w3mantra/



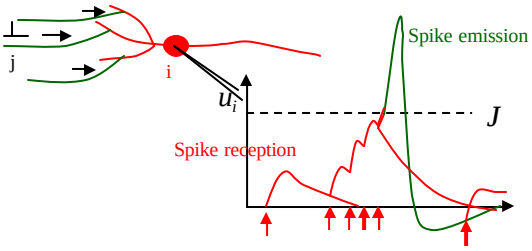
10 000 neurons
3 km wires

Signal:
action potential (spike)



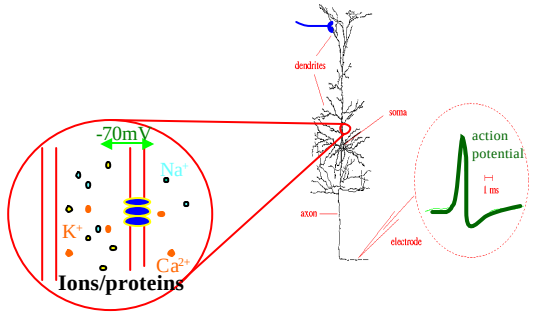


Integrate-and-fire type models

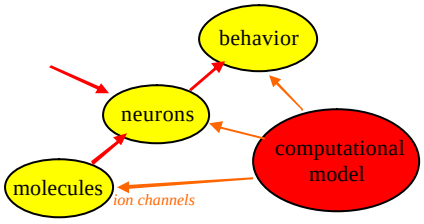


-spikes are events
-threshold
-spike/reset/refractoriness

Hodgkin-Huxley type models



Computational Neuroscience



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1: Introduction to Hodgkin-Huxley models

Wulfram Gerstner
http://diwww.epfl.ch/w3mantra/

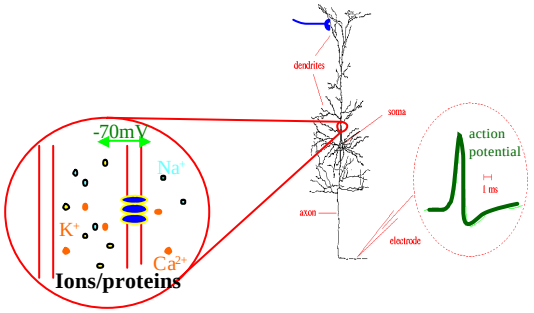
Chapter 2: Detailed neuron models

Hodgkin-Huxley model

BOOK: Spiking Neuron Models,
W. Gerstner and W. Kistler
Cambridge University Press, 2002

Chapter 2

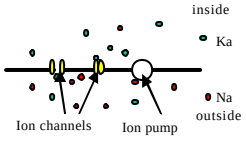
Biophysics of neurons

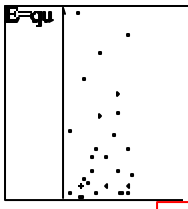


Hodgkin-Huxley Model

Ion density

$$n \propto e^{-\frac{E}{kT}}$$



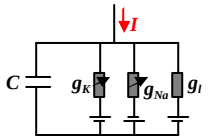


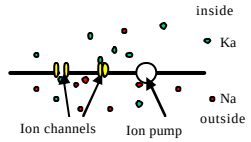
Reversal potential

Blackboard

Concentration difference ⇔ voltage difference

Hodgkin-Huxley Model



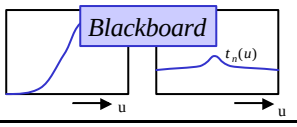


stimulus

$$C\frac{du}{dt} = \overbrace{-g_{Na}m^3h(u-E_{Na})}^{I_{Na}} \overbrace{-g_K n^4(u-E_K)}^{I_K} \overbrace{-g_l(u-E_l)}^{I_{leak}} + I(t)$$

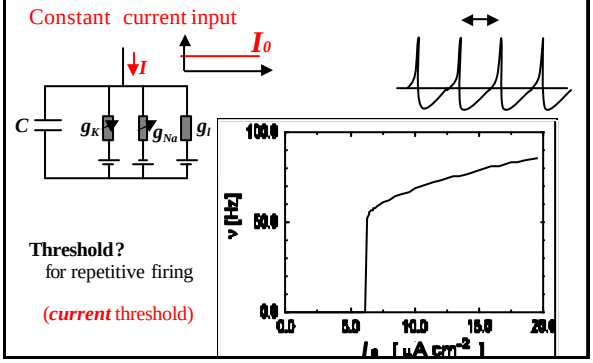
$$\frac{dm}{dt} = \frac{m_0(u) - m}{\tau_m(u)}$$

Blackboard



Hodgkin-Huxley Model

Constant current input

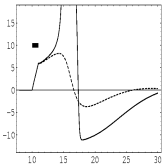


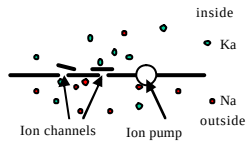
Threshold?
for repetitive firing

(***current*** threshold)

Hodgkin-Huxley Model

pulse input
$I(t)$





Threshold?

- AP if amplitude 7.0 units
- No AP if amplitude 6.9 units

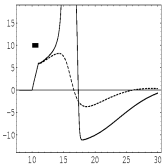
(pulse with 1ms duration)

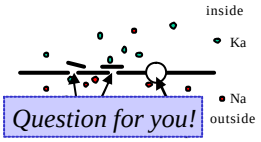
(and pulse with 0.5 ms duration?)

Hodgkin-Huxley Model

pulse input

$I(t)$

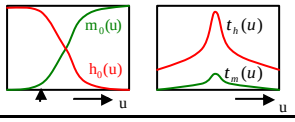




$$C\frac{du}{dt} = \overbrace{-g_{Na}m^3h(u-E_{Na})}^{I_{Na}} \overbrace{-g_K n^4(u-E_K)}^{I_K} \overbrace{-g_l(u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{t_m(u)}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{t_h(u)}$$



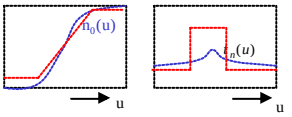
Exercise 1 now: dynamics of ion channels

$$C\frac{du}{dt} = -g_{Na}m^3h(u-E_{Na}) - g_K n^4(u-E_K) - g_l(u-E_l) + I(t)$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{t_h(u)}$$

$$\frac{dn}{dt} = -\frac{n-n_0(u)}{t_n(u)}$$

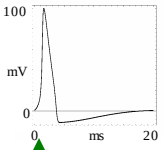
$$\frac{dm}{dt} = -\frac{m-m_0(u)}{t_m(u)}$$



Hodgkin-Huxley Model

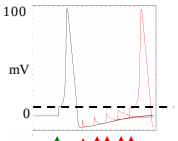
Where is the firing threshold?

Action potential



Strong stimulus

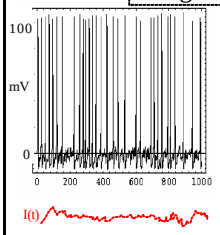
refractoriness



strong stimuli

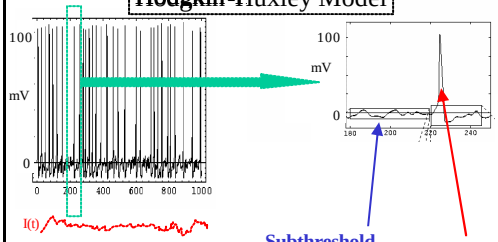
Strong stimulus

Hodgkin-Huxley Model



Stimulation with time-dependent input current

Hodgkin-Huxley Model



Subthreshold response

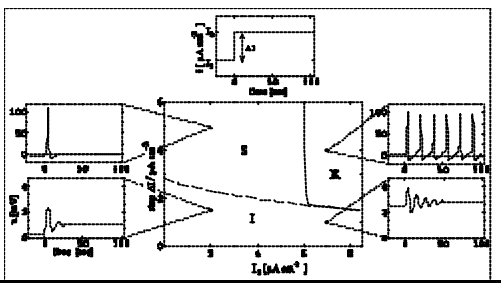
Spike

Hodgkin-Huxley Model

Step current input

I_2

ΔI



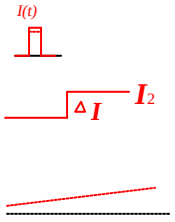
Hodgkin-Huxley Model

Where is the firing threshold?

pulse input

step input

ramp input



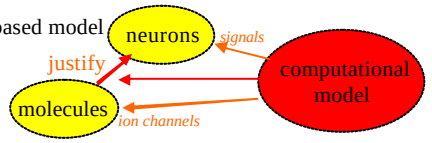
There is no threshold
- no current threshold
- no voltage threshold

'effective' threshold
- depends on typical input

$$C\frac{du}{dt} = -g_{Na}m^3h(u - E_{Na}) - \dots$$

Detour: from molecules to neuron models

Ion channel based model



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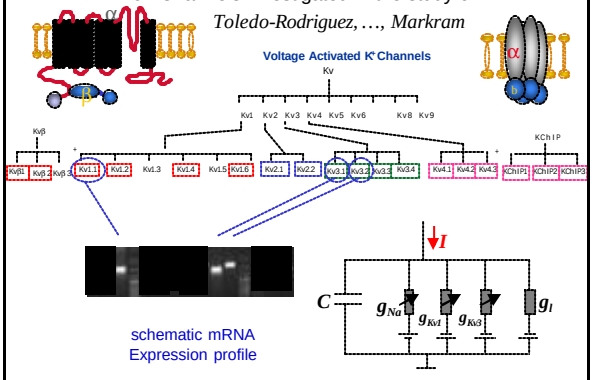
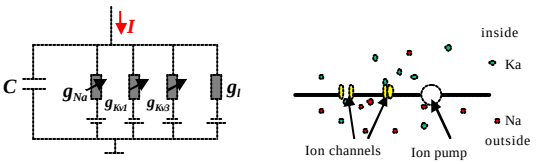
Laboratory of Computational Neuroscience, LCN, CH 1015 Lausanne

Ion Channels investigated in the study of

Toledo-Rodriguez, …, Markram



Model of fast spiking interneuron



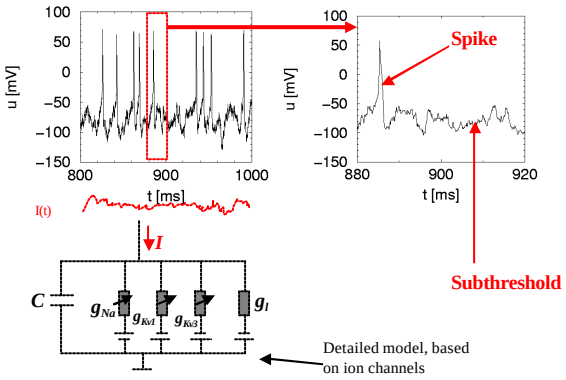
$$C\frac{du}{dt} = \overbrace{-g_{Na}m^3h(u-E_{Na})}^{I_{Na}} \overbrace{-g_{Kv1}n_{Kv1}^4(u-E_K)-g_{Kv3}n_{Kv3}^2(u-E_K)}^{I_K} \overbrace{-g_l(u-E_l)}^{I_{leak}} + I(t)$$

stimulus

$$\frac{dm}{dt} = \frac{m_\infty(u) - m}{\tau_m(u)}$$

Erisir et al, 1999
Hodgkin and Huxley, 1952

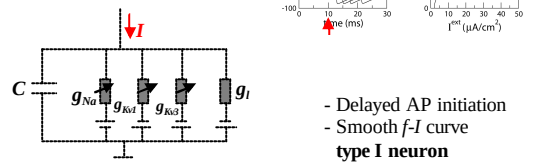
Model of fast spiking interneuron



Model of fast spiking interneuron

Current pulse

Detailed model, based on ion channels



- Delayed AP initiation
- Smooth *f-I* curve

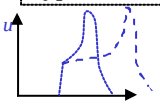
type I neuron

Type I and type II models

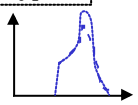
Response at firing threshold?

pulse input

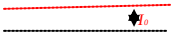
Type I

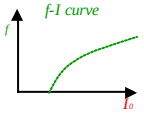


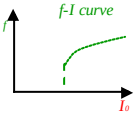
type II



ramp input/
constant input







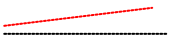
Hodgkin-Huxley type models

Where is the firing threshold?

pulse input

step input

ramp input

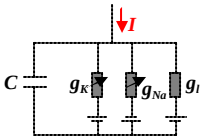


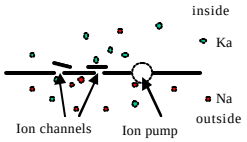
There is no threshold
- no current threshold
- no voltage threshold

'effective' threshold
- depends on typical input

threshold process not immediate
- delayed AP possible

Hodgkin-Huxley type models - **EXERCISES**

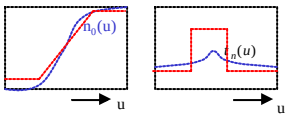




stimulus

$$C\frac{du}{dt} = -g_K n^4 (u - E_K) + I(t)$$

Exercises 2 now!



From Hodgkin-Huxeley to Integrate&Fire: Where is the firing threshold?

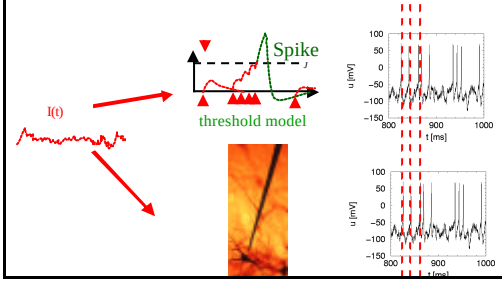
1: Introduction to Hodgkin-Huxley models

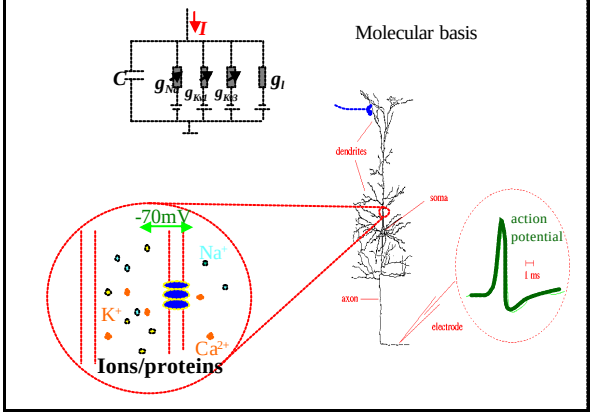
√ 2: threshold in the Hodgkin-Huxeley model

→ 3. Comparison to Integrate-and-Fire model

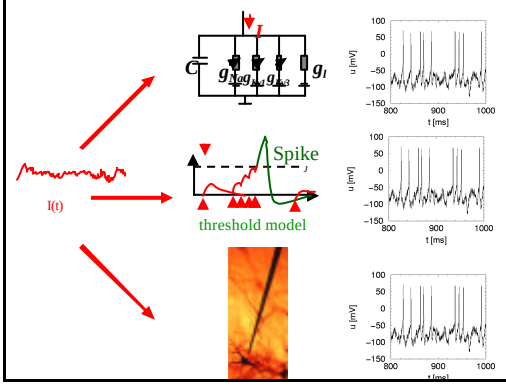
Wulfram Gerstner
http://diwww.epfl.ch/w3mantra/

Validation of neuron models



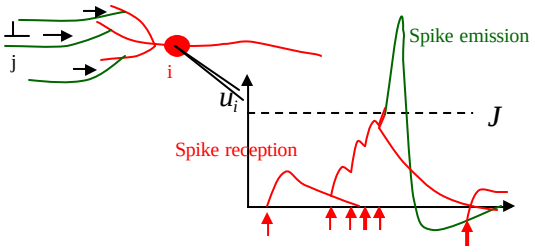


Validation of neuron models



Hodgkin-Huxley vs. SRM

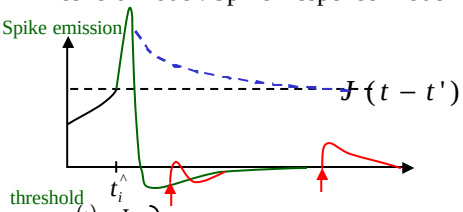
Threshold models: Integrate-and-fire type models



-spikes are events
-threshold
-spike/reset/refractoriness

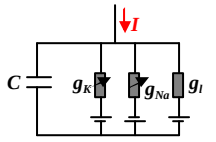
Subthreshold regime
- linear
- passive membrane

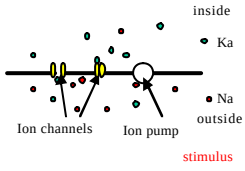
Threshold model: Spike Response Model



$$u(t) = J$$
$$\frac{du(t)}{dt} \geq 0 \rightarrow \text{Firing: } t' = t$$

Hodgkin-Huxley Model

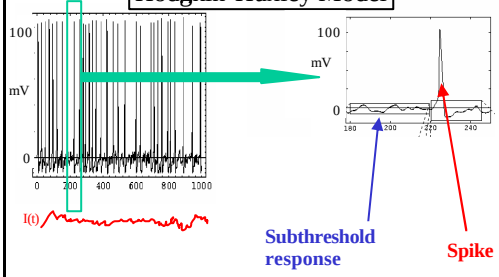




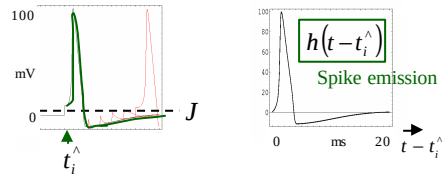
$$C\frac{du}{dt} = \overbrace{g_{Na}m^3h(u-E_{Na})}^{I_{Na}} + \overbrace{g_K n^4(u-E_K)}^{I_K} + \overbrace{g_l(u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = \frac{m_0(u) - m}{\tau_m(u)}$$

Hodgkin-Huxley Model



Threshold model adapted to Hodgkin-Huxley Model



threshold

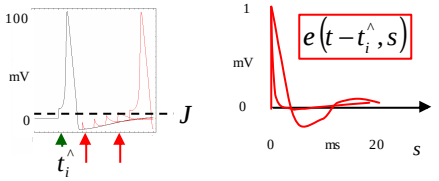
$$u_i(t) = J \quad \frac{du_i(t)}{dt} \geq 0 \rightarrow \text{Firing: } t_i^\wedge = t$$

Full Spike Response Model

potential

$$u_i(t) = \underbrace{h(t-t_i^\wedge)}_{\text{Last spike of i}} + \int_0^\infty \underbrace{e(t-t_i^\wedge, s)}_{} I(t-s)ds + \ldots$$

Hodgkin-Huxley Model



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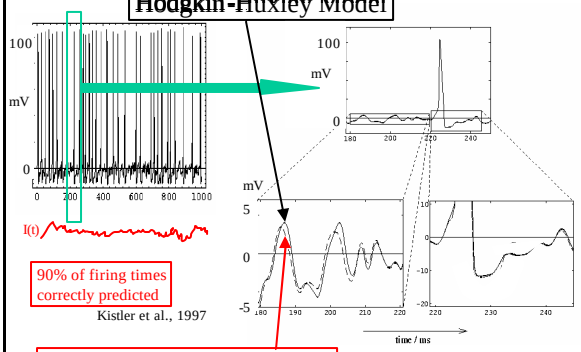
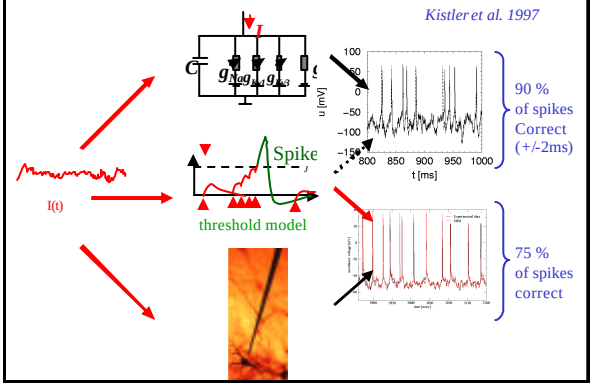


Validation of neuron models



Hodgkin-Huxley Model

Where is the firing threshold?

There is no threshold

- no current threshold
- no voltage threshold

'effective' threshold
- depends on typical input

BUT:

- threshold model is a good approximation
- (voltage threshold)

Detailed neuron models:
Hodgkin-Huxeley model

1: Introduction to Hodgkin-Huxley models

2: threshold in the Hodgkin-Huxeley model

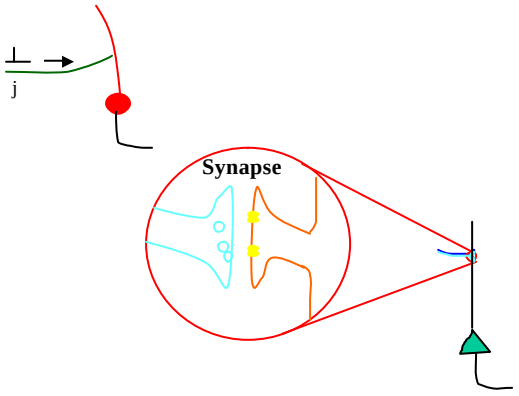
√ 3. Comparison to Integrate-and-Fire model

→ 4. Synaptic input (conductance input)

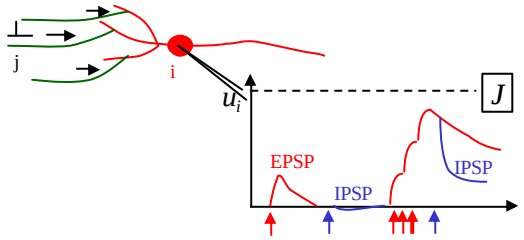
5. Dendrite model and cable equation

Wulfram Gerstner
http://diwww.epfl.ch/w3mantra/

Model of Synaptic input: conductance change



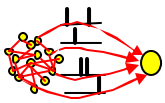
Model of Synaptic input: conductance change



$$t \cdot \frac{d}{dt}u = -(u-u_{rest}) - \sum_{k,f} g_e(t-t_k^f)[u-E_e] - \sum_{k',f'} g_i(t-t_{k'}^{f'})[u-E_i]$$

EPSC IPSC

Synaptic input: conductance change



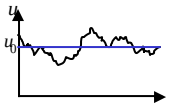
Stochastic spike arrival:
excitation, total rate R_e
inhibition, total rate R_i

conductance pulses

$$t \frac{d}{dt}u = -(u-u_{rest}) - \sum_{k,f} g_e(t-t_k^f)[u-E_e] - \sum_{k',f'} g_i(t-t_{k'}^{f'})[u-E_i]$$

$$t^{eff}(R_e,R_i)\frac{d}{dt}u = -(u-u_0) - \sum_{k,f} g_e(t-t_k^f)[u_0-E_e] - \sum_{k',f'} g_i(t-t_{k'}^{f'})[u_0-E_i] + \text{corr}$$

EPSC IPSC

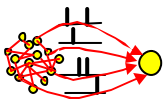


current pulses

effective current model
(effective time constant approximation)

Exercise:

-calculate correction term
-calculate effective time constant

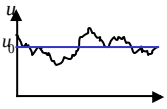


conductance pulses

$$t \frac{d}{dt}u = -(u-u_{rest}) - \sum_{k,f} g_e(t-t_k^f)[u-E_e] - \sum_{k',f'} g_i(t-t_{k'}^{f'})[u-E_i]$$

$$t^{eff}\frac{d}{dt}u = -(u-u_0) - \sum_{k,f} g_e(t-t_k^f)[u_0-E_e] - \sum_{k',f'} g_i(t-t_{k'}^{f'})[u_0-E_i] + \text{corr}$$

EPSC IPSC



current pulses

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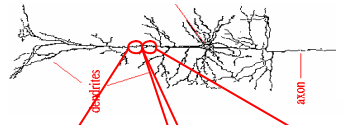
Spatial structure, Dendrite model, axon model



Blackboard