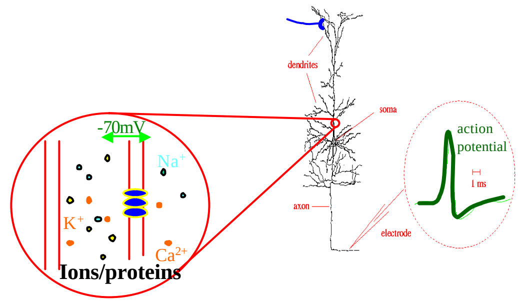
Lecture 3: Two-dimensional neuron models

BOOK: Spiking Neuron Models,

W. Gerstner and W. Kistler
Cambridge University Press, 2002

Chapter 3

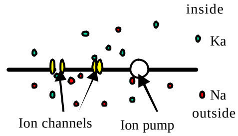
Biophysics of neurons

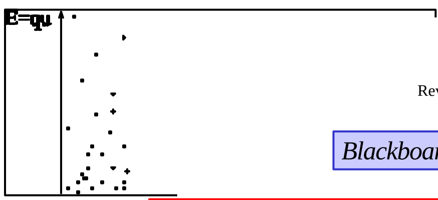


Hodgkin-Huxley Model

Ion density

$$n \propto e^{-\frac{E}{kT}}$$



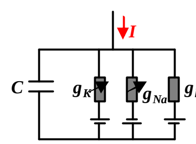


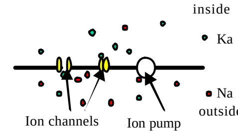
Reversal potential

Blackboard

Concentration difference ⇔ voltage difference

Hodgkin-Huxley Model



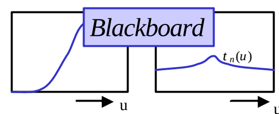


stimulus

$$C\frac{du}{dt} = \underbrace{-g_{Na}m^3h(u-E_{Na})}_{I_{Na}}\underbrace{-g_K n^4(u-E_K)}_{I_K}\underbrace{-g_l(u-E_l)}_{I_{leak}}+I(t)$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)} \qquad \frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

Blackboard



Two-dimensional neuron models

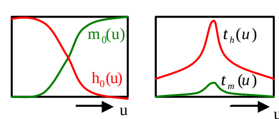
-Reduction of Hodgkin-Huxley
-step 1: separation of time scales

Reduction of Hodgkin-Huxley Model

stimulus

$$C\frac{du}{dt} = \underbrace{g_{Na}m^3h(u-E_{Na})}_{I_{Na}}+\underbrace{g_K n^4(u-E_K)}_{I_K}+\underbrace{g_l(u-E_l)}_{I_{leak}}+I(t)$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)} \qquad \frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$



1) dynamics of m is fast $\longrightarrow$ $m(t) = m_0(u(t))$

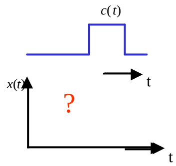
Exercise: separation of time scales

stimulus

$$C\frac{du}{dt} = \underbrace{g_{Na}m^3h(u-E_{Na})}_{I_{Na}} + \underbrace{g_K n^4(u-E_K)}_{I_K} + \underbrace{g_l(u-E_l)}_{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

$$\frac{dx}{dt} = -\frac{x-c(t)}{\tau}$$



$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m}$$

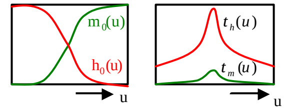
$$\frac{du}{dt} = f(u) - m$$

Reduction of Hodgkin-Huxley Model

stimulus

$$C\frac{du}{dt} = \underbrace{g_{Na}m^3h(u-E_{Na})}_{I_{Na}} + \underbrace{g_K n^4(u-E_K)}_{I_K} + \underbrace{g_l(u-E_l)}_{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)} \qquad \frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$



1) dynamics of m is fast → $m(t) = m_0(u(t))$

2) dynamics of h and n is similar → $1-h(t) = a\,n(t)$

Blackboard

Reduction of Hodgkin-Huxley Model

stimulus

$$C\frac{du}{dt} = \underbrace{g_{Na}m_0(u)^3(1-w)(u-E_{Na})}_{I_{Na}} + \underbrace{g_K\left(\frac{w}{a}\right)^4(u-E_K)}_{I_K} + \underbrace{g_l(u-E_l)}_{I_{leak}} + I(t)$$

$$\frac{dw}{dt} = -\frac{w-w_0(u)}{\tau_w(u)}$$

1) dynamics of m is fast → $m(t) = m_0(u(t))$

2) dynamics of h and n is similar → $\underbrace{1-h(t)}_{w} = \underbrace{a\,n(t)}_{w}$

Blackboard

Two-dimensional neuron models

-Reduction of Hodgkin-Huxley

-step 1: separation of time scales

-step 2: effective variable w

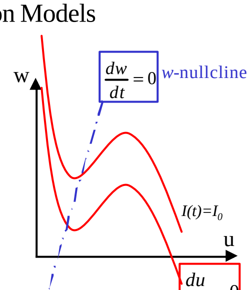
- Phase space analysis

Reduction of Hodgkin-Huxley Model: 2 dimensional Neuron Models

stimulus

$$\tau\frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w\frac{dw}{dt} = G(u,w)$$

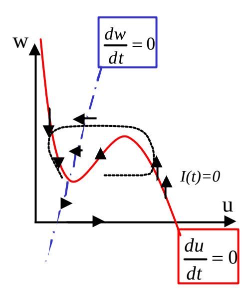


FitzHugh-Nagumo Model 2-dimensional Neuron Models

stimulus

$$\tau\frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w\frac{dw}{dt} = G(u,w)$$



FitzHugh Nagumo Model – limit cycle

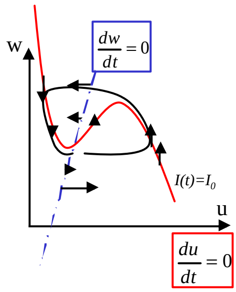
stimulus

$$t\frac{du}{dt} = F(u,w) + I(t)$$

$$t_w \frac{dw}{dt} = G(u,w)$$

-unstable fixed point
-closed boundary
 with arrows pointing inside

→ limit cycle



limit cycle

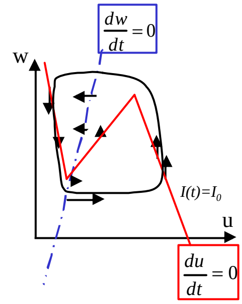
Stability of fixed points -Exercise

stimulus

$$t\frac{du}{dt} = au + bw + I_0$$

$$t_w \frac{dw}{dt} = c\,u + d\,w$$

Exercise:
Calculate stability for parameters a,b,c,d

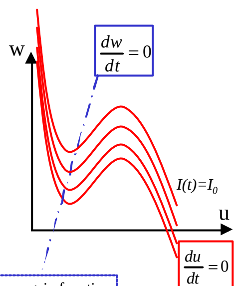


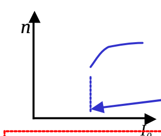
type II Model – constant input

stimulus

$$t\frac{du}{dt} = F(u,w) + I(t)$$

$$t_w \frac{dw}{dt} = G(u,w)$$





Stability lost → oscillation with finite frequency

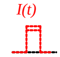
type II Model - pulse input

stimulus

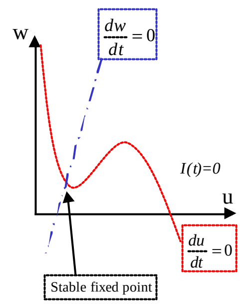
$$t\frac{du}{dt} = F(u,w) + I(t)$$

$$t_w \frac{dw}{dt} = G(u,w)$$

pulse input



Blackboard

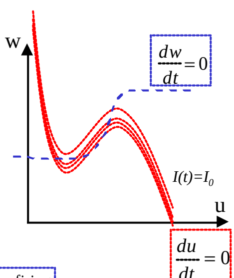


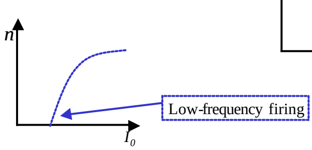
type I Model – constant input

stimulus

$$t\frac{du}{dt} = F(u,w) + I(t)$$

$$t_w \frac{dw}{dt} = G(u,w)$$





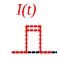
type I Model-pulse input

stimulus

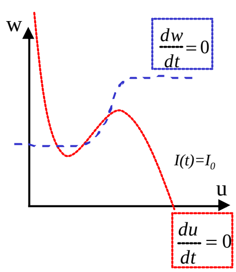
$$t\frac{du}{dt} = F(u,w) + I(t)$$

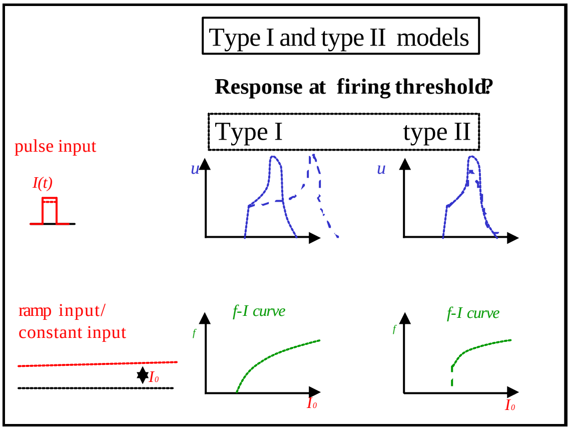
$$t_w \frac{dw}{dt} = G(u,w)$$

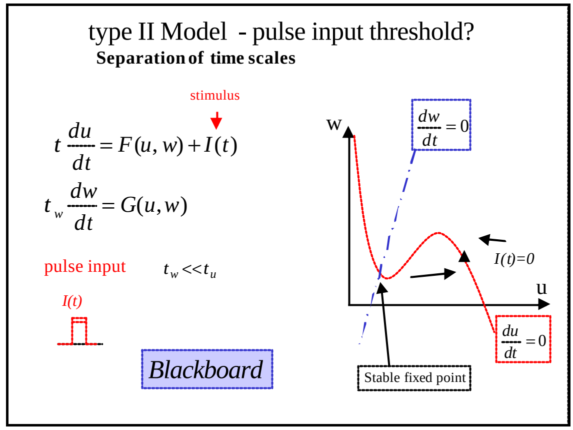
pulse input

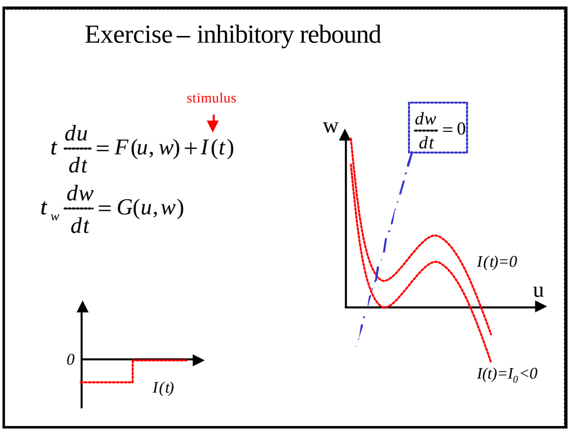


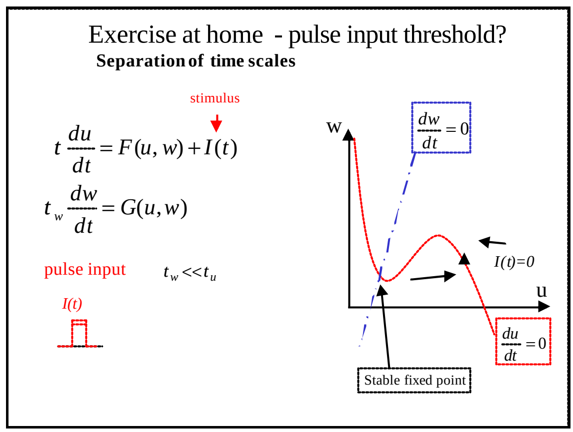
Blackboard




Type I and type II models
Response at firing threshold?
Type I
type II
pulse input
I(t)
u
u
ramp input/
constant input
I₀
f-I curve
f-I curve
f
f
I₀
I₀


type II Model - pulse input threshold?
Separation of time scales
stimulus
$t \frac{du}{dt} = F(u,w) + I(t)$
$t_w \frac{dw}{dt} = G(u,w)$
pulse input
$t_w << t_u$
I(t)
Blackboard
w
$\frac{dw}{dt} = 0$
I(t)=0
u
$\frac{du}{dt} = 0$
Stable fixed point


Exercise – inhibitory rebound
stimulus
$t \frac{du}{dt} = F(u,w) + I(t)$
$t_w \frac{dw}{dt} = G(u,w)$
0
I(t)
w
$\frac{dw}{dt} = 0$
I(t)=0
u
$I(t)=I_0<0$


Exercise at home - pulse input threshold?
Separation of time scales
stimulus
$t \frac{du}{dt} = F(u,w) + I(t)$
$t_w \frac{dw}{dt} = G(u,w)$
pulse input
$t_w << t_u$
I(t)
w
$\frac{dw}{dt} = 0$
I(t)=0
u
$\frac{du}{dt} = 0$
Stable fixed point