Week 11 – Continuum models –Part 1: Transients

Biological Modeling of Neural Networks:

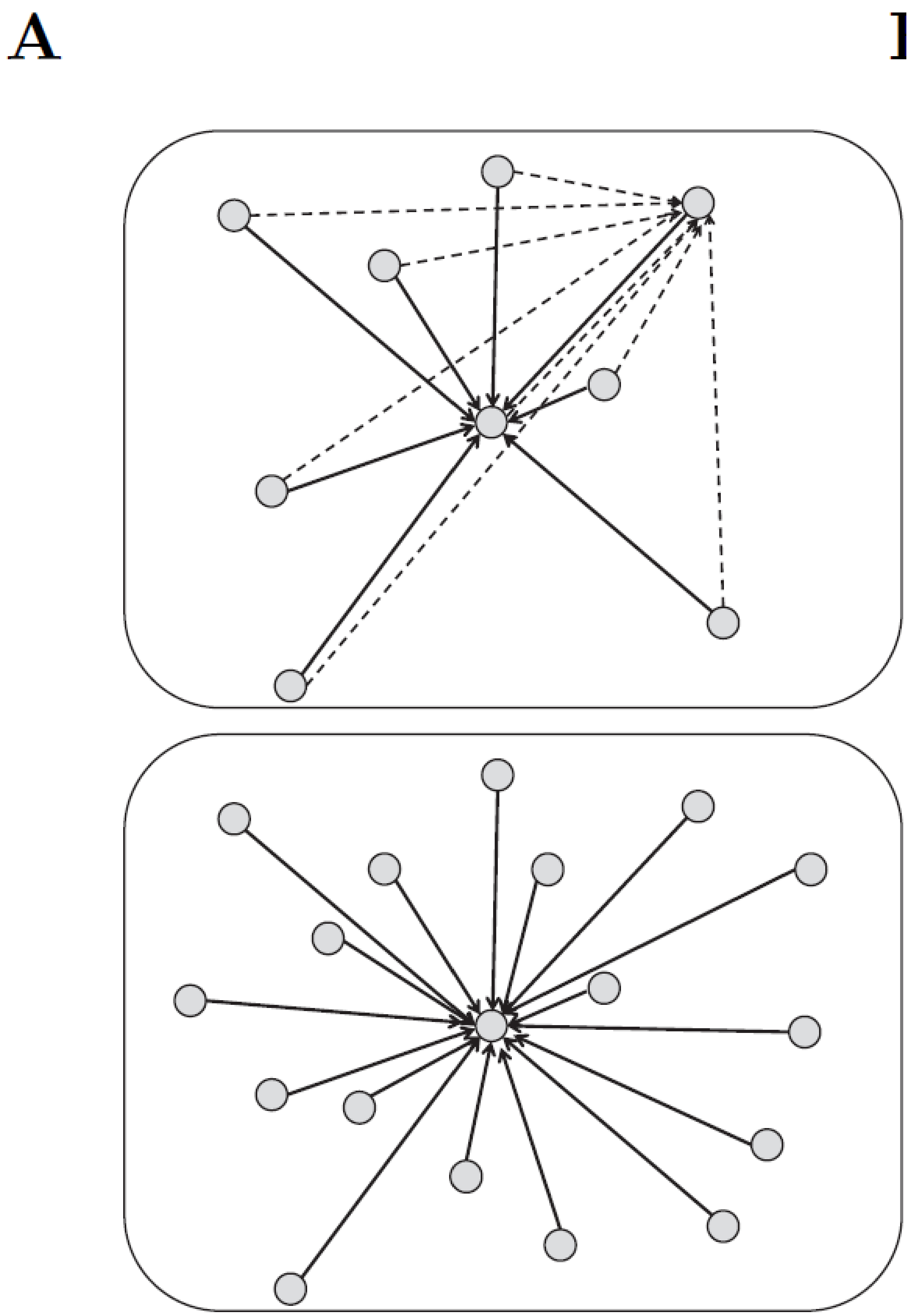
Week 11 – Continuum models: Cortical fields and perception

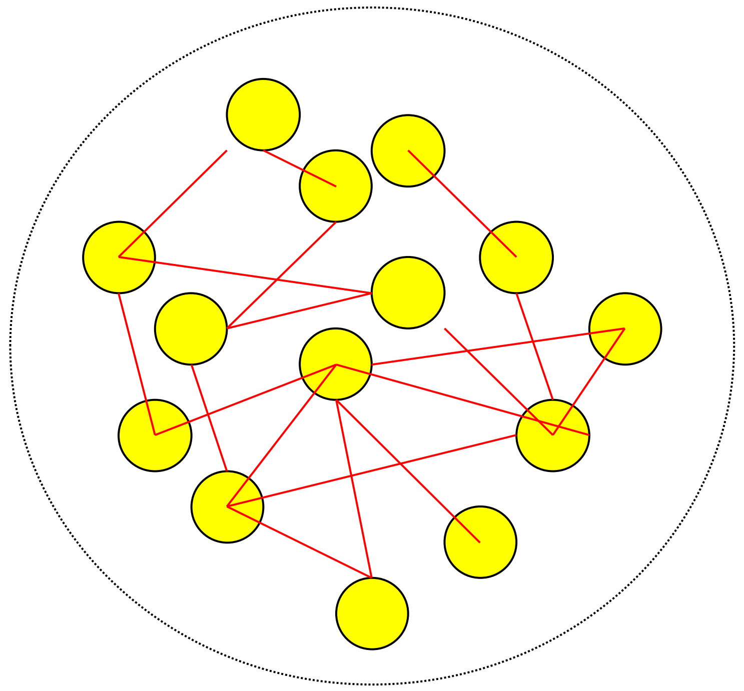
Wulfram Gerstner

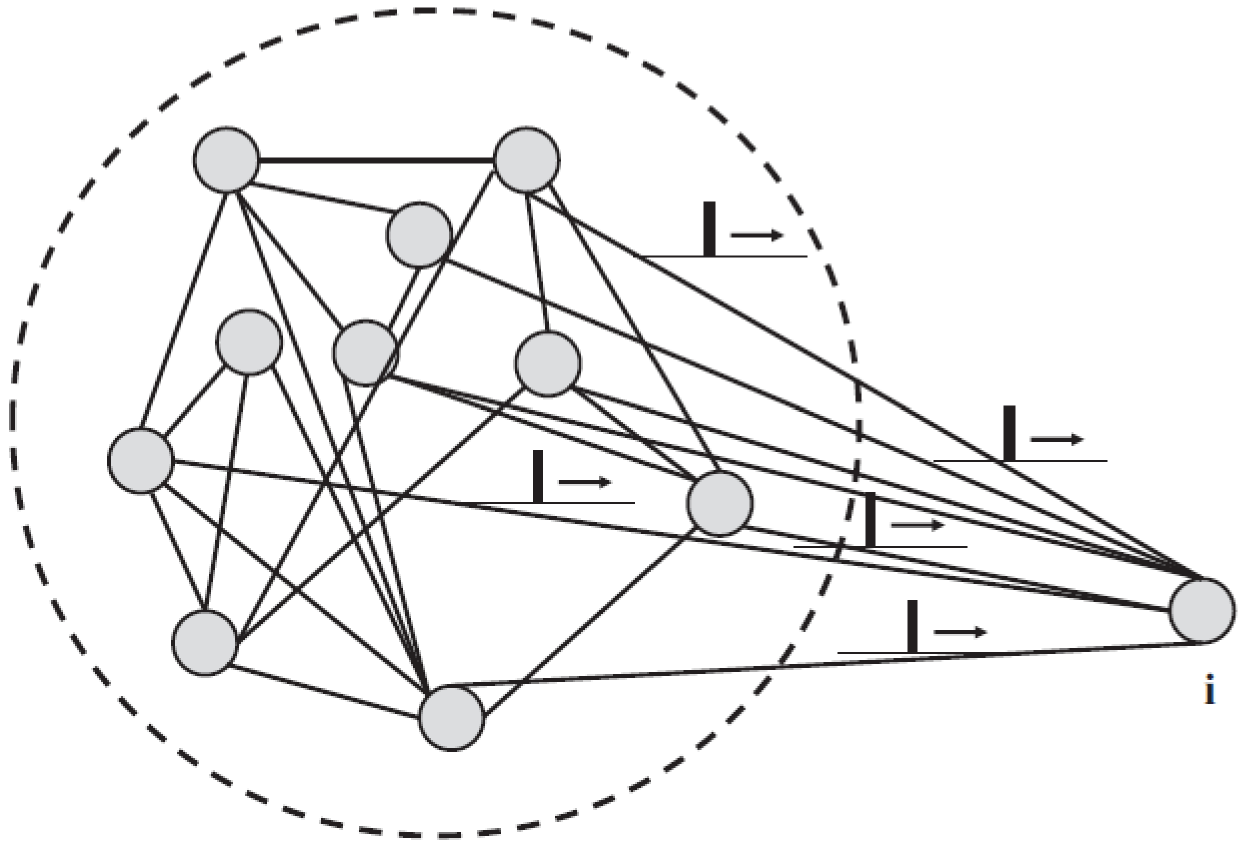
EPFL, Lausanne, Switzerland

review from Week 10-part 3: mean-field arguments

Single population full connectivity







All neurons receive the same total input current ('mean field')

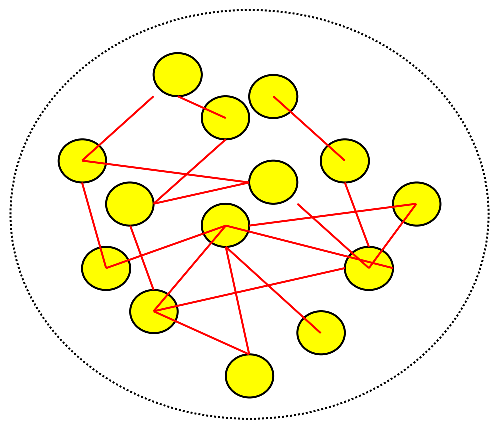
Review from Week 10: stationary state/asynchronous activity

Homogeneous network

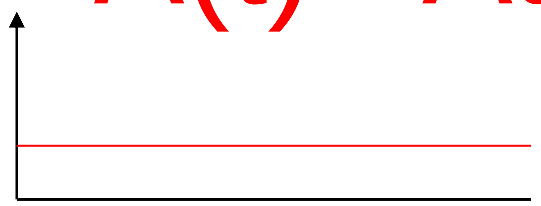
All neurons are identical,

Single neuron rate = population rate

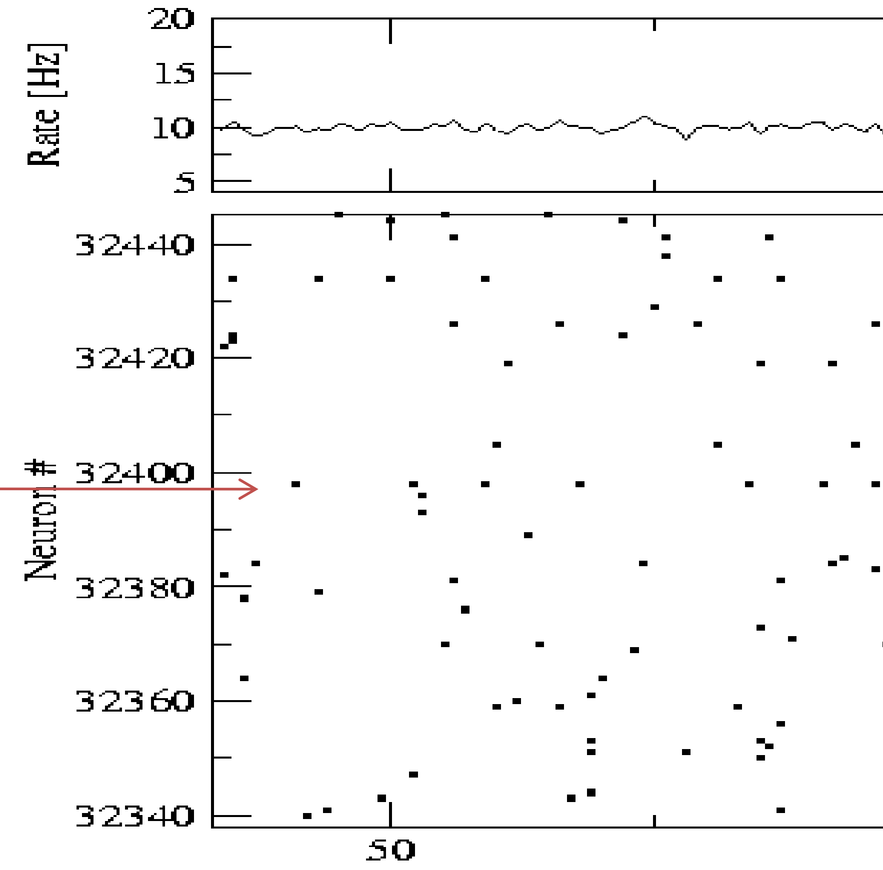
$$\nu = g(I_0) = A_0$$



A(t)= A_0= const



Single neuron



frequency (single neuron)

$$\nu = \langle s \rangle^{-1} = \left[\int_0^\infty s \, P_I(\hat{t} + s \,|\, \hat{t}) \, ds \right]^{-1} = g(h_0)$$

Week 10-part 3: mean-field arguments

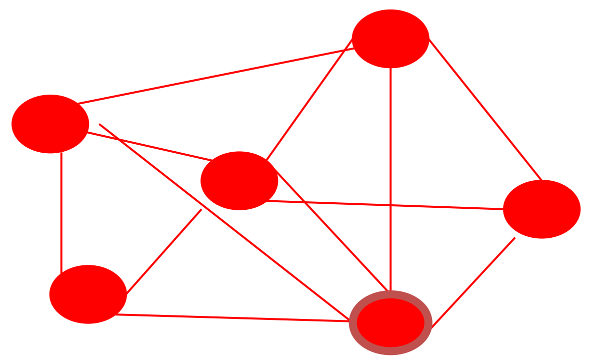
All neurons receive the same total input current ('mean field')

$$I_0 = [J_0 q A_0 + I_0^{ext}]$$

$$I_i(t) = J_0 \int \alpha(s) A(t-s) ds + I^{ext}(t)$$

Index i disappears

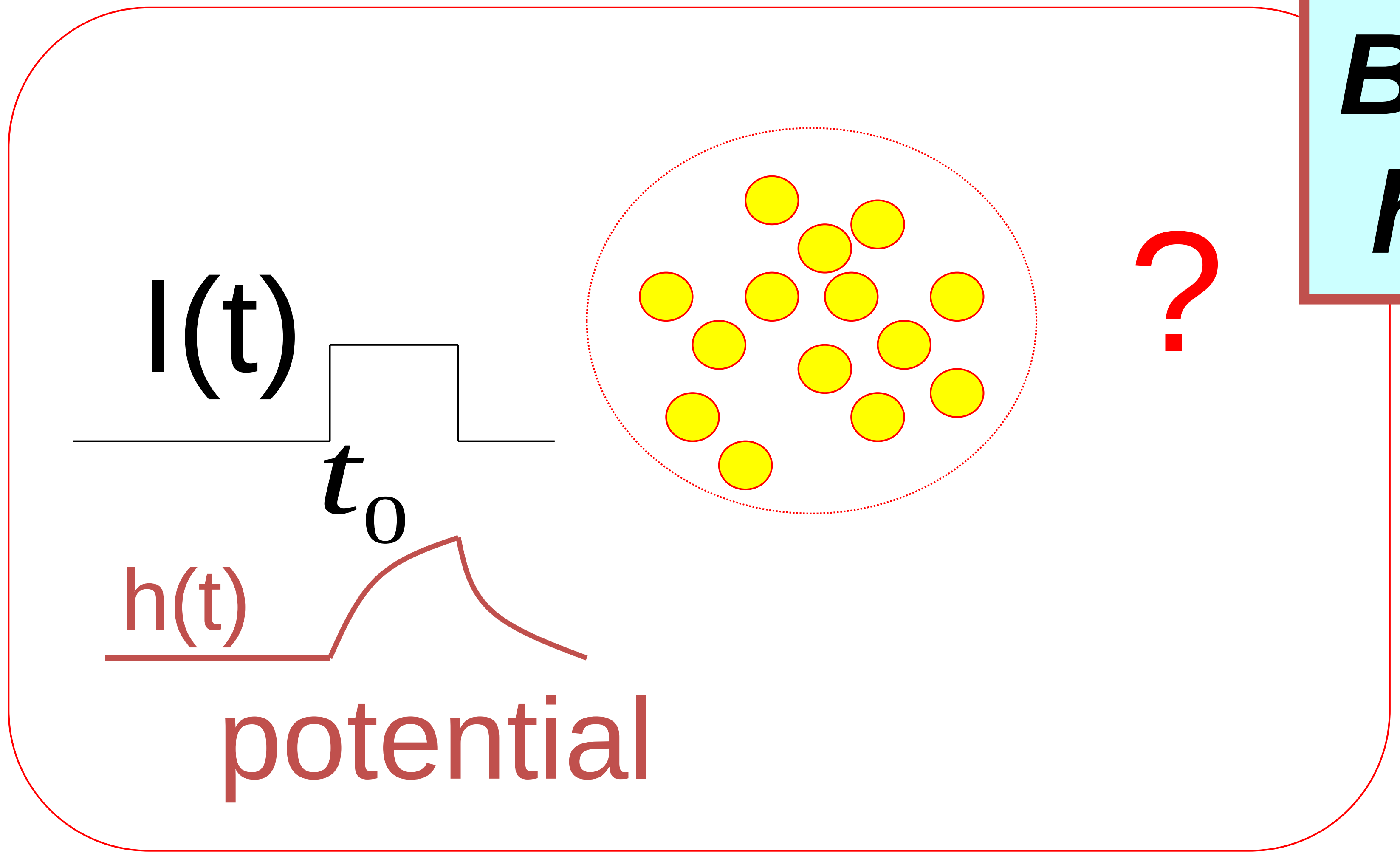
$$w_{ij} = \frac{J_0}{N}$$



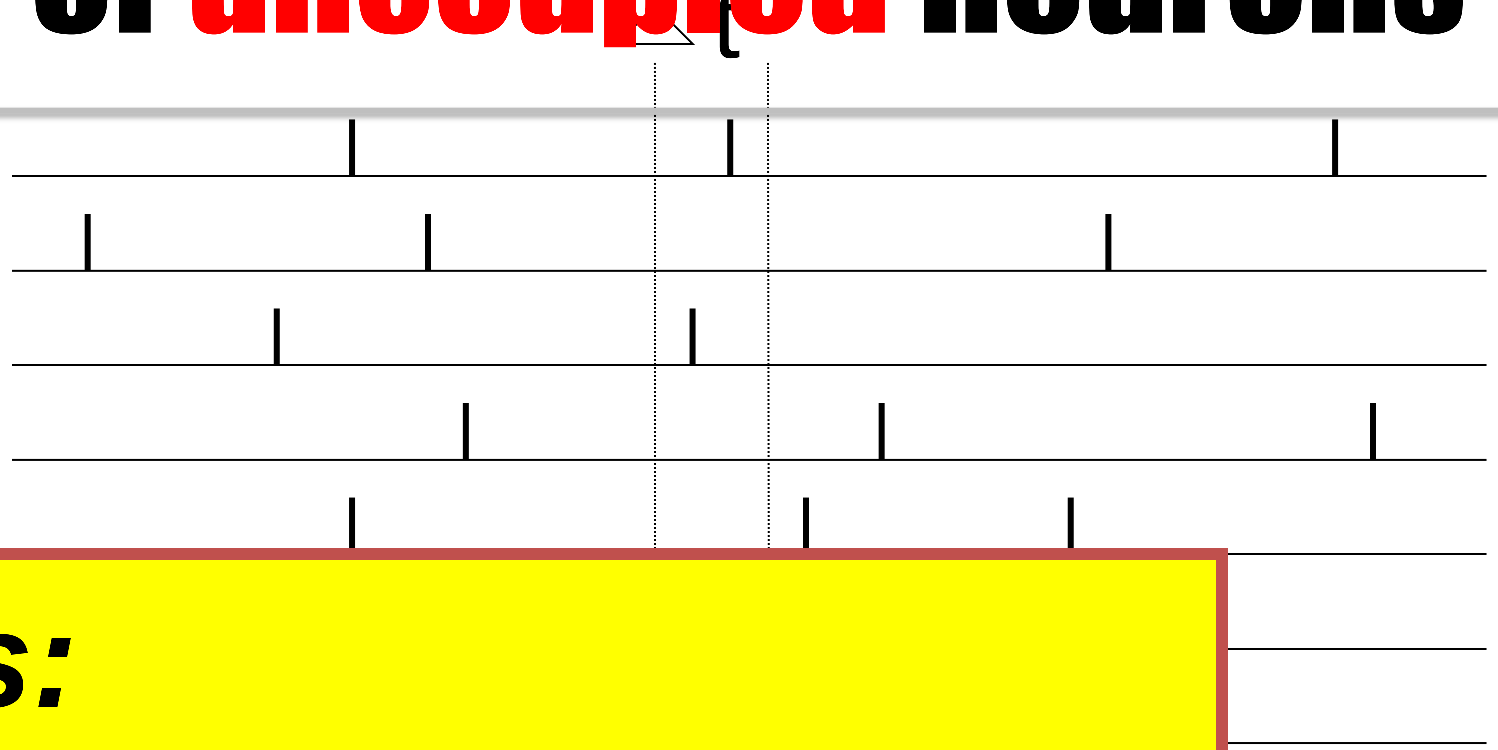
All spikes, all neurons

$$I^{net}(t) = \sum_j \sum_f w_{ij} \alpha(t - t_j^f) + I^{ext}$$

Week 11-part 1: Transients in a population of uncoupled neurons



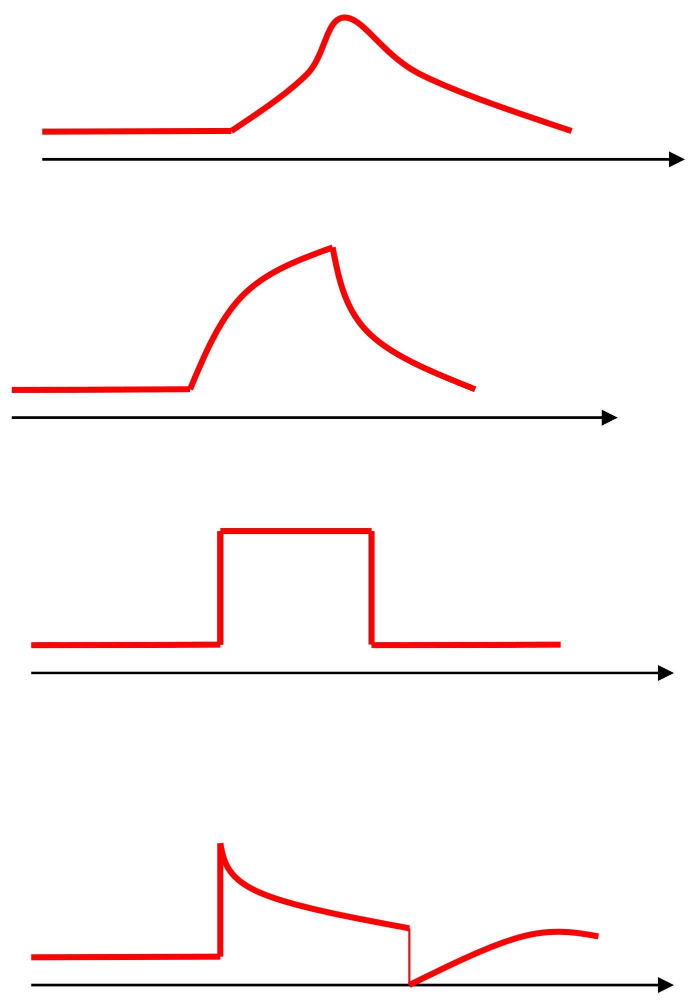
Blackboard:
h(t)



Students:
Which would you choose?

population activity

$$A(t) = \frac{n(t-\frac{\Delta t}{2};t+\frac{\Delta t}{2})}{N\Delta t}$$



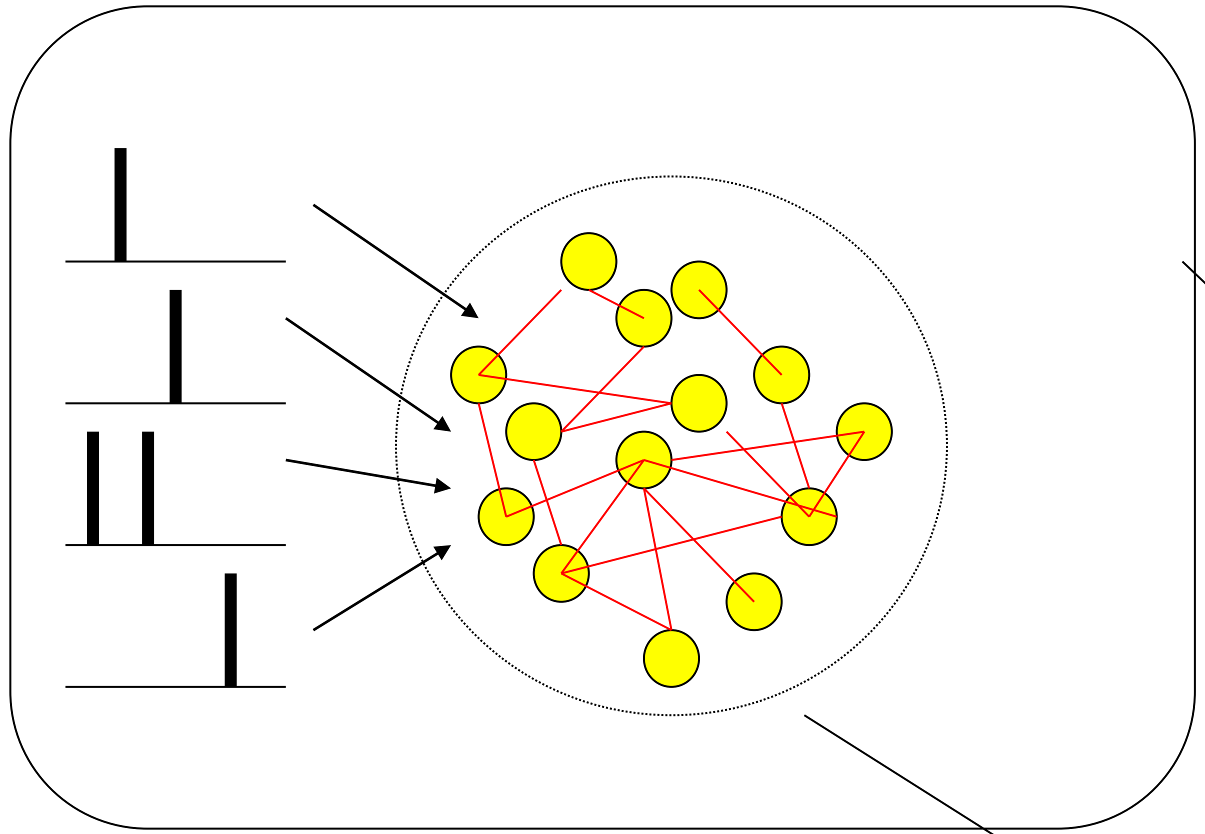
$$\tau \frac{d}{dt} A(t) = -A(t) + g(h(t))$$

$$A(t) = g(h(t)) = g(\int \kappa(s) I(t-s) ds)$$

$$A(t) = g(I(t))$$

$$A(t) = g(I(t), I'(t))$$

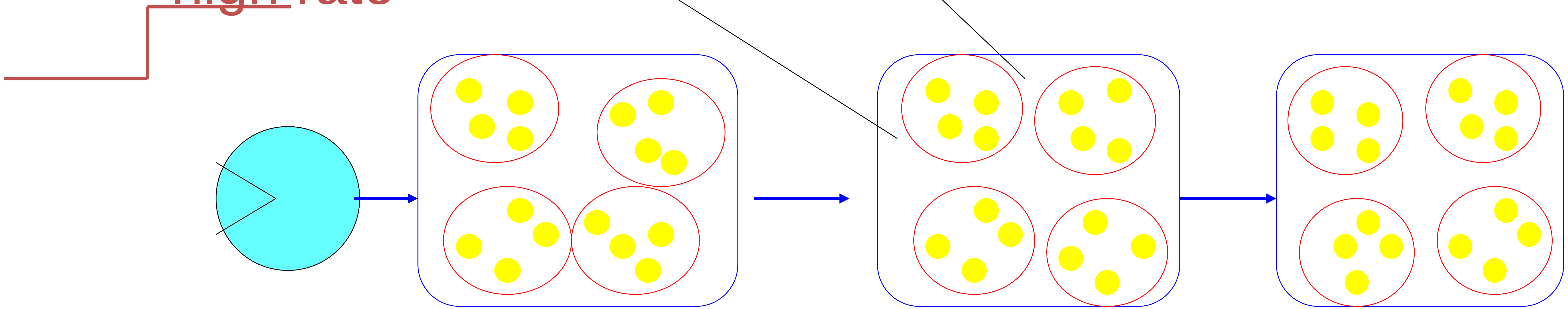
Week 11-part 1: Transients in a population of neurons



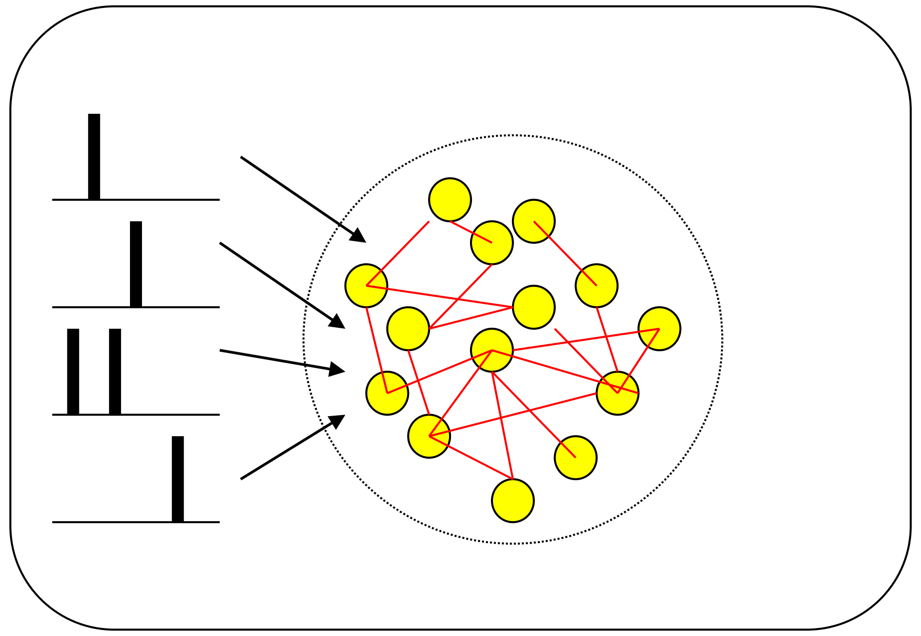
Population
- 50 000 neurons
- 20 percent inhibitory
- randomly connected

Connections
4000 external
4000 within excitatory
1000 within inhibitory

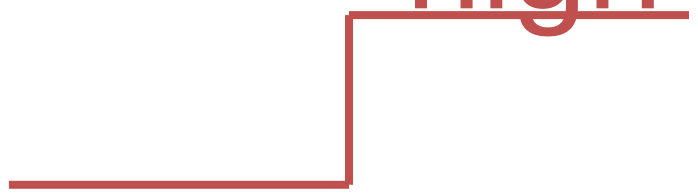
input {low rate / high rate}



Week 11-part 1: Transients in a population of neurons



input
- low rate
- high rate



Population
- 50 000 neurons
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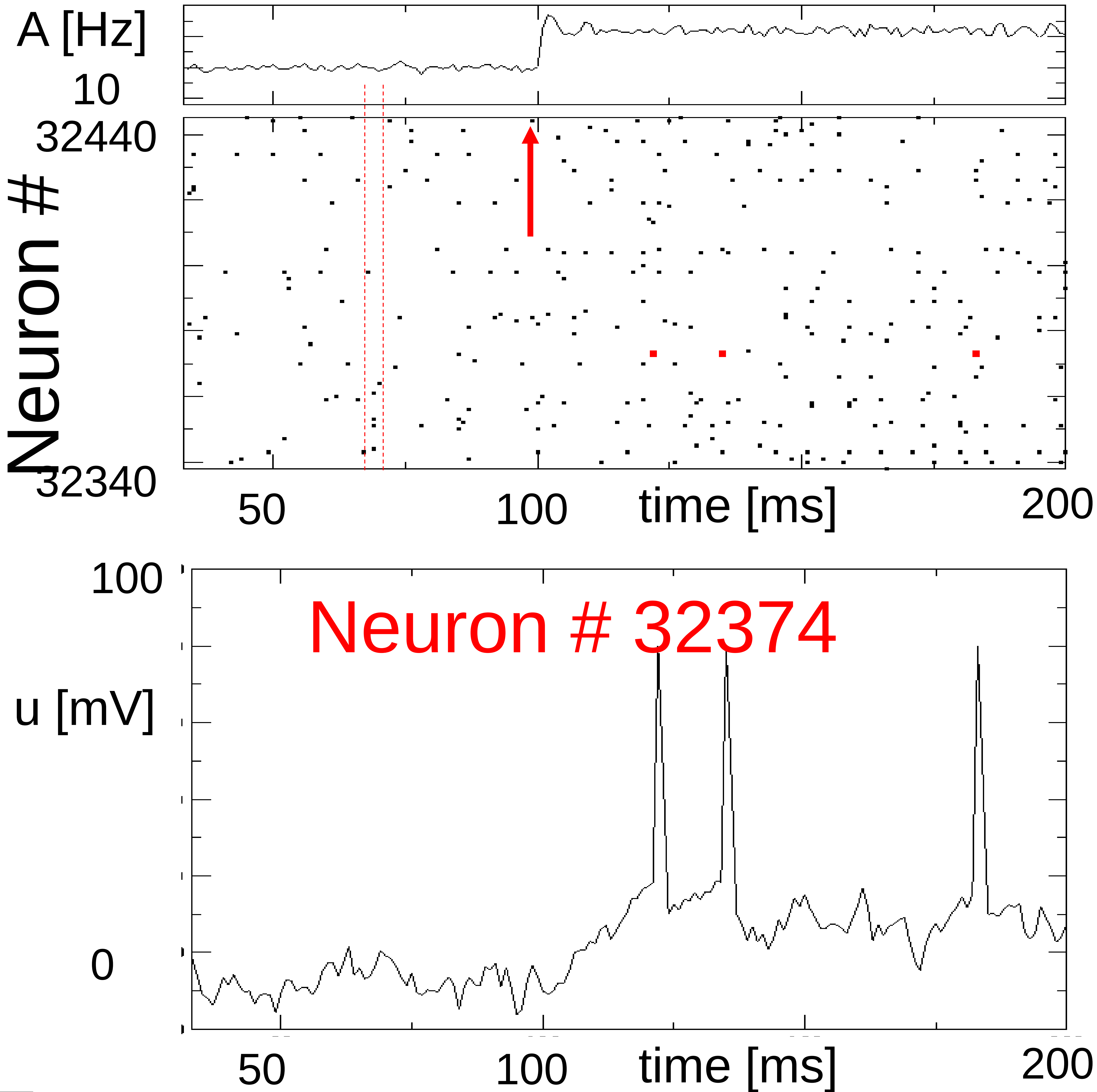
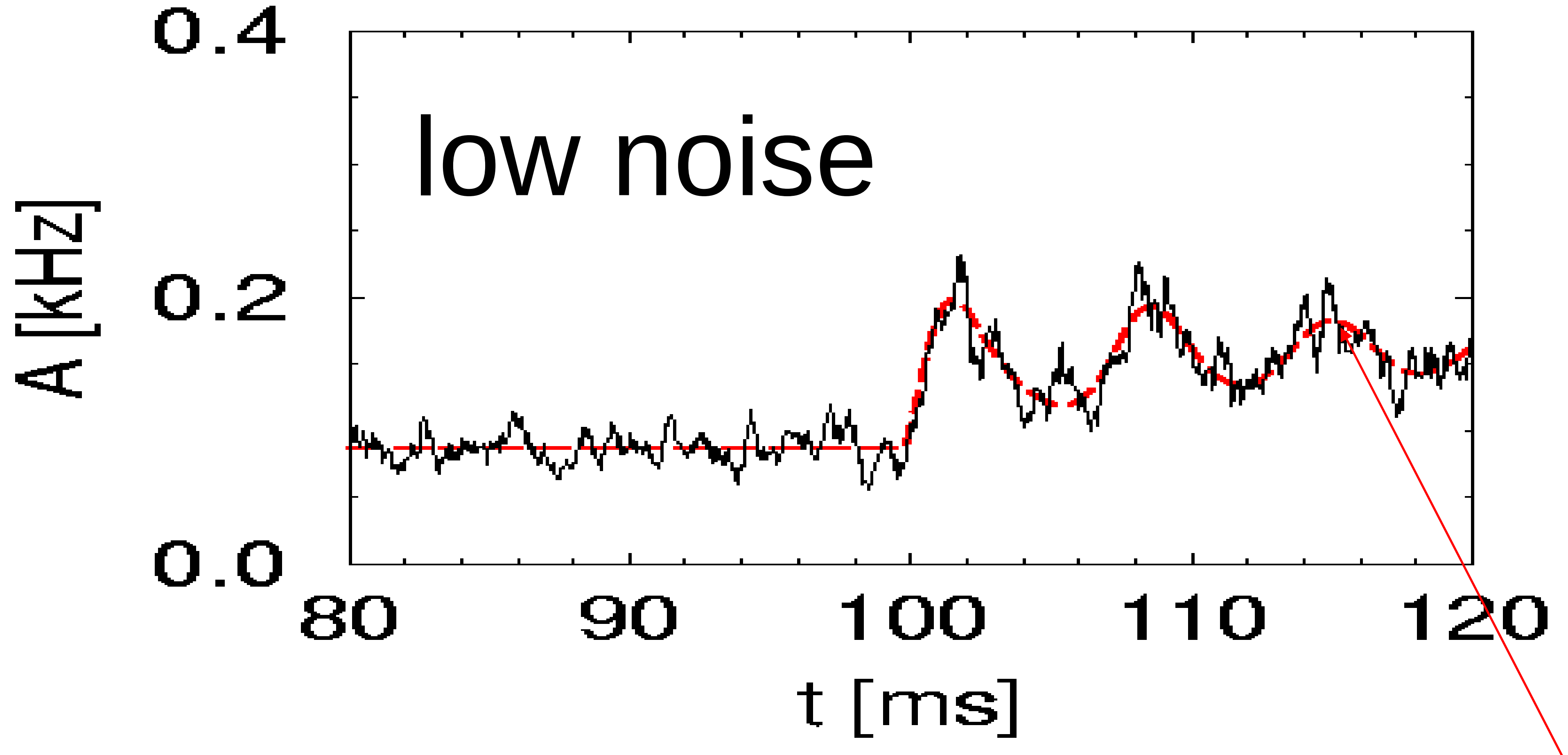
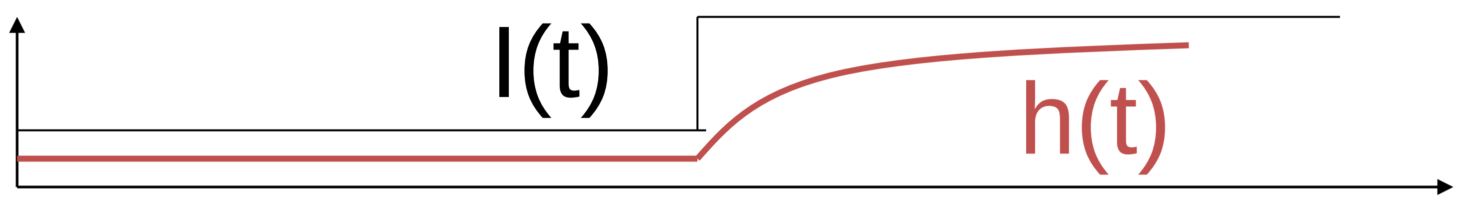


Week 11-part 1: Theory of transients for escape noise models

uncoupled population

(escape noise/fast noise)





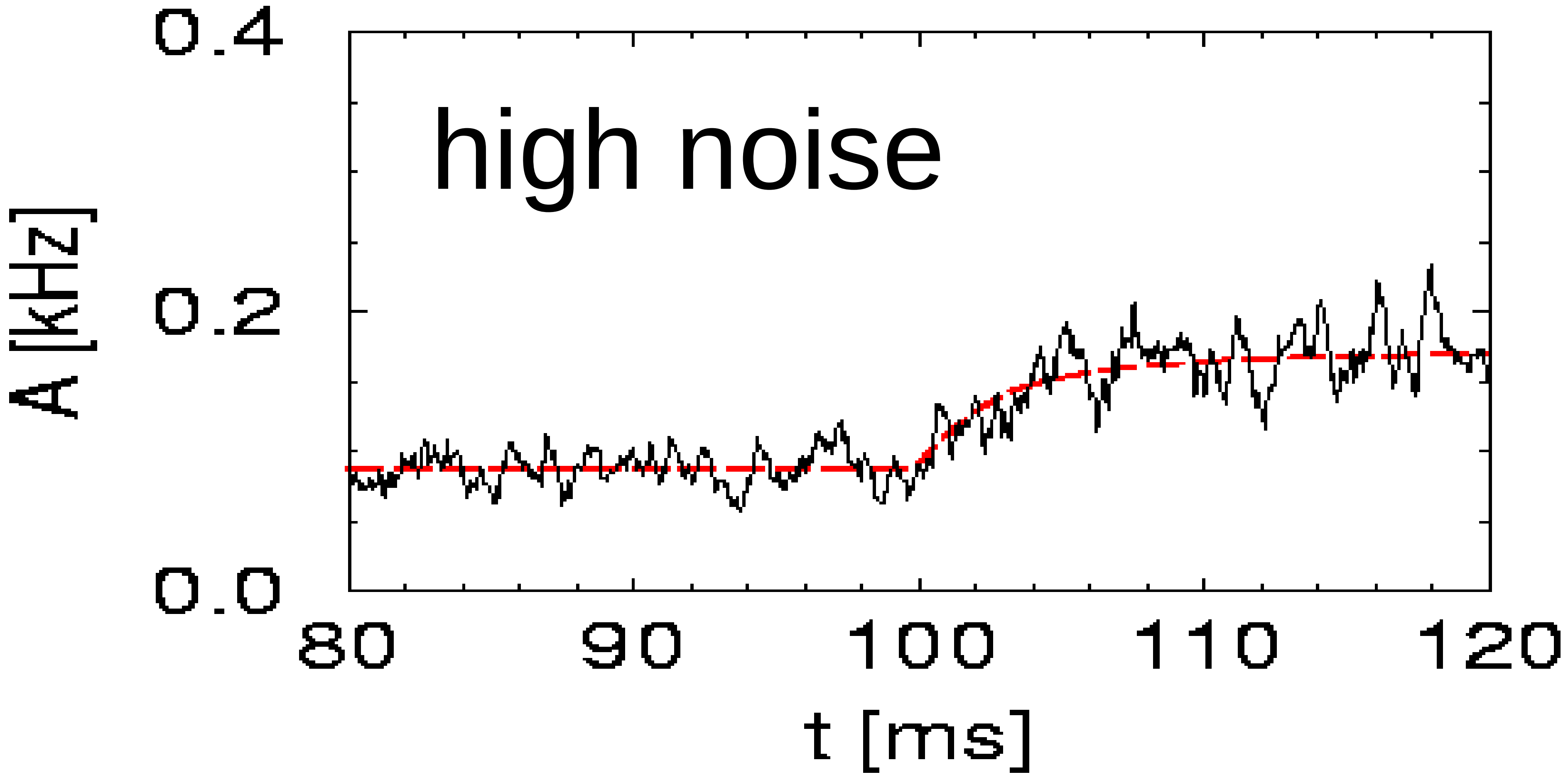
fast transient

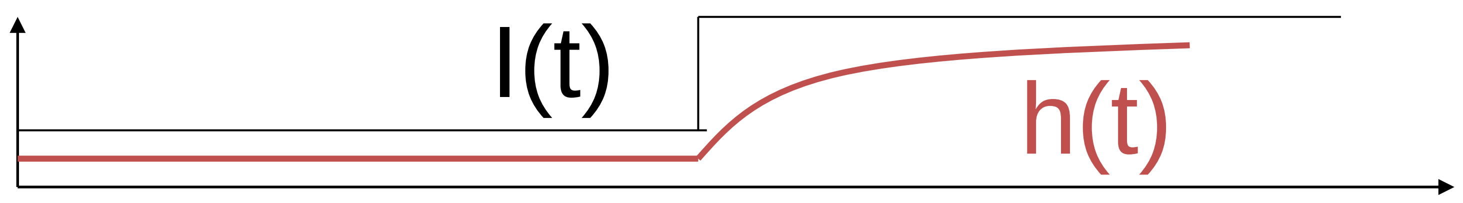
$$A(t) \approx g(I(t))$$

$$A(t) \approx g(I(t), I'(t))$$

But transient oscillations

(escape noise/fast noise)





slow transient

$$A(t) = F(h(t))$$

Week 11-part 1: High-noise activity equation

blackboard

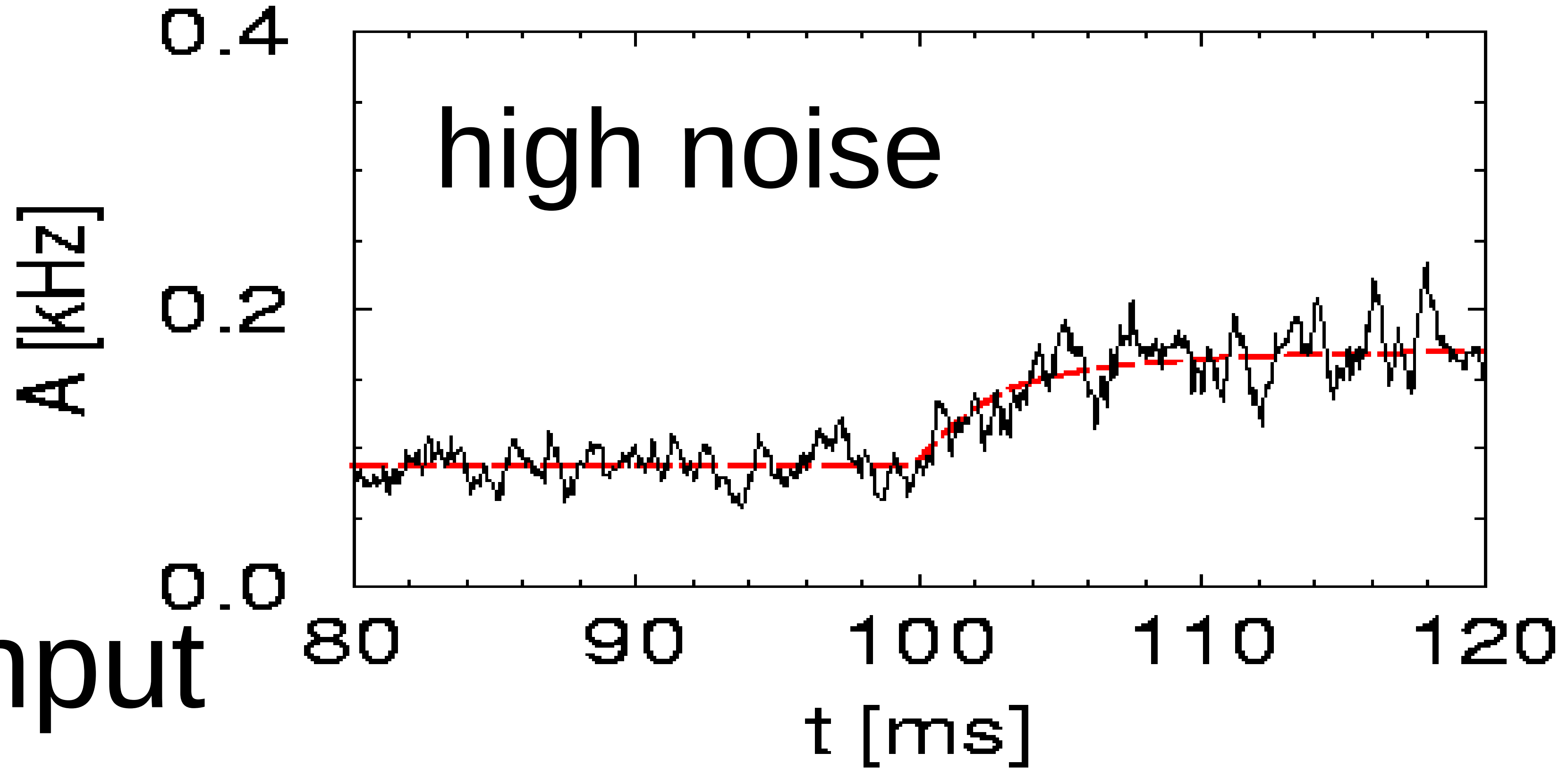
In the limit of **high noise**,
Population activity

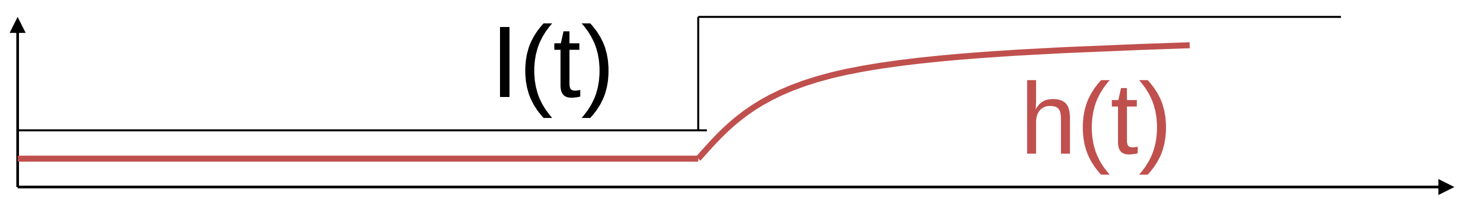
$$A(t) = F(h(t))$$

Membrane potential caused by input

$$\tau \frac{d}{dt} h(t) = -h(t) + R\ I(t)$$

noise model A
(escape noise/fast noise)





slow transient

$$A(t) = F(h(t))$$

Week 11-part 1: High-noise activity equation

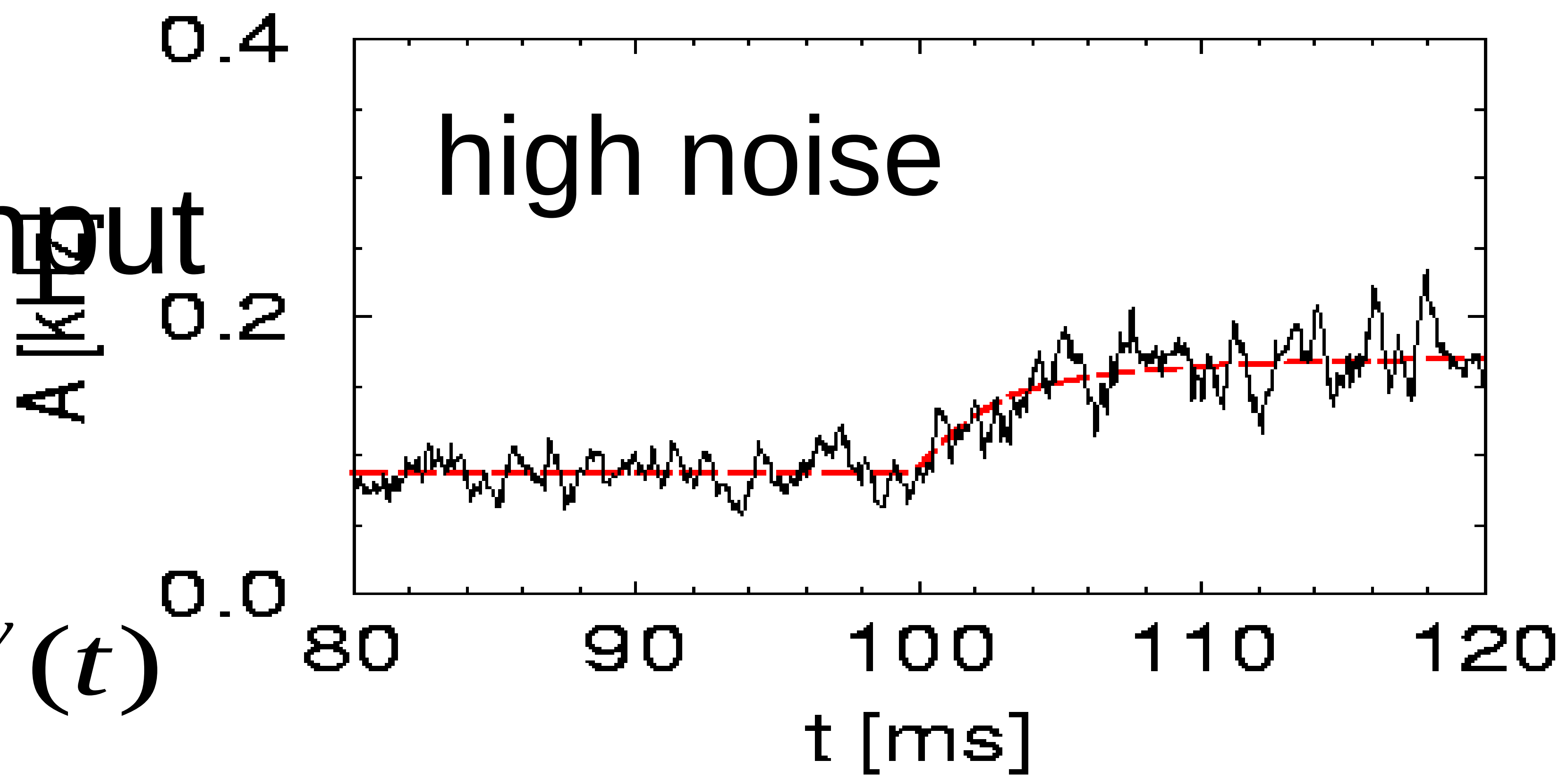
noise model A
(escape noise/fast noise)

Population activity

$$A(t) = F(h(t))$$

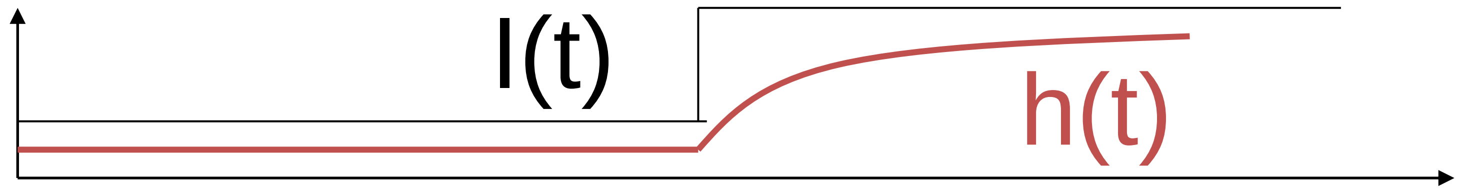
Membrane potential caused by input

$$\tau \frac{d}{dt} h(t) = -h(t) + R\ I(t)$$



$$I(t) = I^{ext}(t) + I^{netw}(t)$$

$$I(t) = I^{ext}(t) + J_0\, q\, A(t)$$



$$I(t) = I^{ext}(t) + J_0\, q\, F(h(t))$$

slow transient

$$\tau \frac{d}{dt} h(t) = -h(t) + R\, I^{ext}(t) + \gamma\, F(h(t))$$

$$A(t) = F(h(t))$$

1 population = 1 differential equation

Week 10-part 2: mean-field also works for random coupling

full connectivity

random: prob p fixed

random: number K of inputs fixed

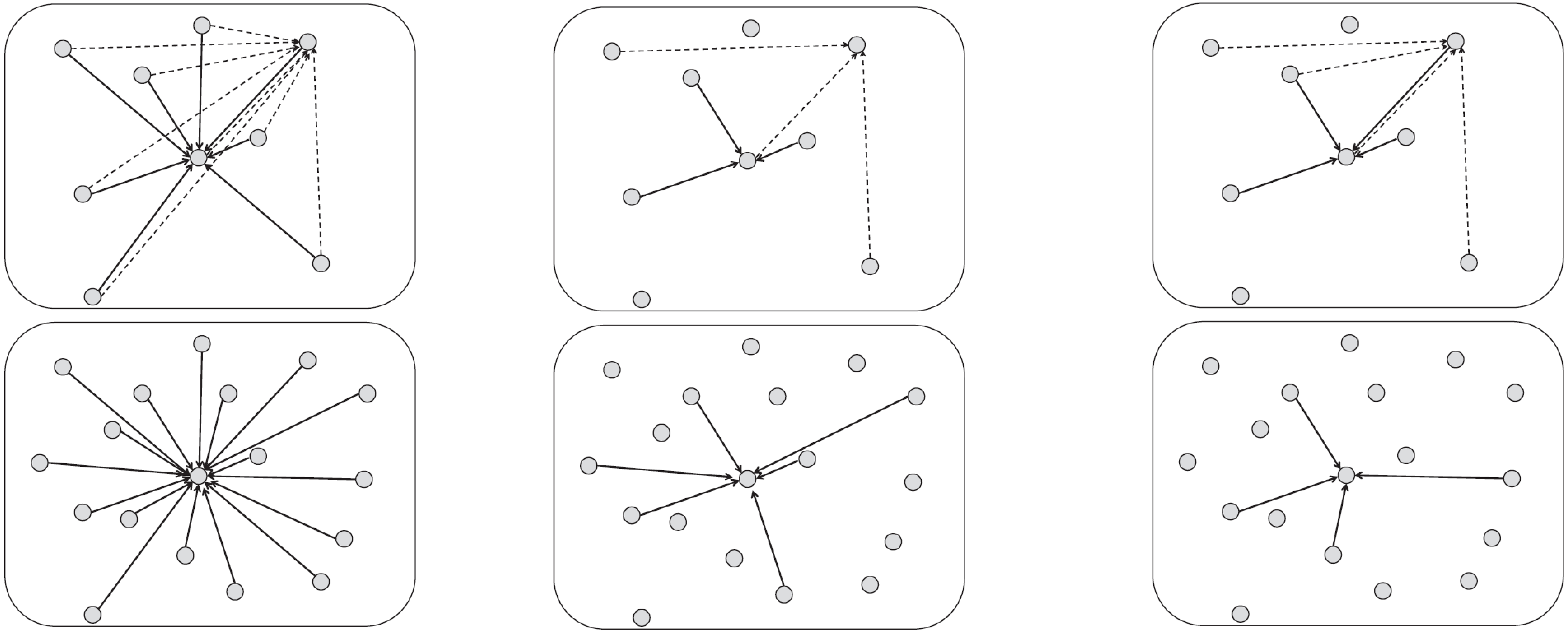


Image: Gerstner et al. Neuronal Dynamics (2014)

Quiz 1, now

Population equations

[ ] A single cortical model population can exhibit transient oscillations

[ ] Transients are always sharp

[ ] Transients are always slow

[ ] in a certain limit transients can be slow

[ ] An escape noise model in the high-noise limit has transients which are always slow

[ ] A single population described by a single first-order differential equation (no integrals/no delays) can exhibit transient oscillations

Week 11 – part 2 :

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 11 – Continuum models: Cortical fields and perception

Wulfram Gerstner

EPFL, Lausanne, Switzerland

11.1 Transients
- sharp or slow

11.2 Spatial continuum
- from multiple to continuous populations
- cortical connectivity

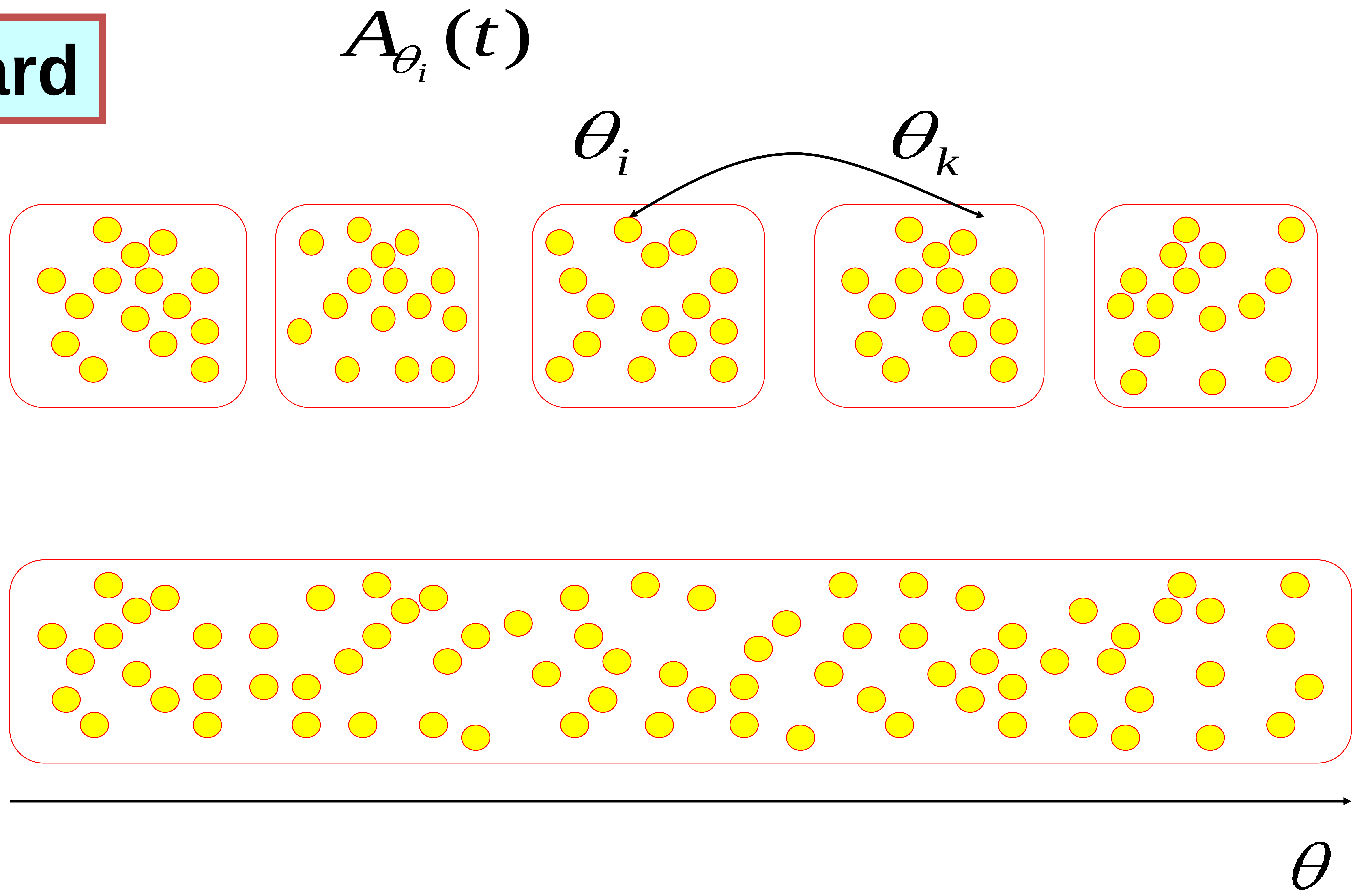
11.3 Solution types
- uniform solution
- bump solution

11.4. Perception

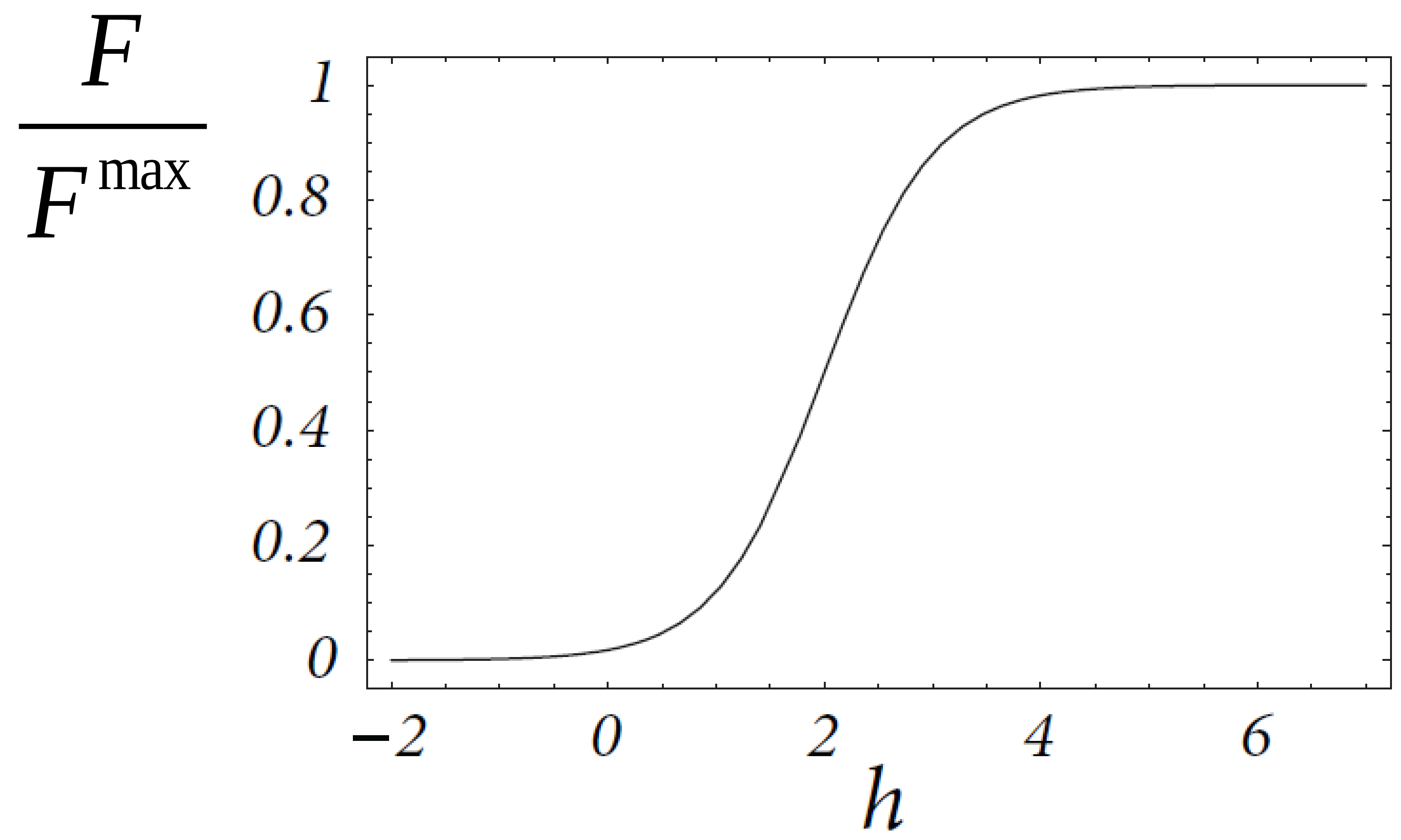
11.5. Head direction cells

Week 11-part 2: multiple populations → continuum

Blackboard



Week 11-part 2: Field equation (continuum model)



Population activity

$$A(x,t) = F(h(x,t))$$

Membrane potential caused by input

$$\tau \frac{d}{dt} h(x,t) = -h(x,t) + R\ I(x,t)$$

$$I(x,t) = I^{ext}(x,t) + I^{netw}(x,t)$$

$$I^{netw}(x,t) = d \int w(x-x',t)\, A(x',t) dx'$$

$$\tau \frac{d}{dt} h(x,t) = -h(x,t) + R I^{ext}(x,t) + d \int w(x-x') F(h(x',t)) dx'$$

1 field = 1 integro-differential equation

Exercise 1.1 now (stationary solution)

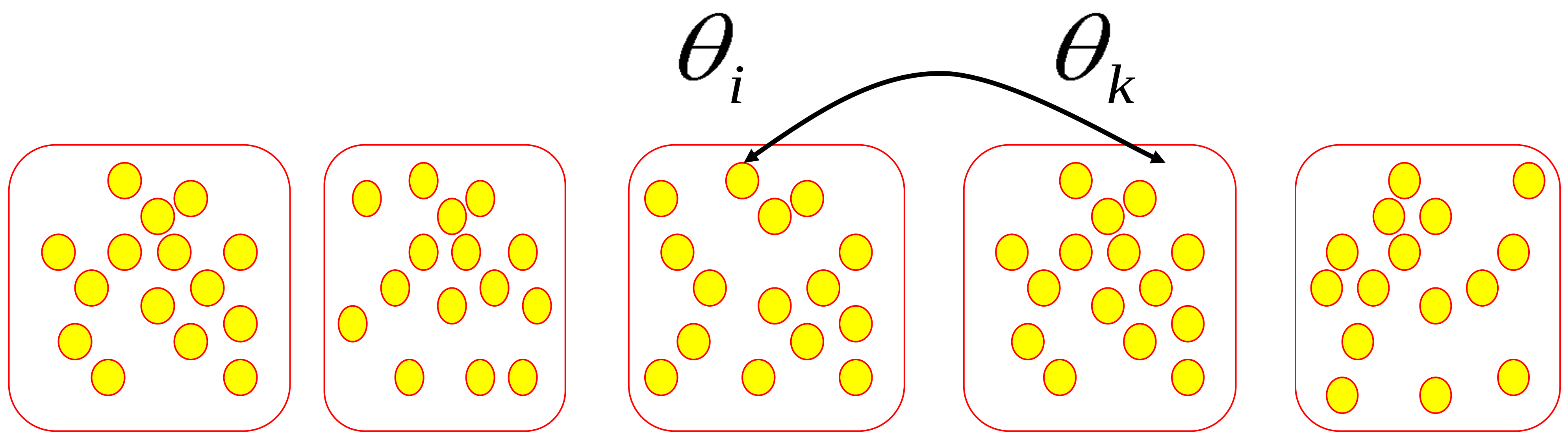
Consider a continuum model,
Find analytical solutions:

- spatially uniform solution $A(x,t) = A_0$

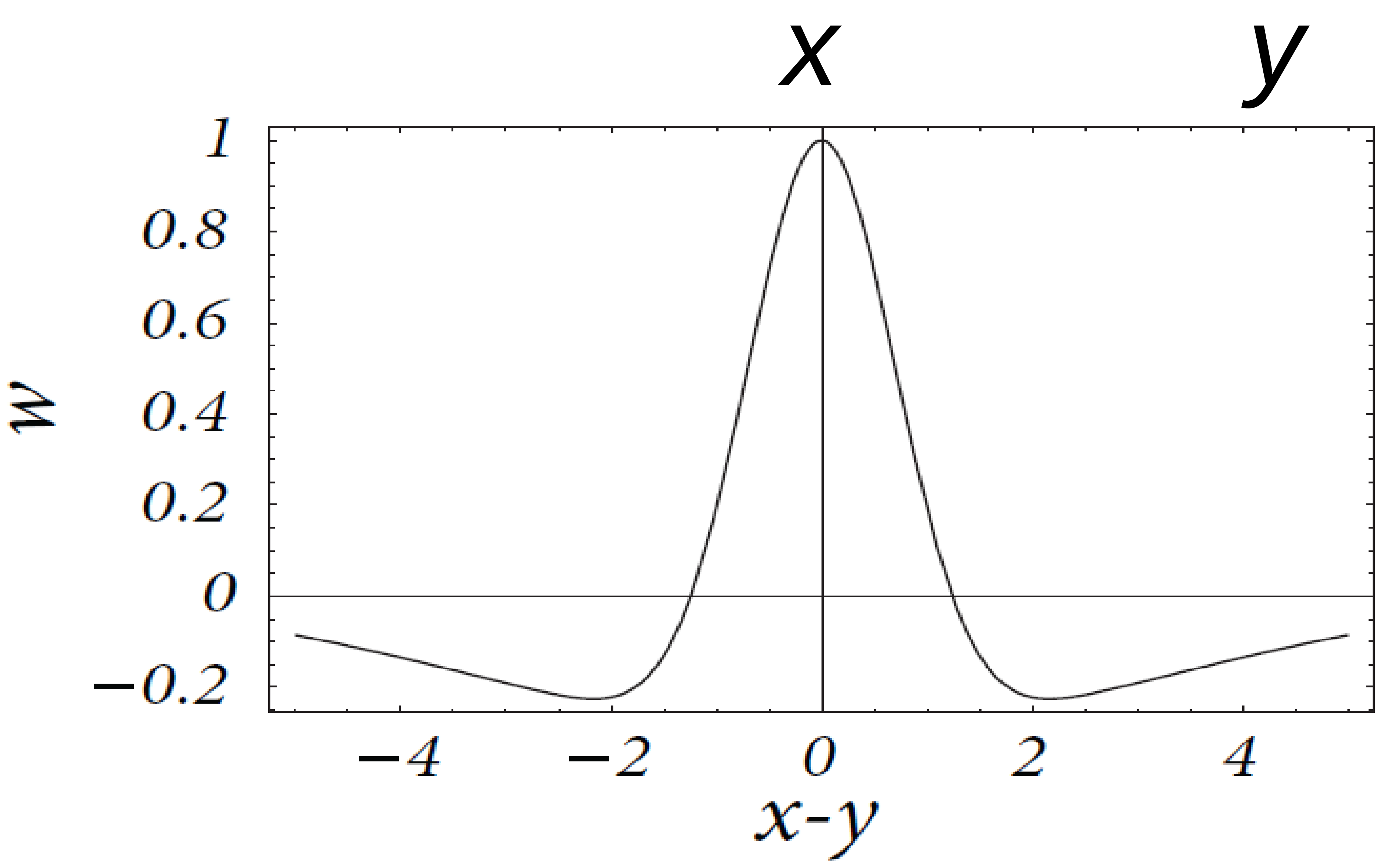
Next lecture at 10:45

If done: start with Exercise 1.2 now (spatial stability)

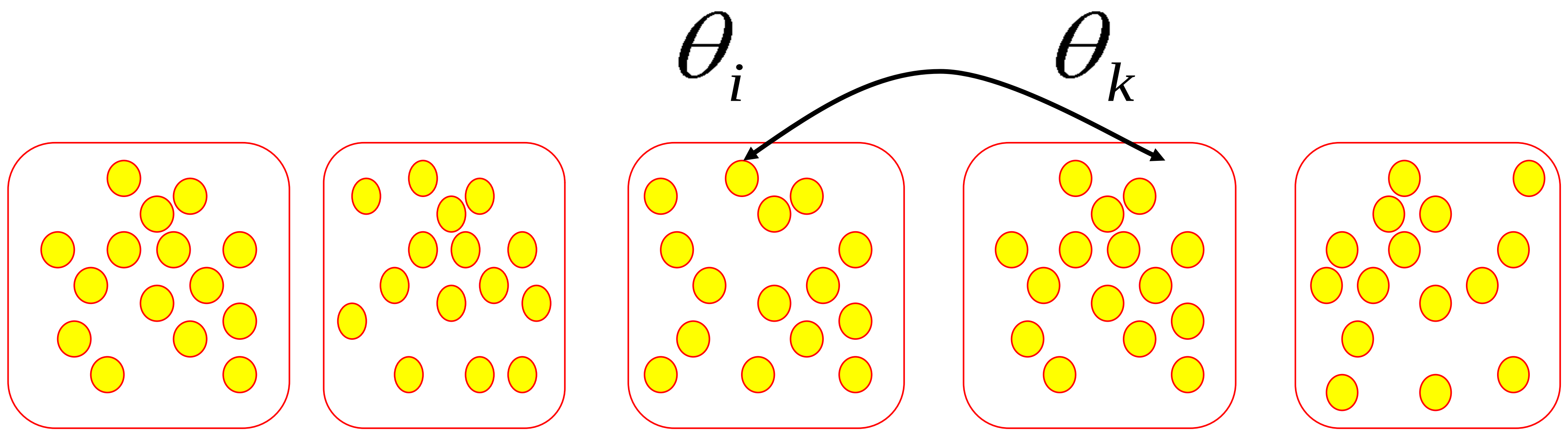
Week 11-part 2: coupling across continuum

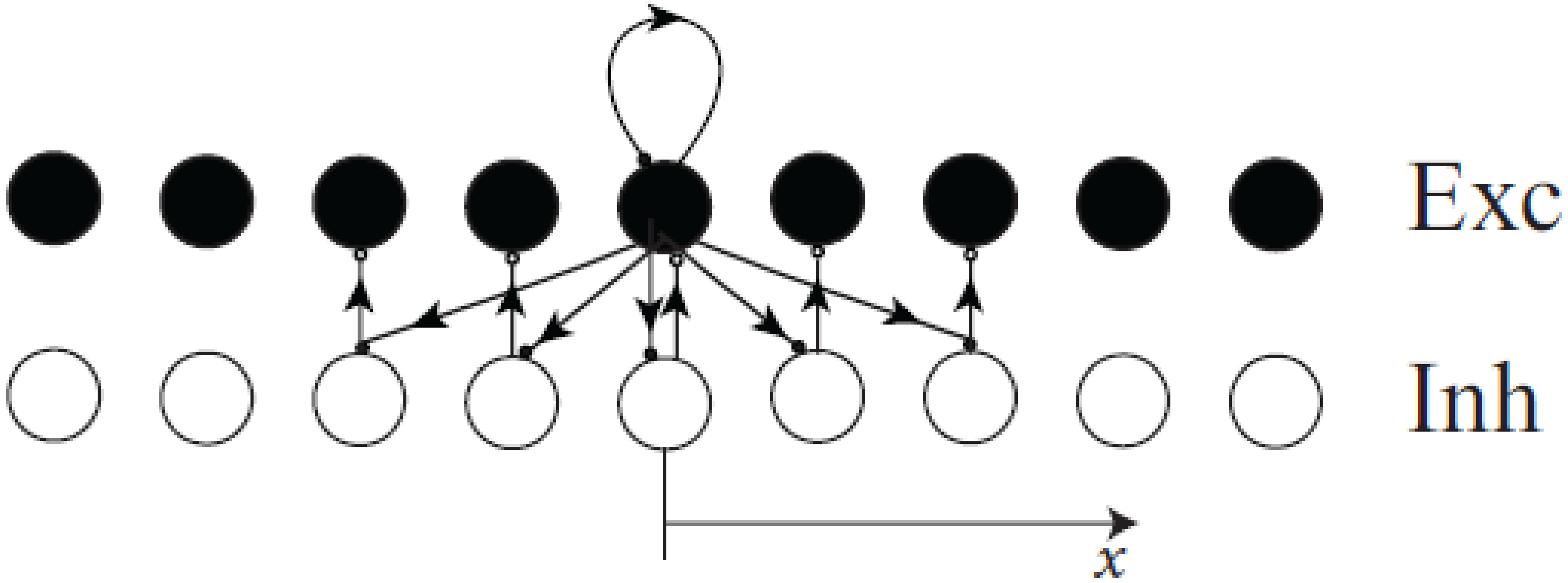


Mexican hat



Week 11-part 2: cortical coupling





Week 11 – part 3 :

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 11 – Continuum models: Cortical fields and perception

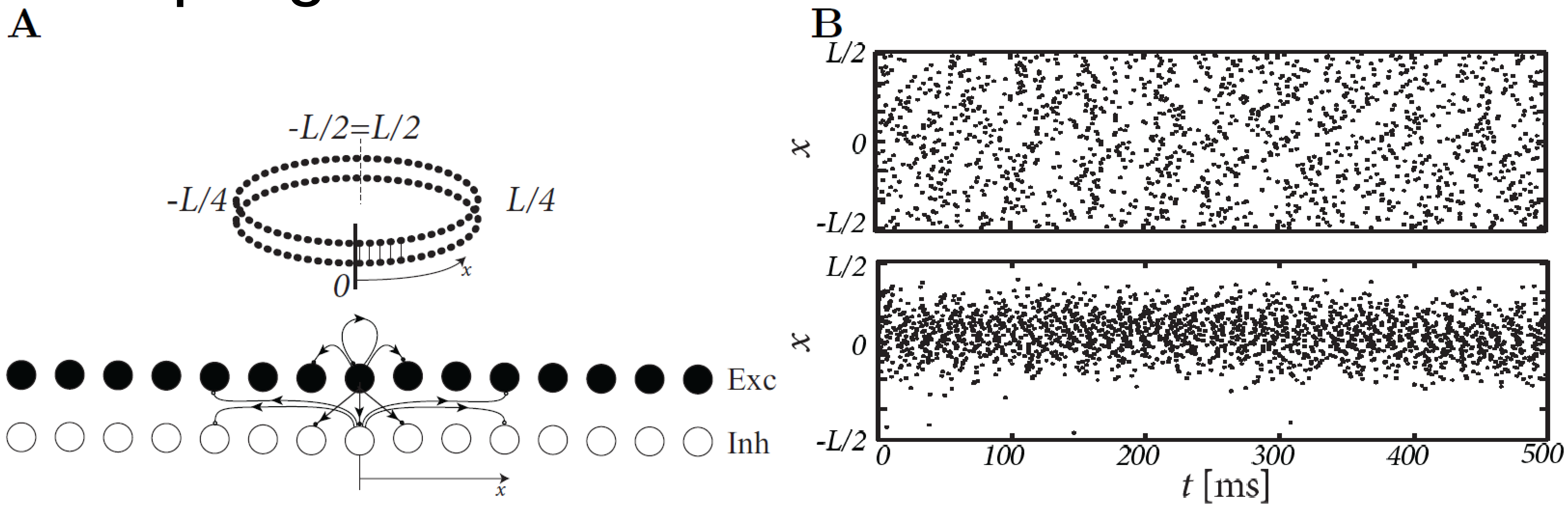
Wulfram Gerstner

EPFL, Lausanne, Switzerland

Week 11-part 3: Solution types (ring model)

Coupling:

Input-driven regime

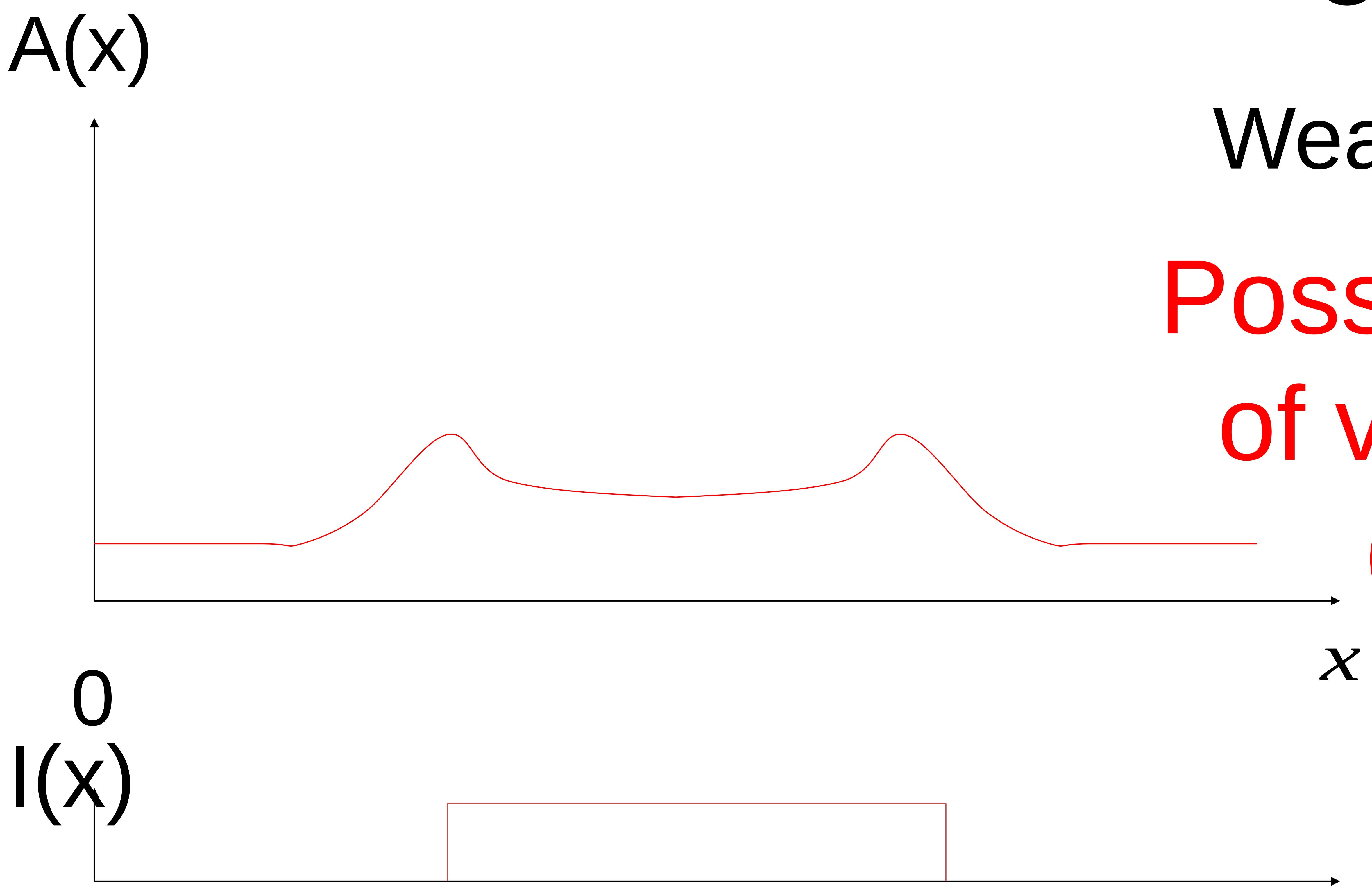


Bump attractor regime

Week 11-part 3: Solution types: input driven regime

Field Equations:
Wilson and Cowan, 1972

I. Edge enhancement



Weaker lateral connectivity

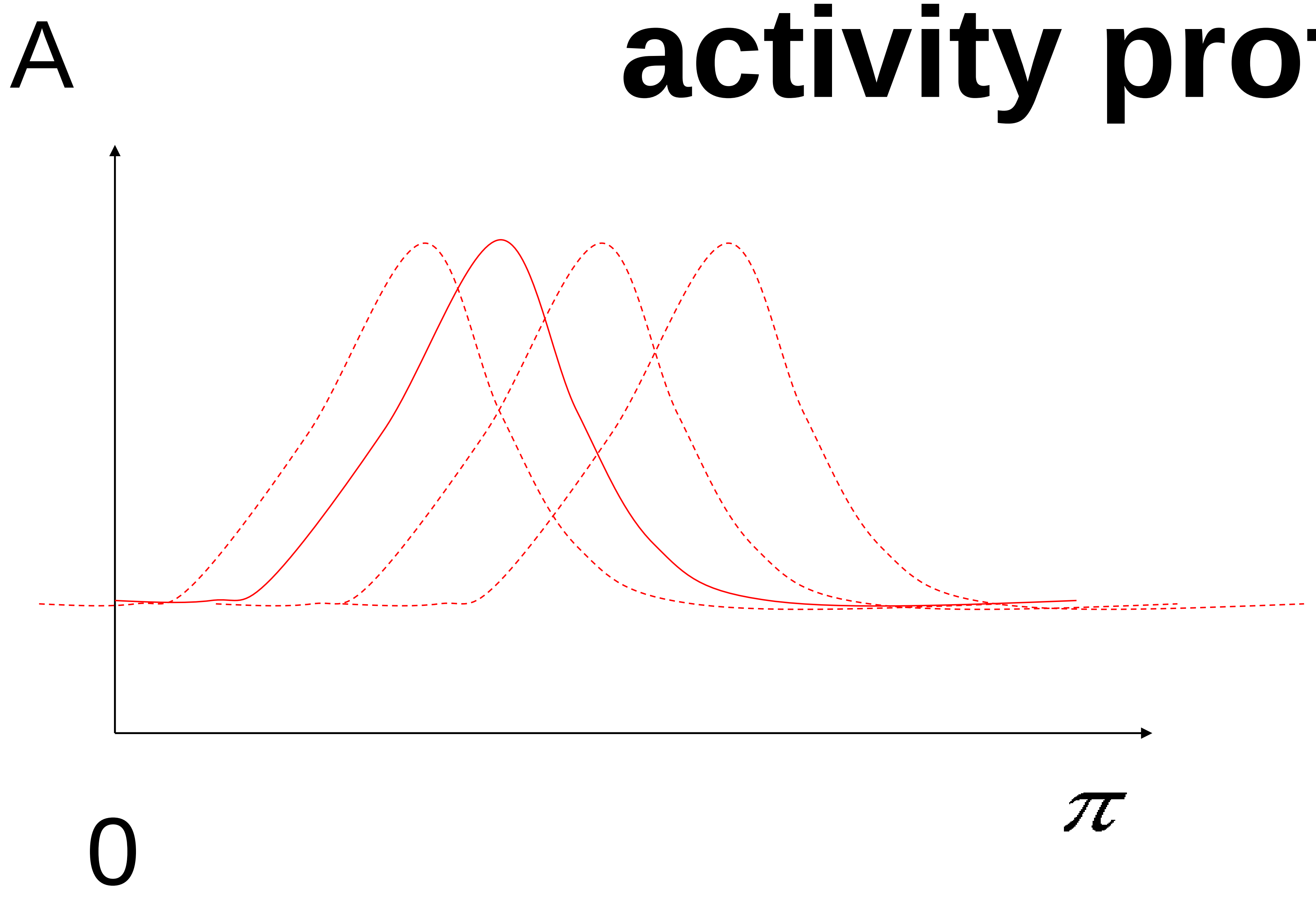
Possible interpretation
of visual cortex cells:
(see final part this week)

Week 11-part 3: Solution types: bump solution

Field Equations:
Wilson and Cowan, 1972

II: Bump formation:
activity profile in the absence of input

strong lateral connectivity

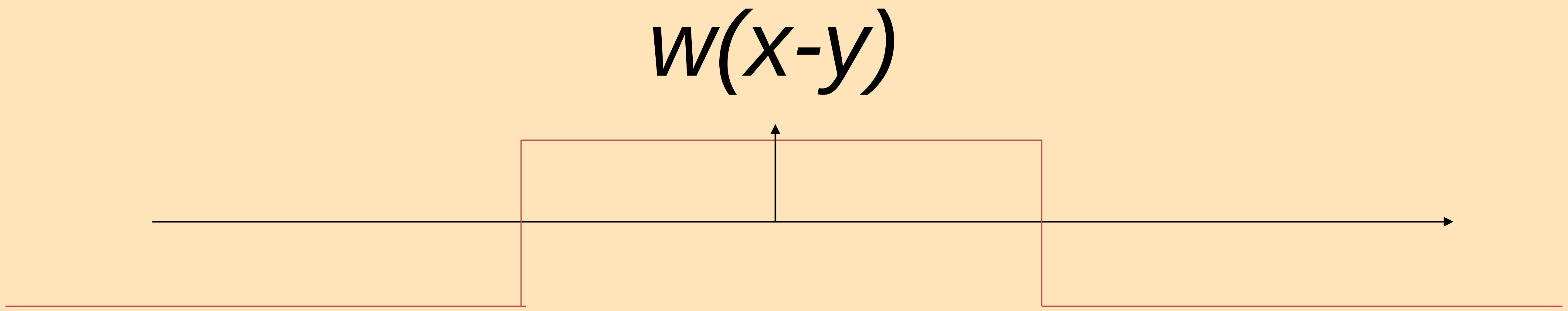


Possible interpretation
of head direction cells:
→ (see later today)

Exercise 2.1+2.2 now (stationary bump solution)

Next lecture at 11:28

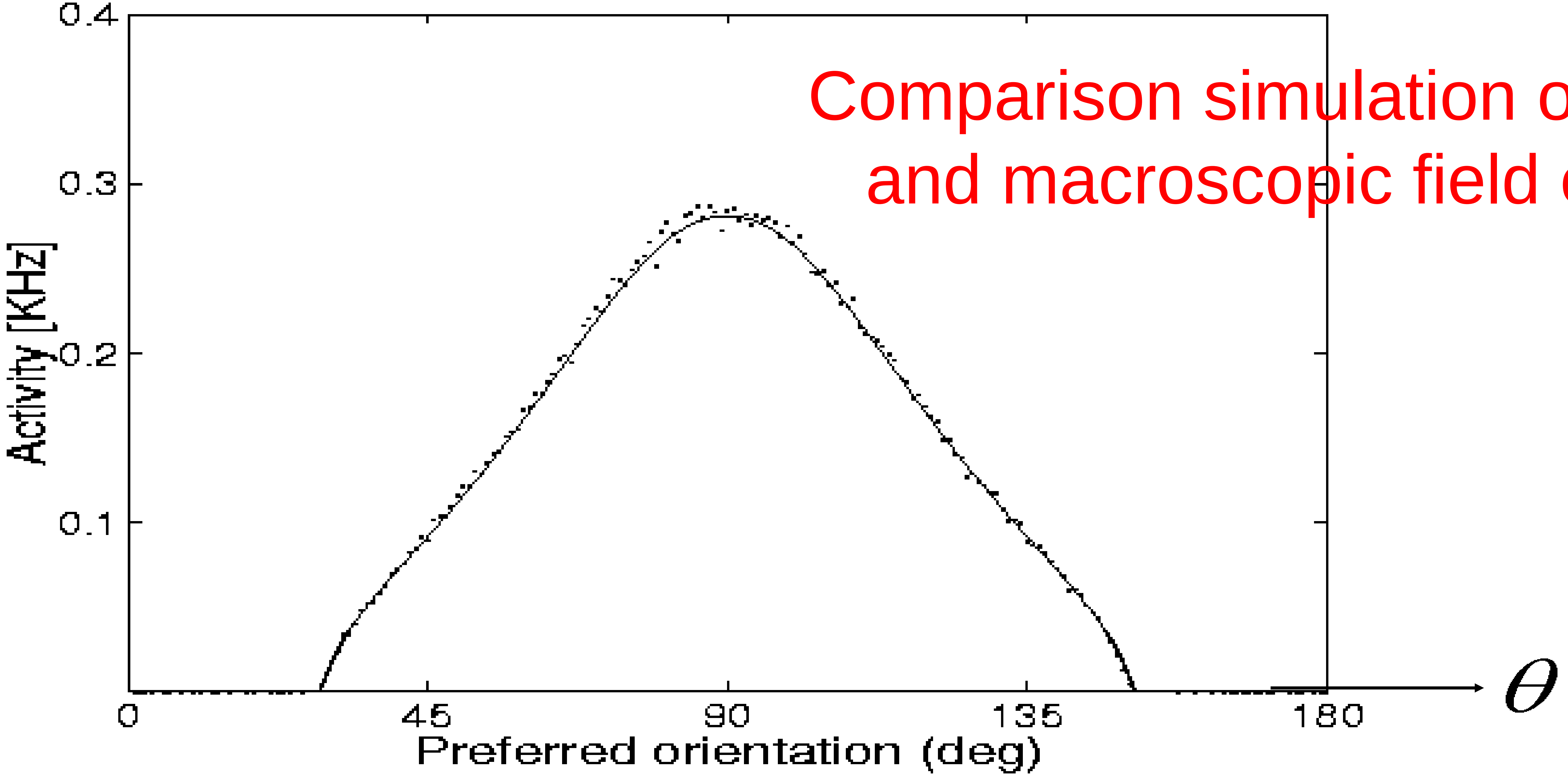
Consider a continuum model,
Find analytically the bump solutions



Week 11-part 3: Solution types: bump solution

Spiridon&Gerstne

$$A(\theta, t) = A(\theta)$$

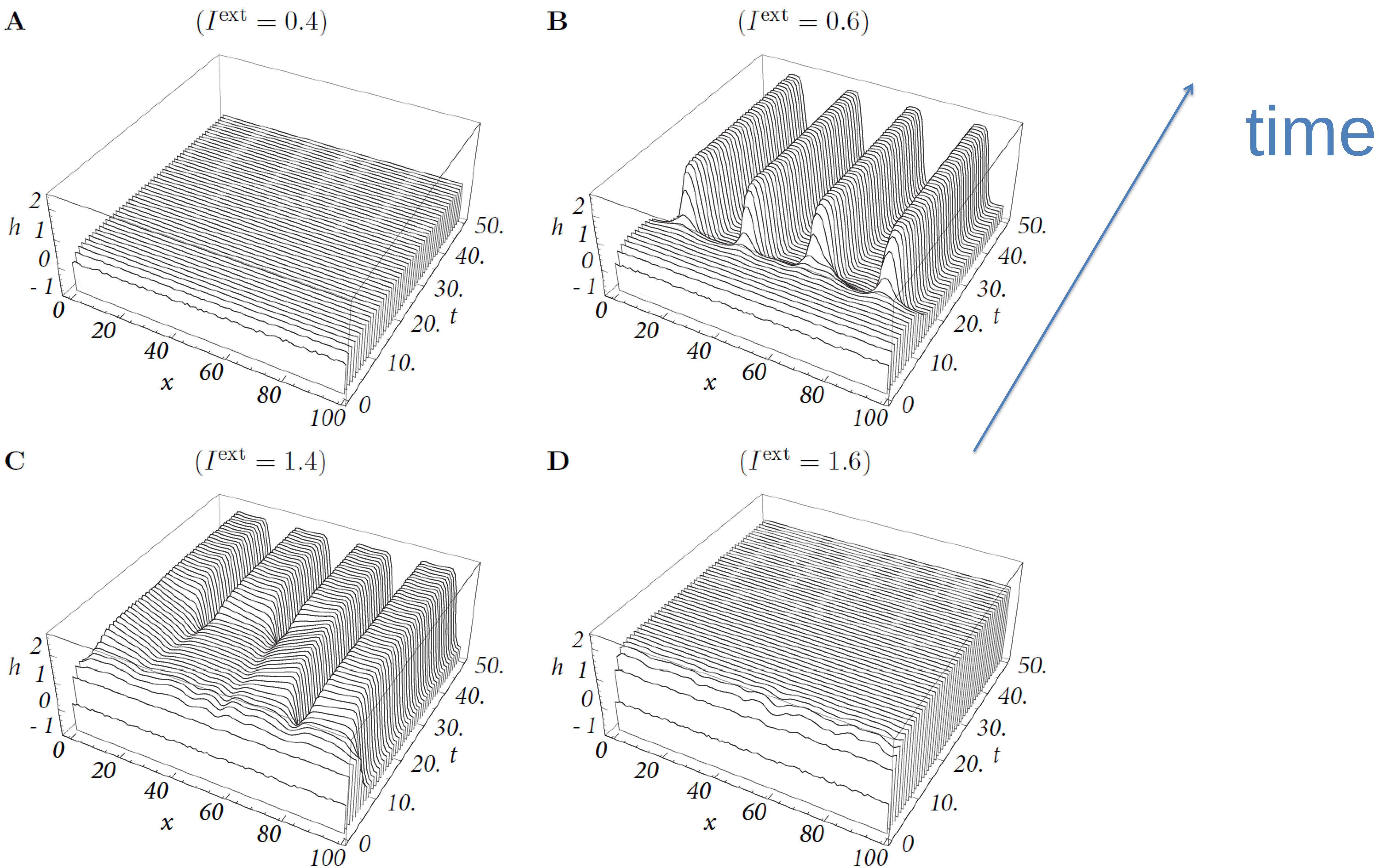


Comparison simulation of neurons and macroscopic field equation

Continuum: stationary profile

See: Chapter 9, book: Spiking Neuron Models, W. Gerstner and W. Kistler, 2002

Week 11-part 3: Solution types (continuum model)



Week 11 – part 4:

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 11 – Continuum models: Cortical fields and perception

Wulfram Gerstner

EPFL, Lausanne, Switzerland

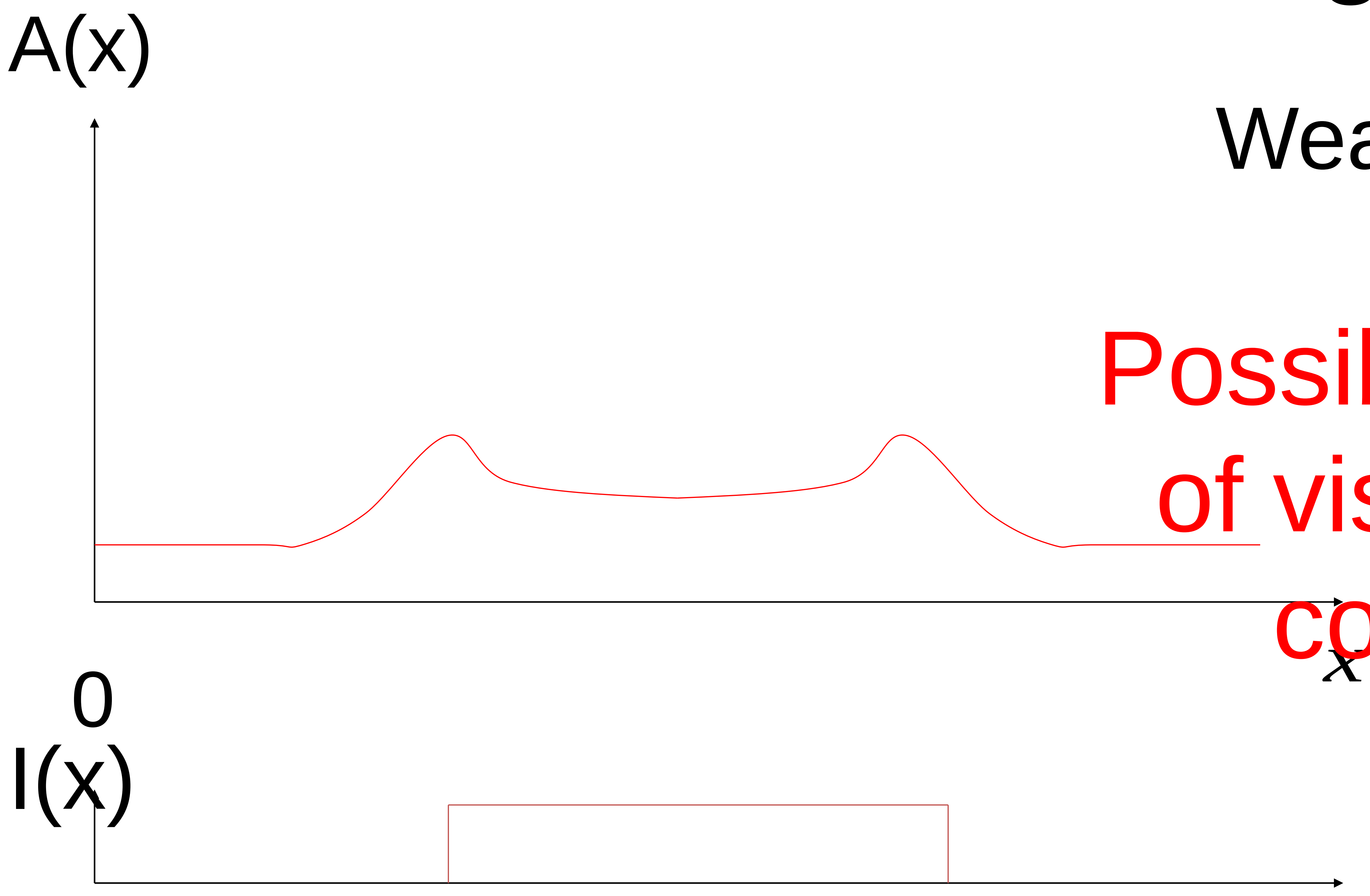
Week 11-part 5: uniform/input driven solution

Field Equations:
Wilson and Cowan, 1972

Basic phenomenology

I. Edge enhancement

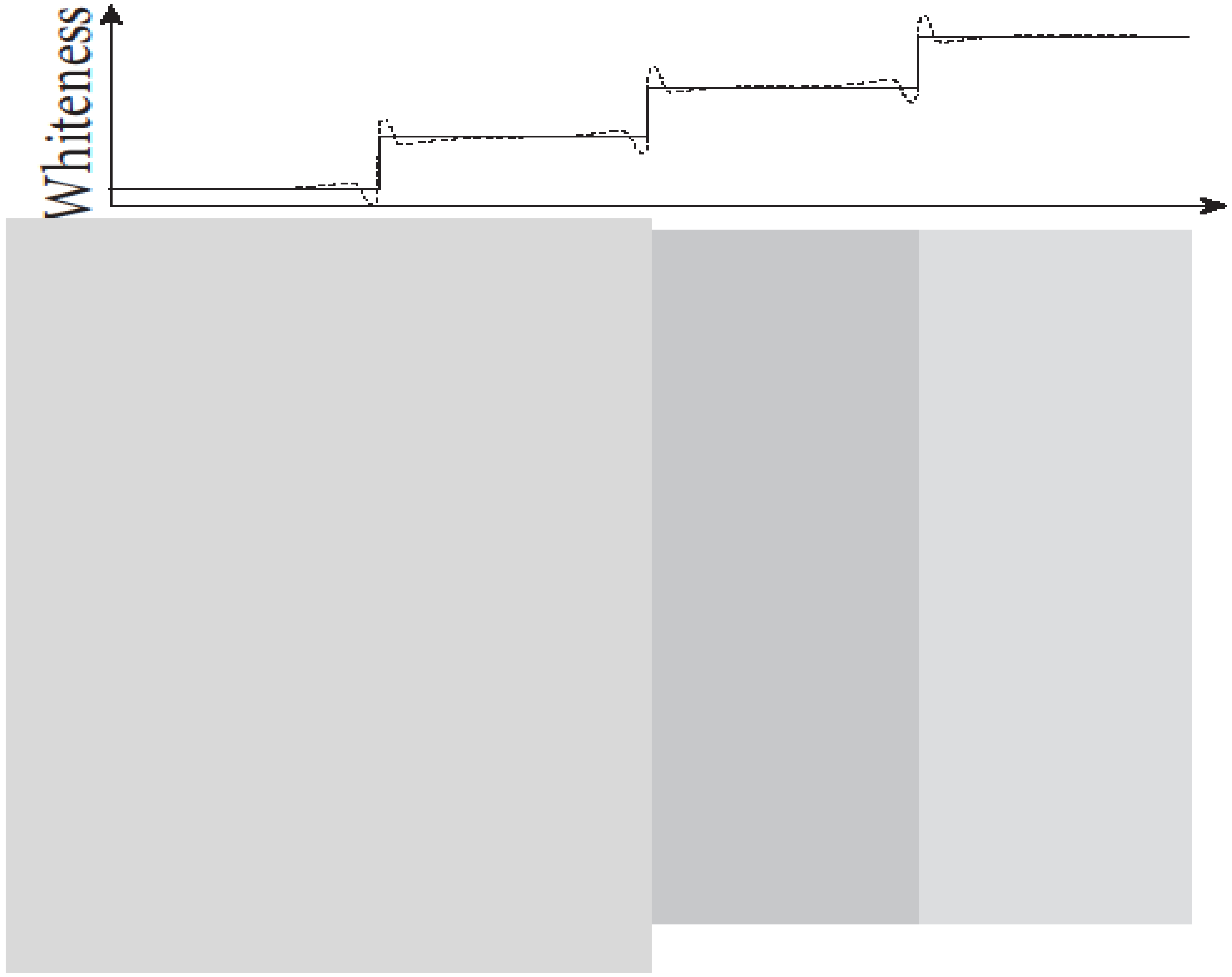
Weaker lateral connectivity



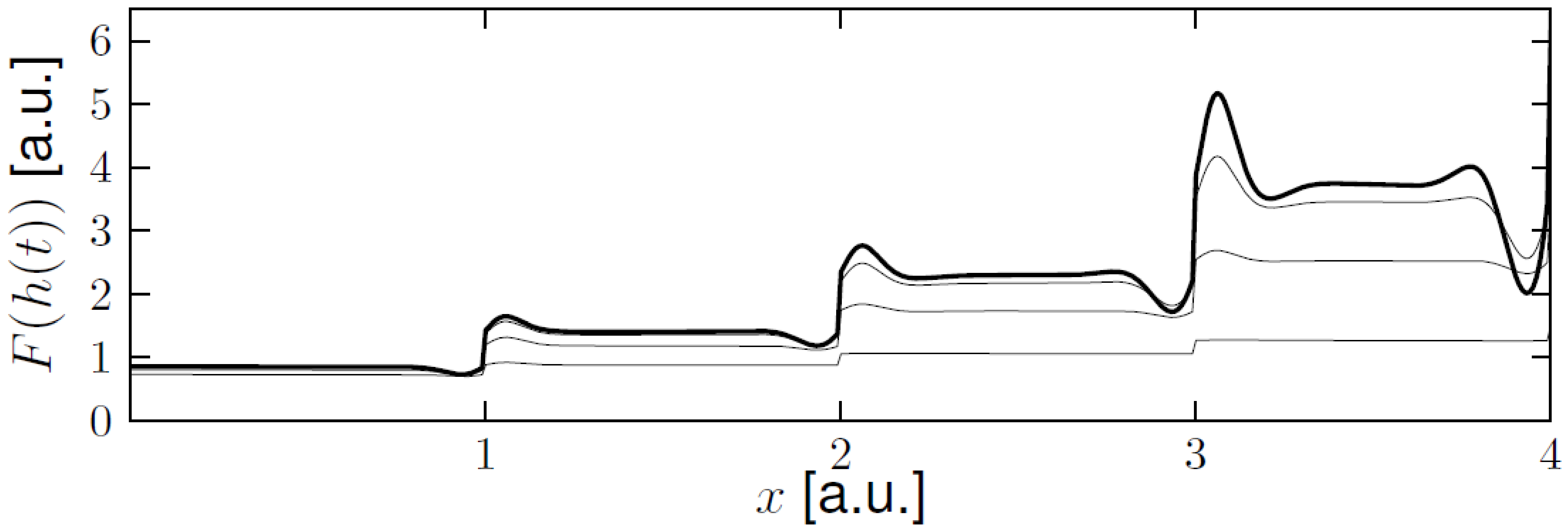
Possible interpretation
of visual cortex cells:
contrast enhancement in
- orientation
- location

Continuum models: grid illusion

Continuum models: Mach Bands

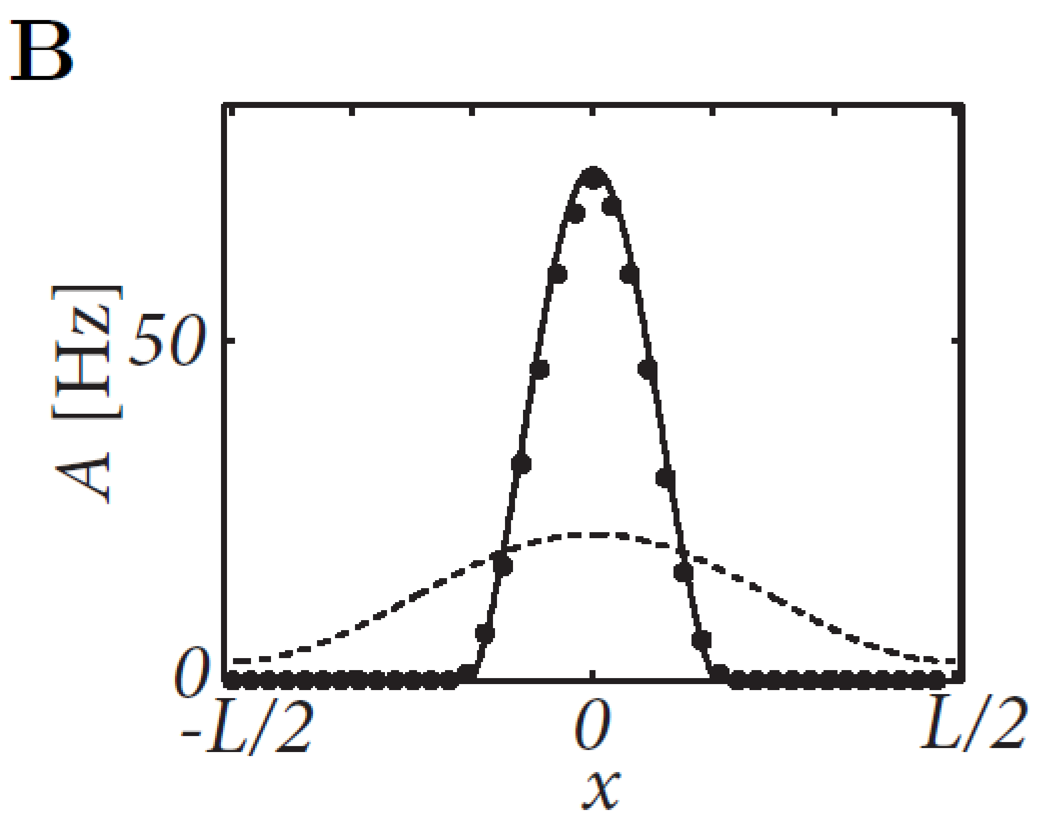


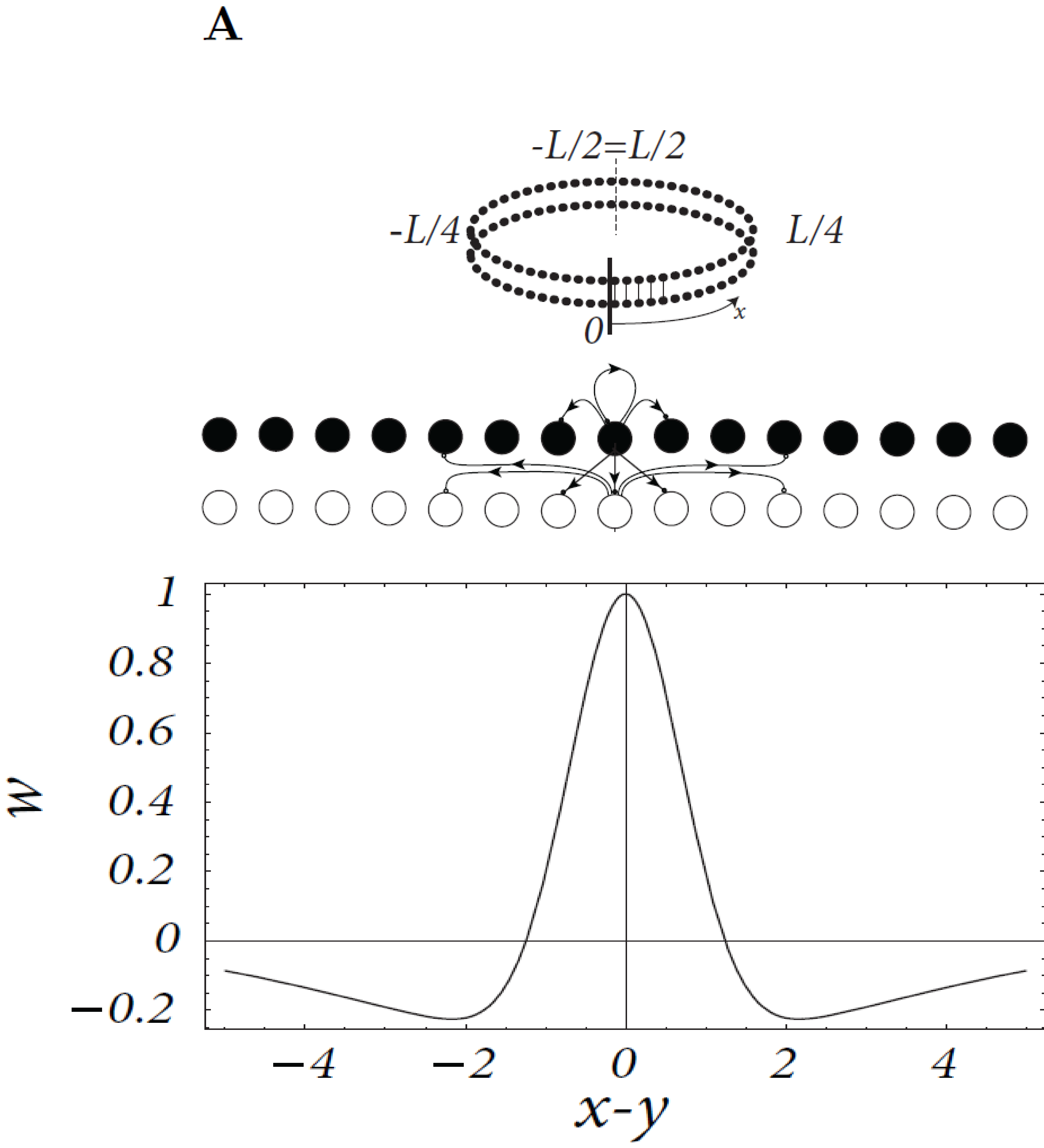
Week 11-part 4: Field models and Perception



ig. 18.9: **A**. Mach bands in a field model with mexican hat

Week 11-part 4: Field models and Perception





Week 11-part 4: Field models and Perception

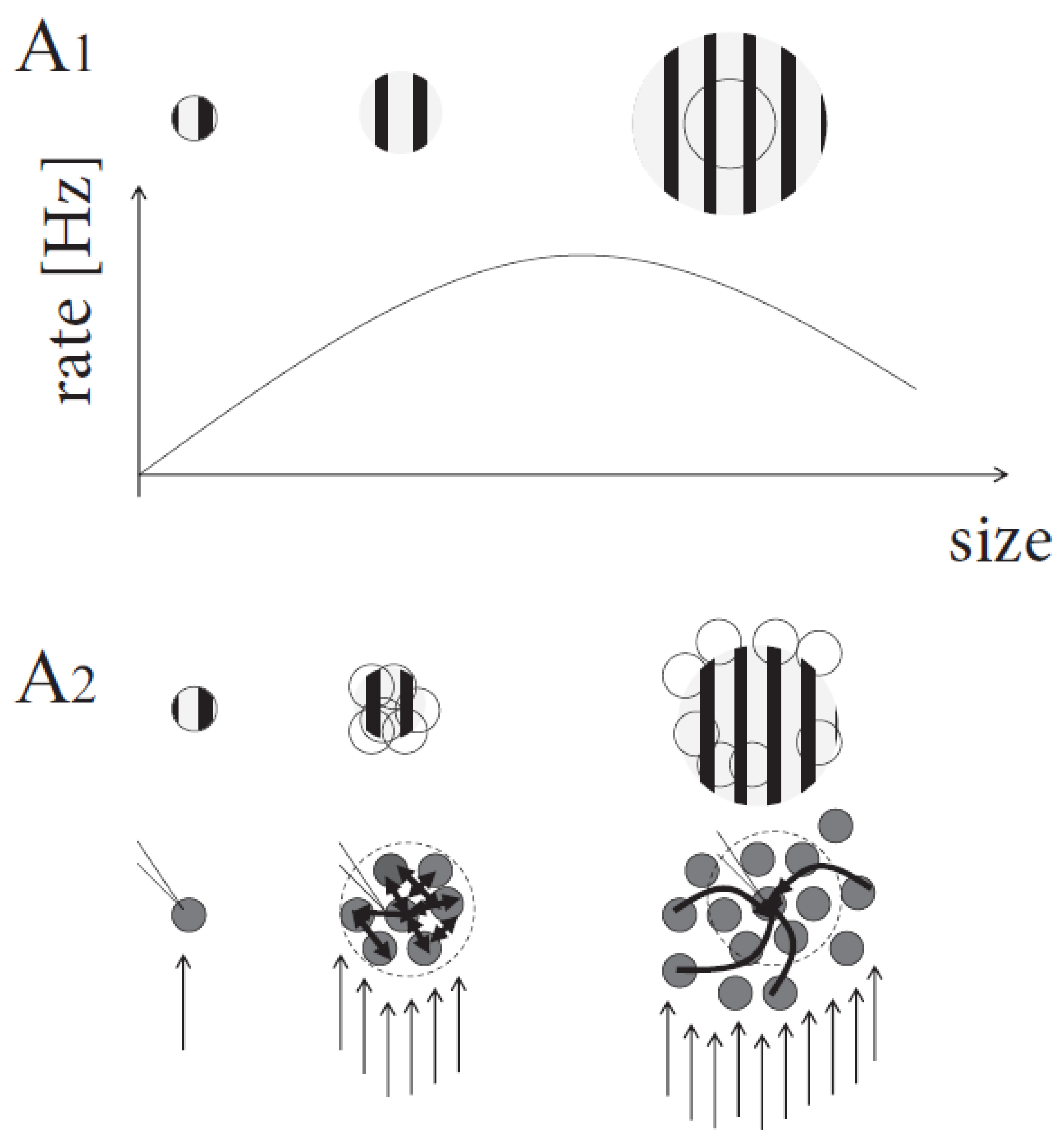


Fig. 18.12: Surround suppression.

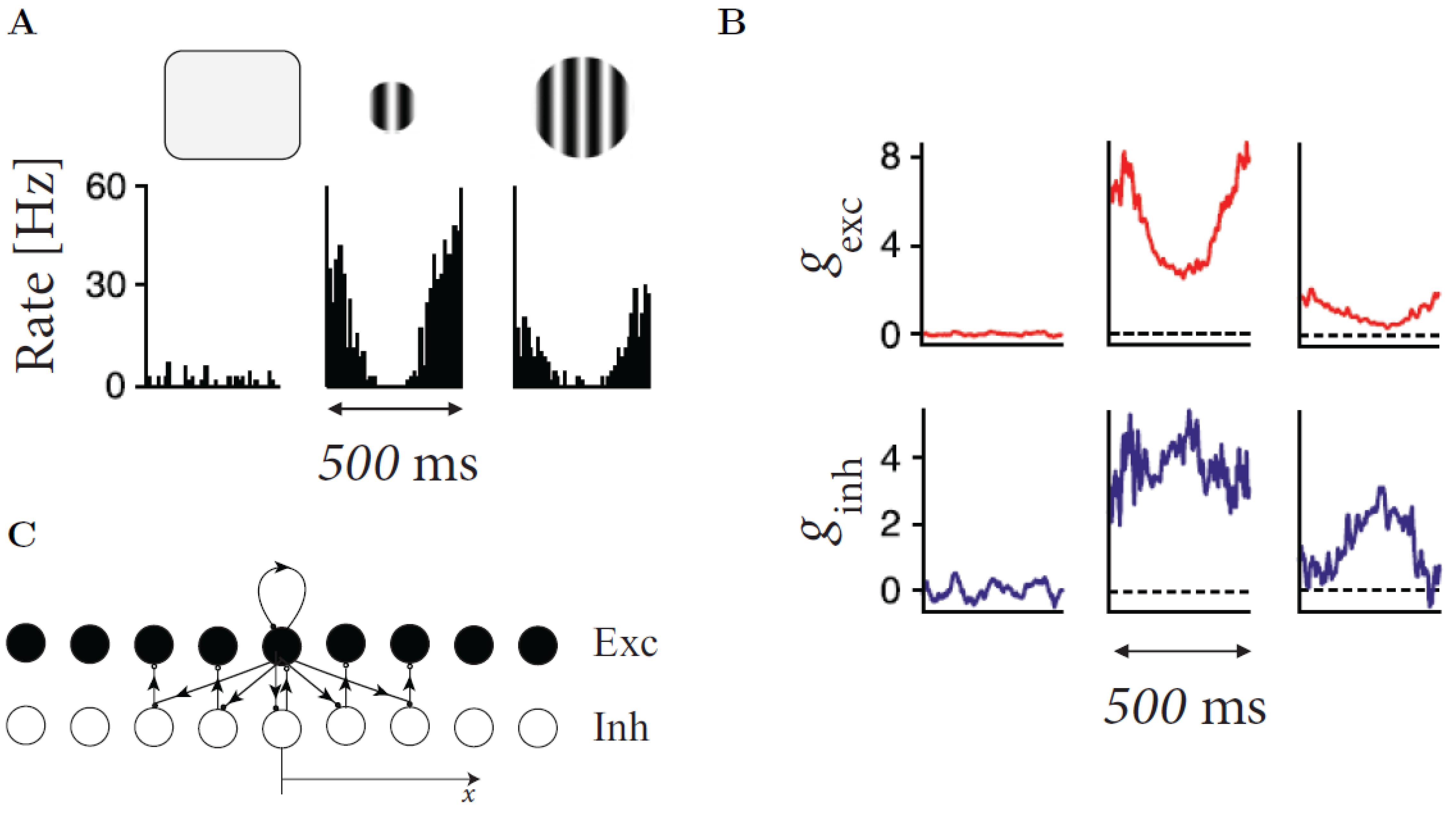


Fig. 18.13: Network stabilized by local inhibition. The schematic model could potentially explain why larger gratings lead not only to less excitatory input g_{exc}, but also to less inhibitory input g_{inh}. **A**. The firing rate as a function of the phase of the moving grating for the three stimulus conditions (blank screen, small and large grating). **B**.Top: Excitatory input into the cell. Bottom: Inhibitory input into the same cell. As in A, left, middle and right correspond to a blank screen, a small grating and or a large grating. Note that the larger grating leads to a reduction of both excitation and inhibition; adapted from (Ozeki et al., 2009). **C**. Network model with long range excitation and local inhibition. Excitatory neurons within a local population excite themselves (feedback arrow), and also send excitatory input to inhibitory cells (downward arrows). Inhibitory neurons project

Week 11 – part 5:

Biological Modeling of Neural Networks:

Week 11 – Continuum models:

Cortical fields and perception

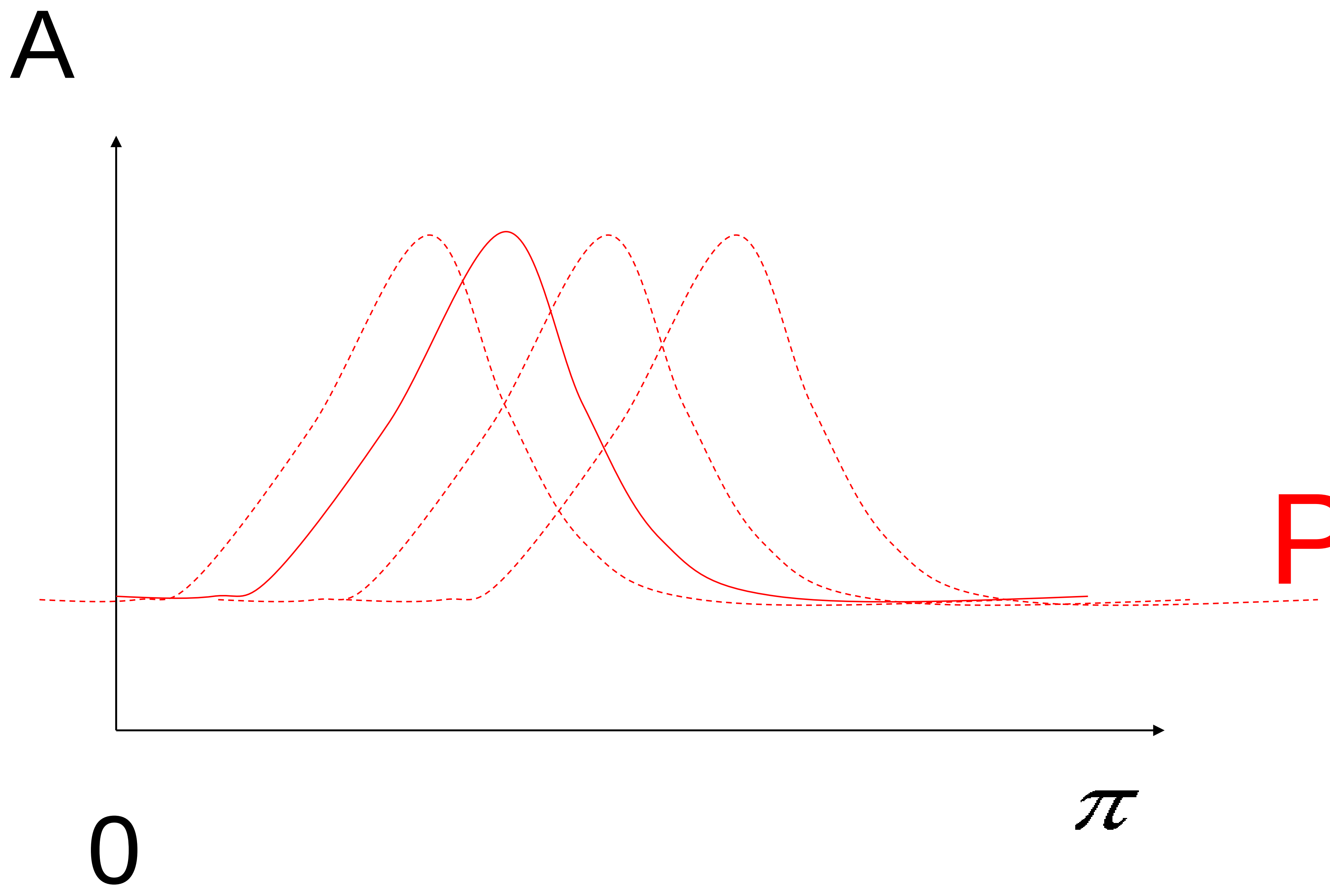
Wulfram Gerstner

EPFL, Lausanne, Switzerland

Week 11-part 5: Bump solution

Basic phenomenology

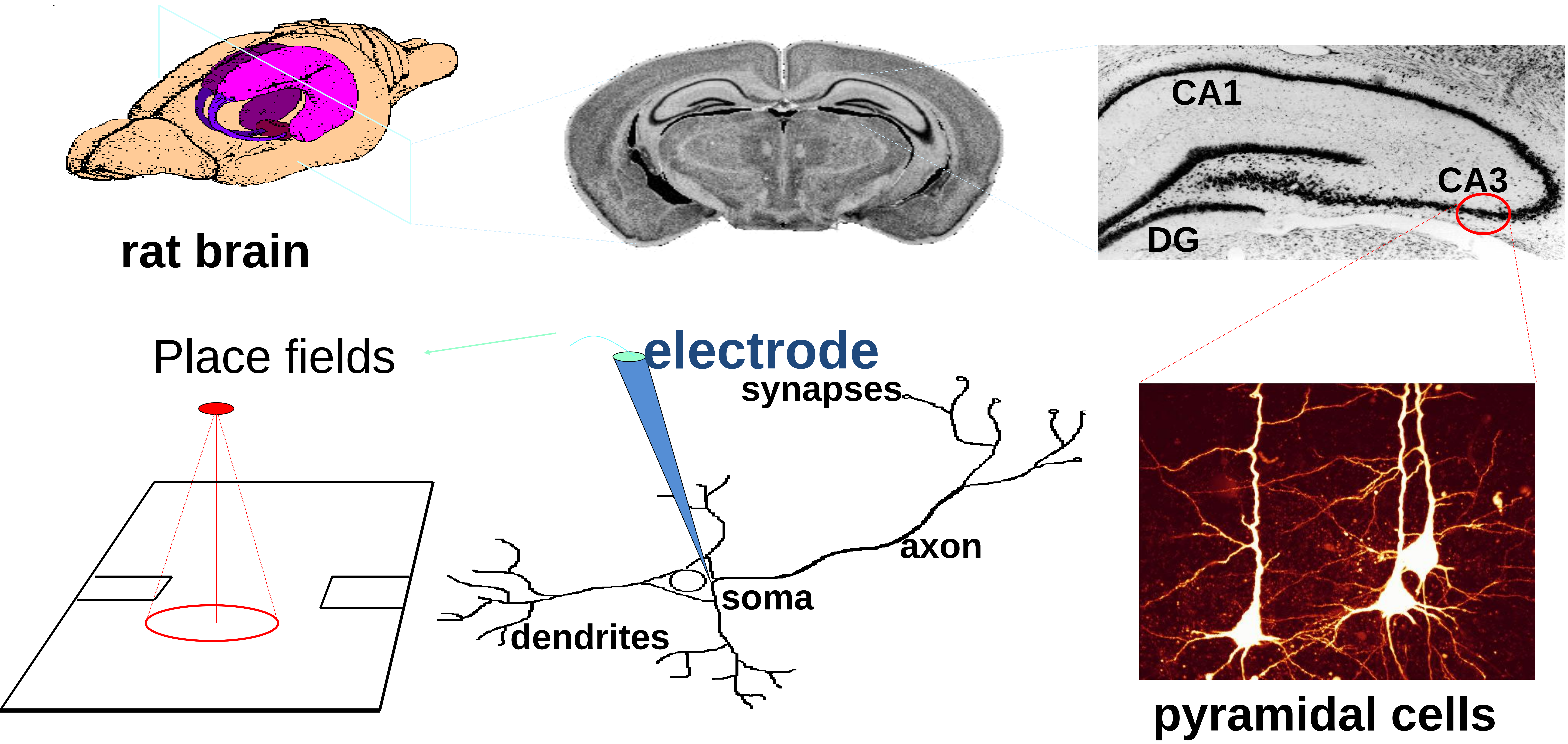
II: Bump formation



strong lateral connectivity

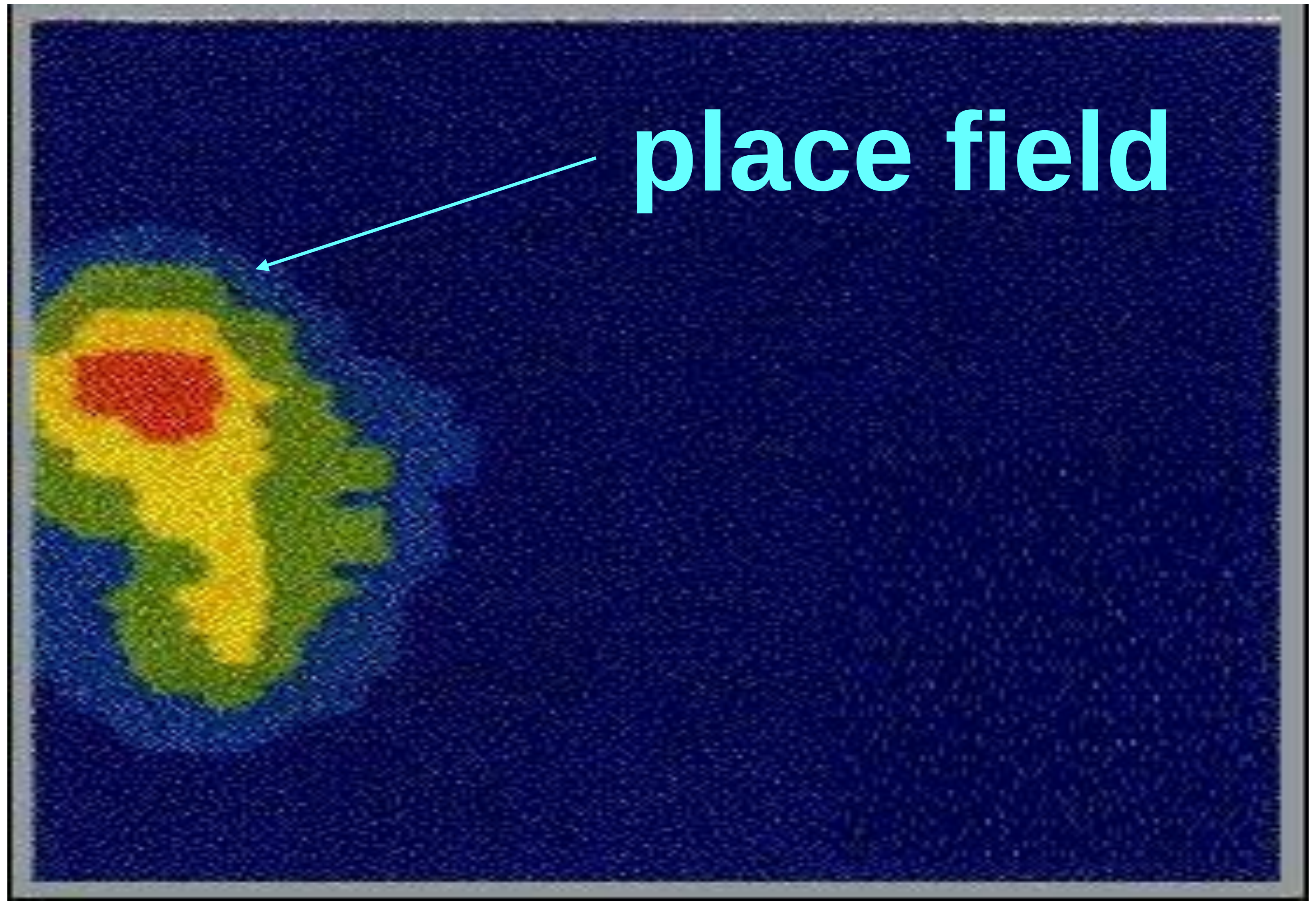
Possible interpretation
of head direction cells:
always some cells active
→ indicate current orientation

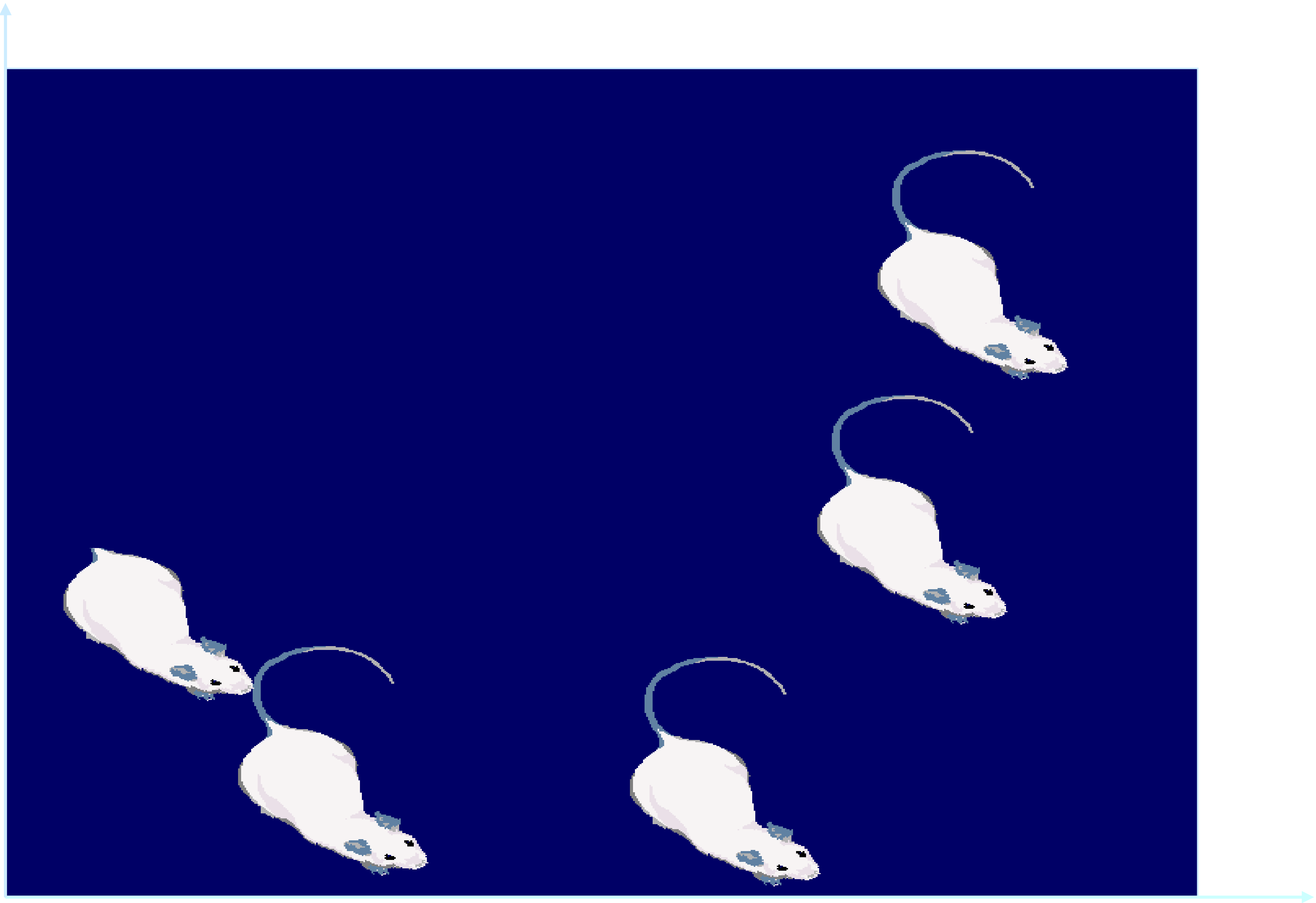
Week 11-part 5: Hippocampal place cells

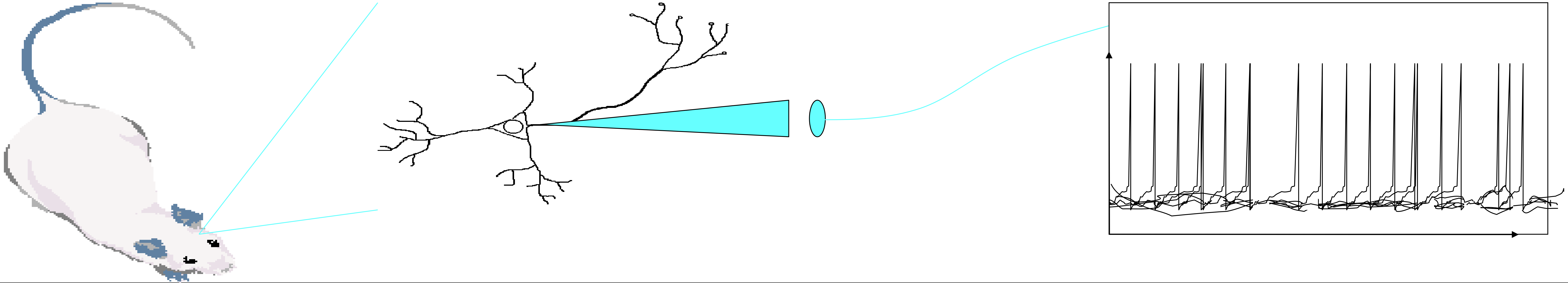


Week 11-part 5: Hippocampal place cells

Main property: encoding the animal's location

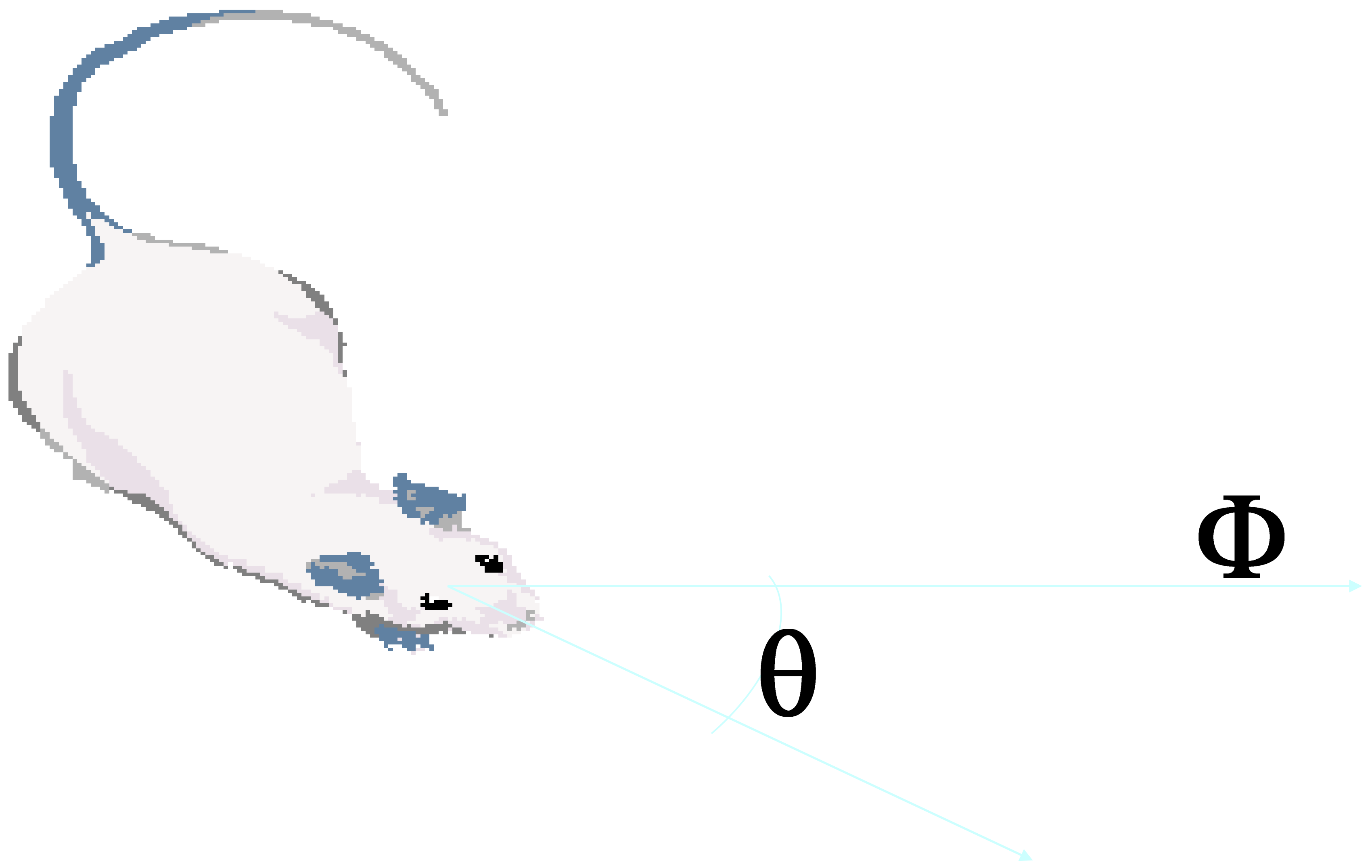




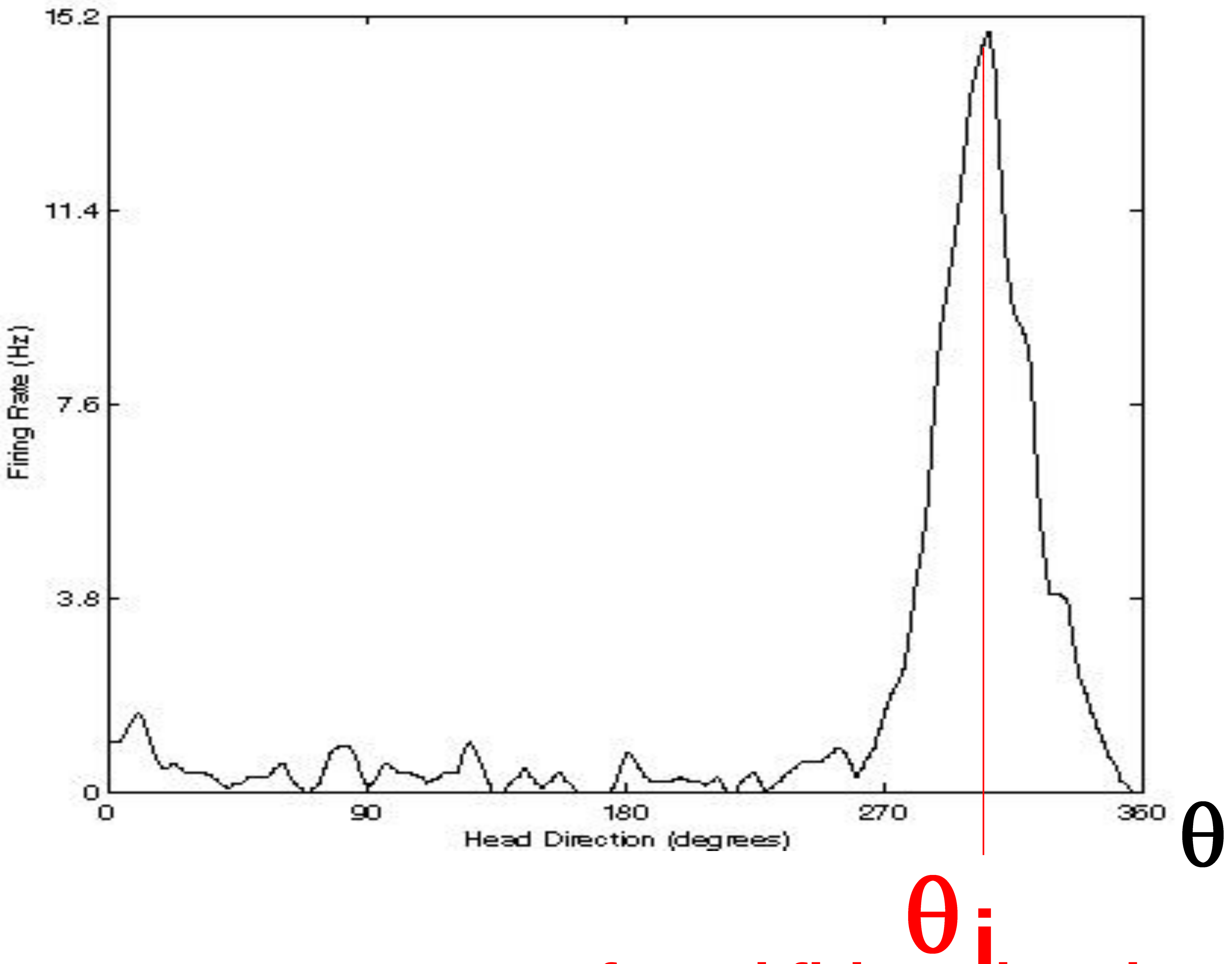


Week 11-part 5: Head direction cells

Main property: encoding the animal 's heading



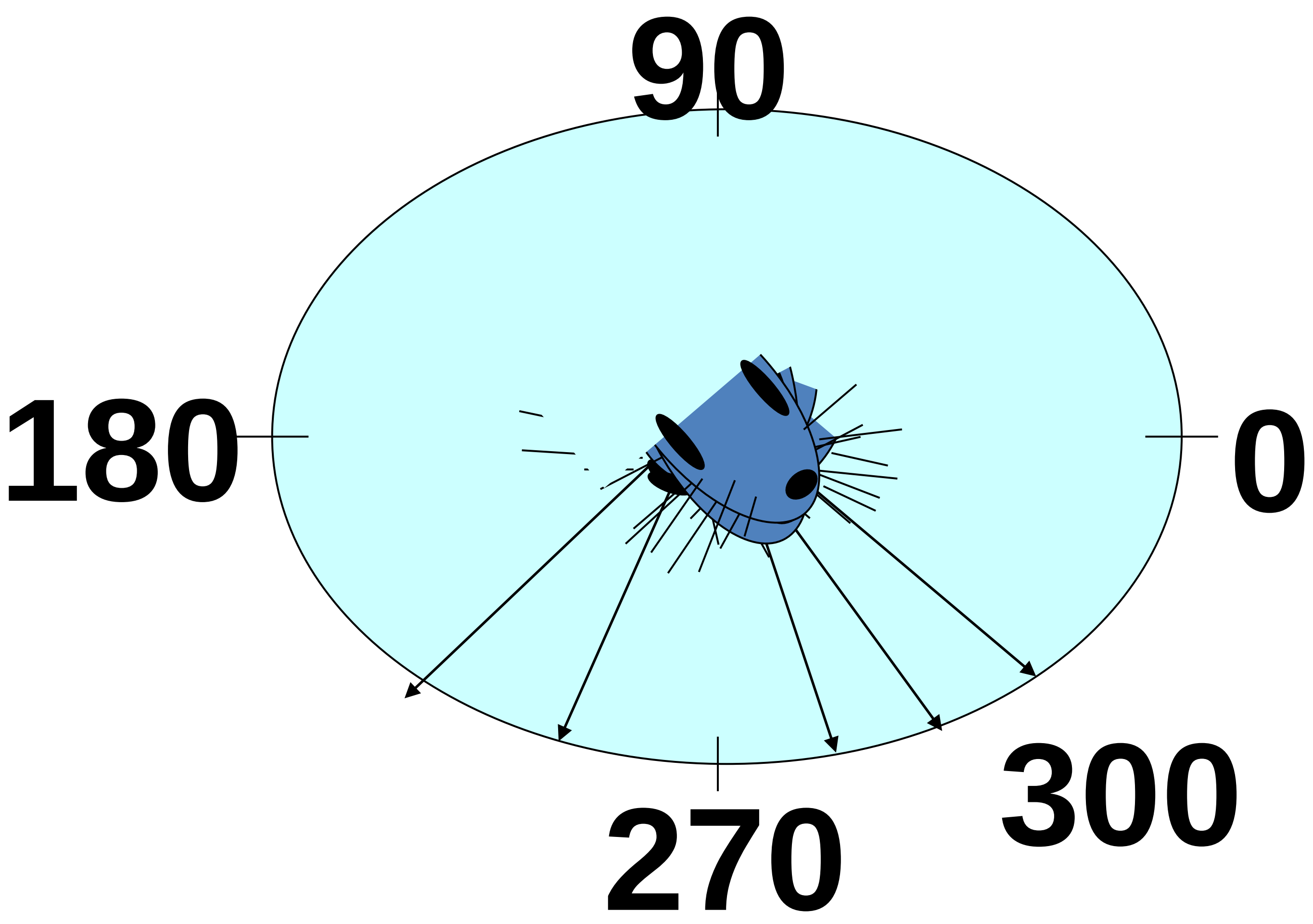
$r_i\ (\theta)$

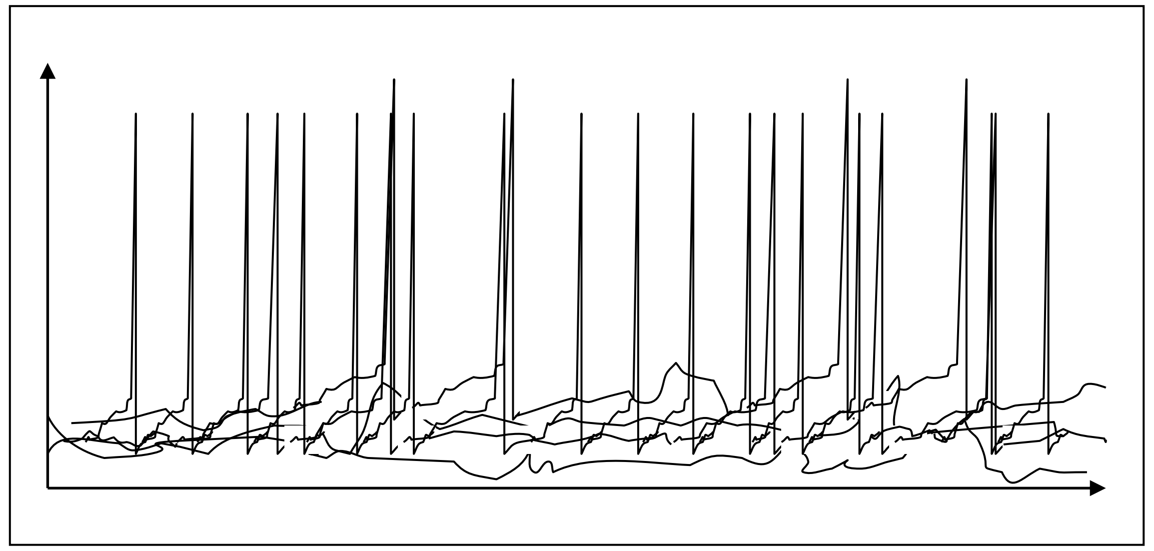


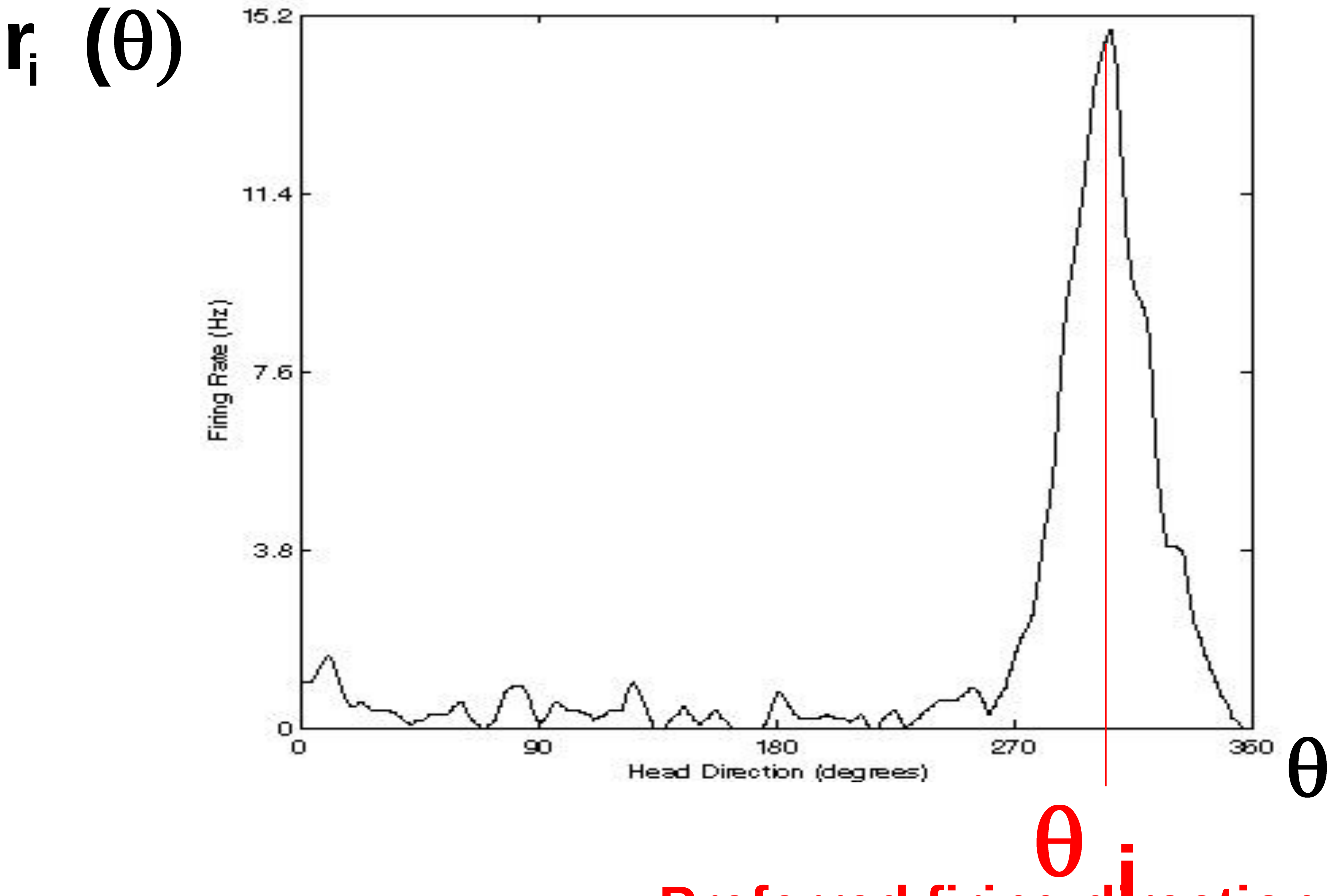
Preferred firing direction

Week 11-part 5: Head direction cells

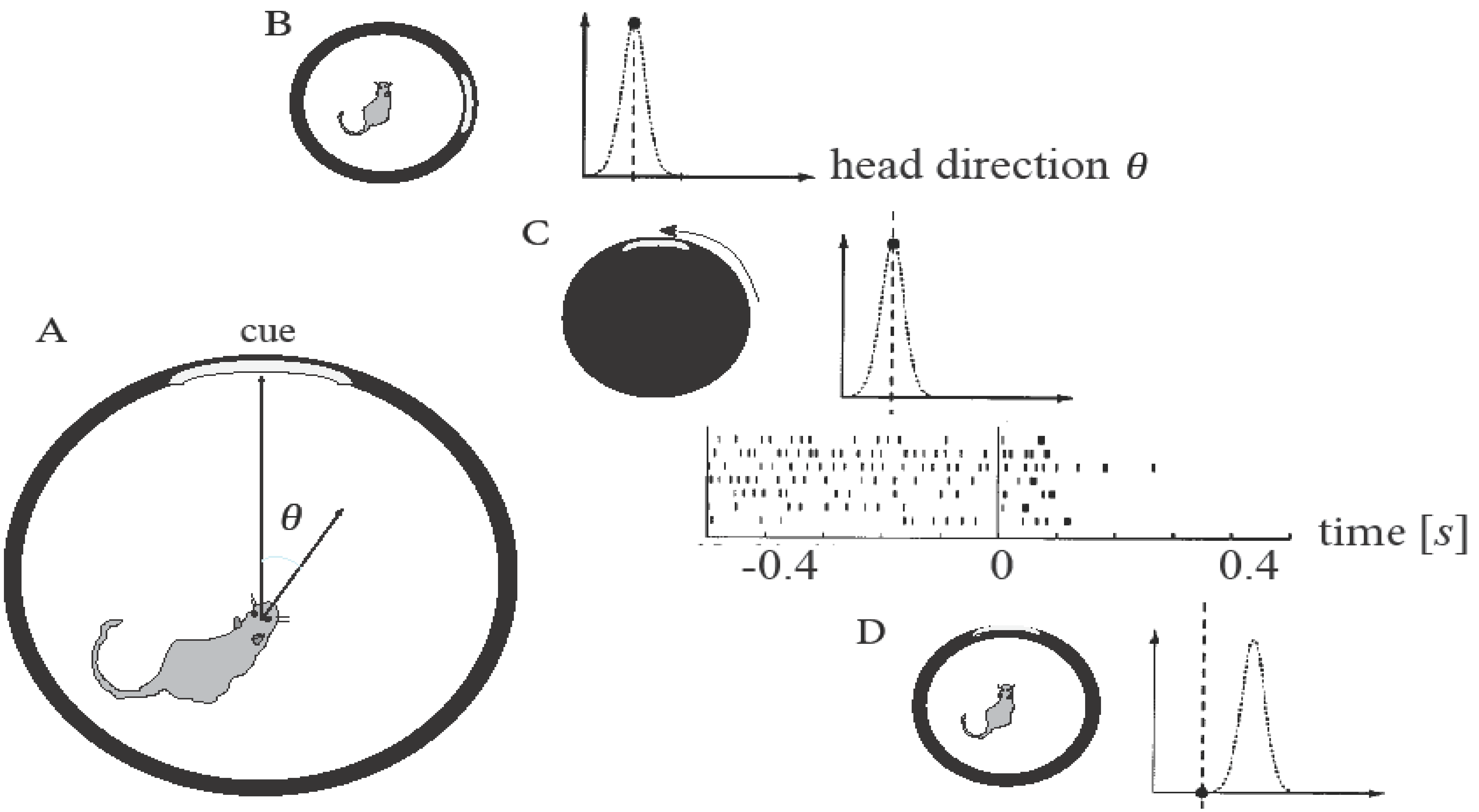
Main property: encoding the animal 's allocentric heading







Week 11-part 5: Head direction cells



Week 11-Continuum models

The END

11.1 Transients

- sharp or slow

11.2 Spatial continuum

- model connectivity
- cortical connectivity

11.3 Solution types

- uniform solution
- bump solution

11.4. Perception

11.5. Head direction cells