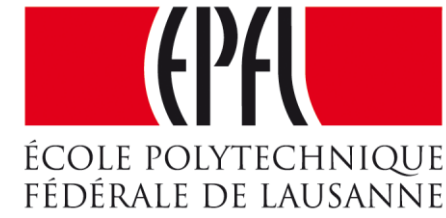


Week 15 – transients and rate models



Biological Modeling of Neural Networks:

Week 15 – Fast Transients and Rate models

Wulfram Gerstner

EPFL, Lausanne, Switzerland

15.1 Review Populations of Neurons

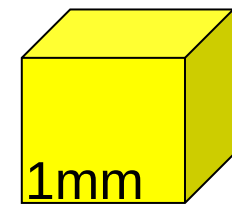
- Transients

15.2 Rate models

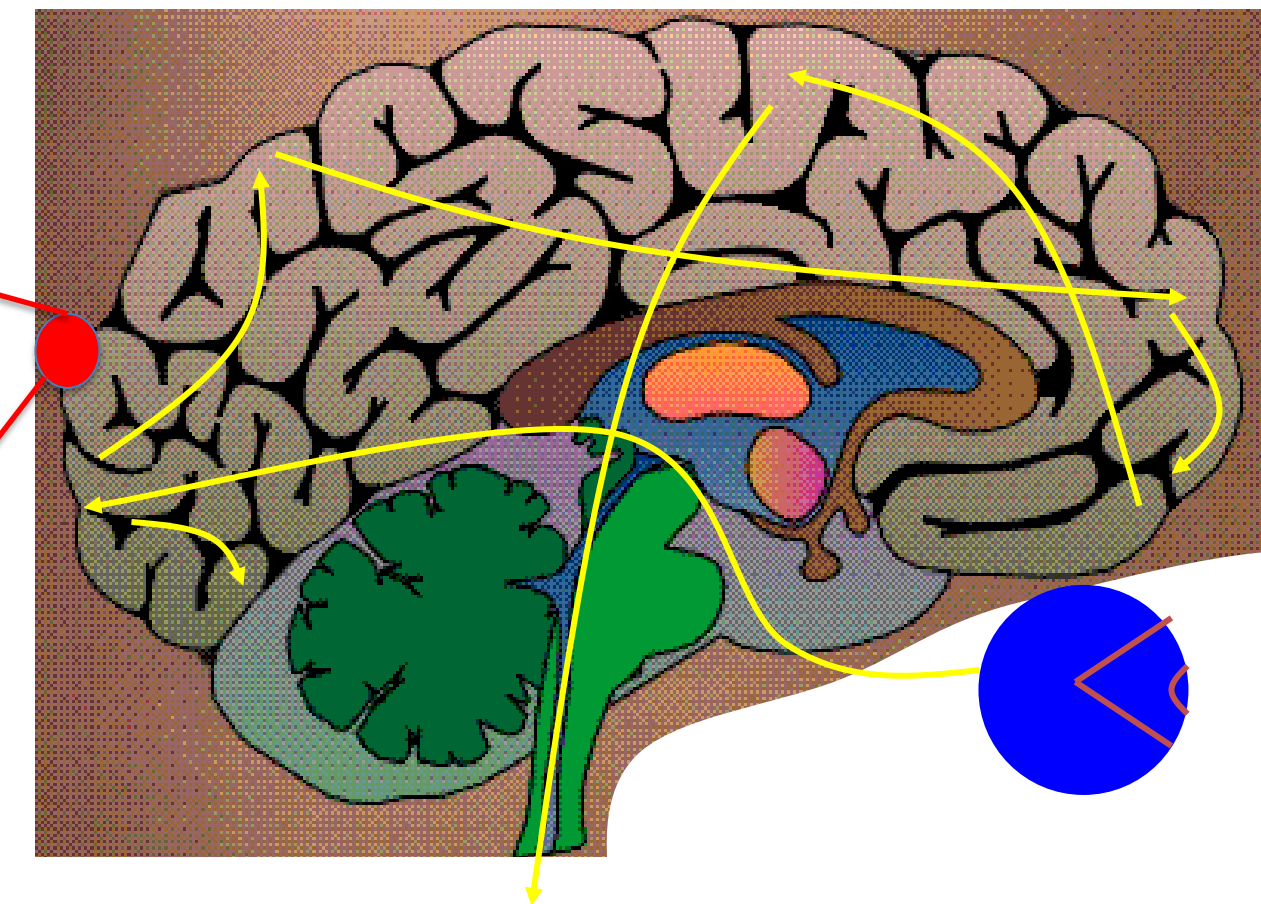
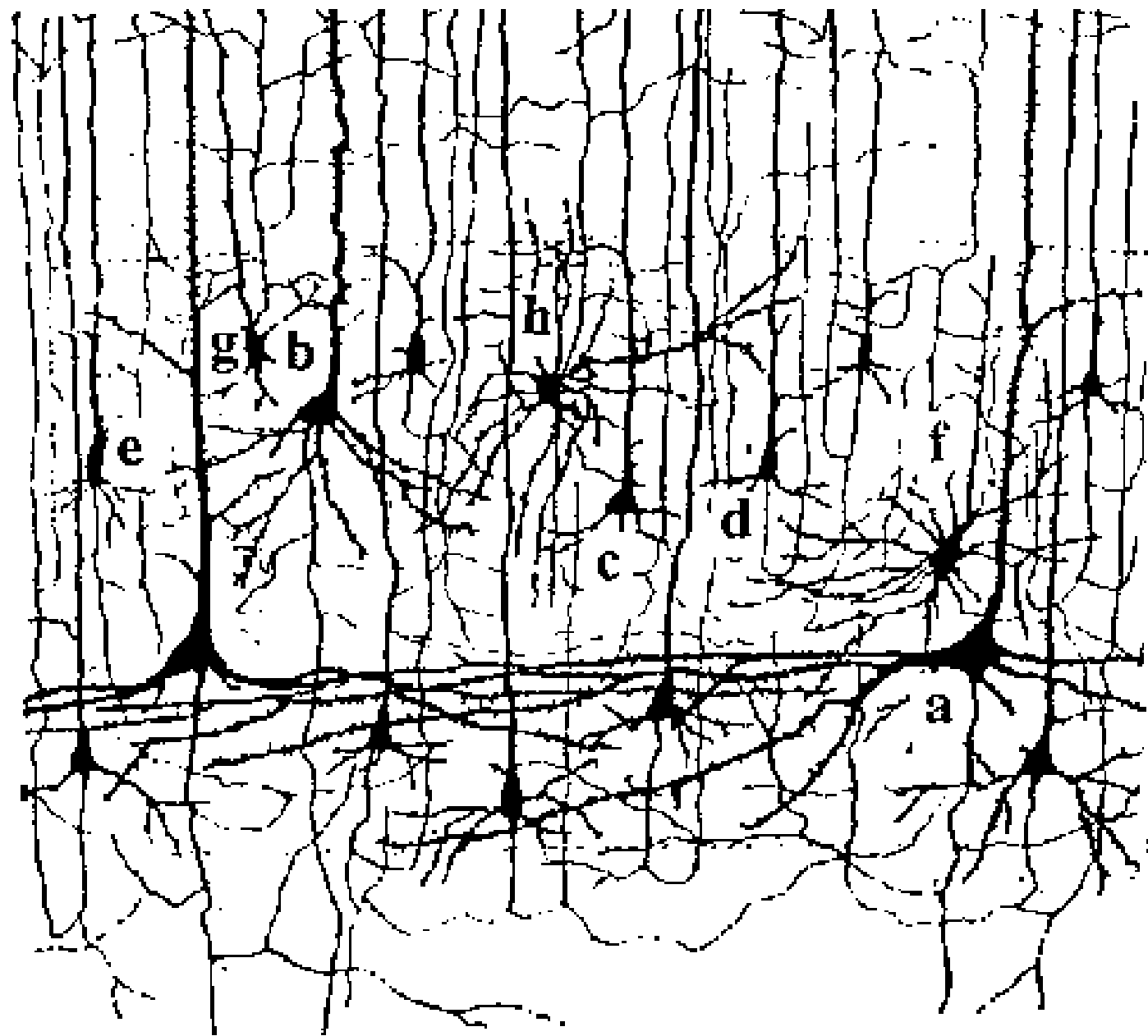
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Week 15-part 1: Review: The brain is complex

Neuronal Dynamics – Brain dynamics is complex



10 000 neurons
3 km of wire

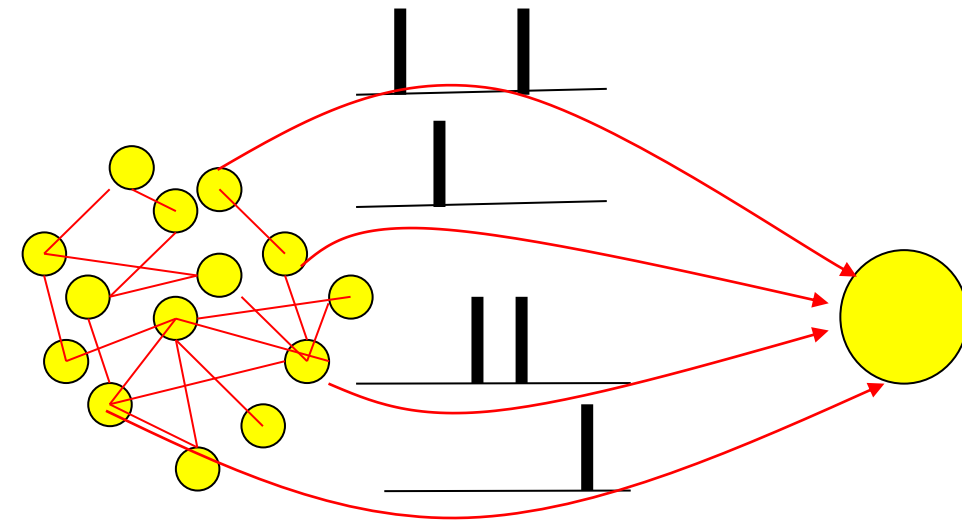


motor
cortex

frontal
cortex

to motor
output

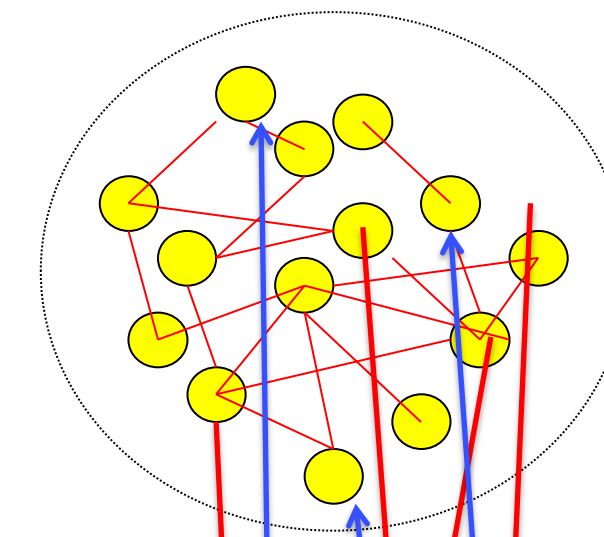
Week 15-part 1: Review: Homogeneous Population



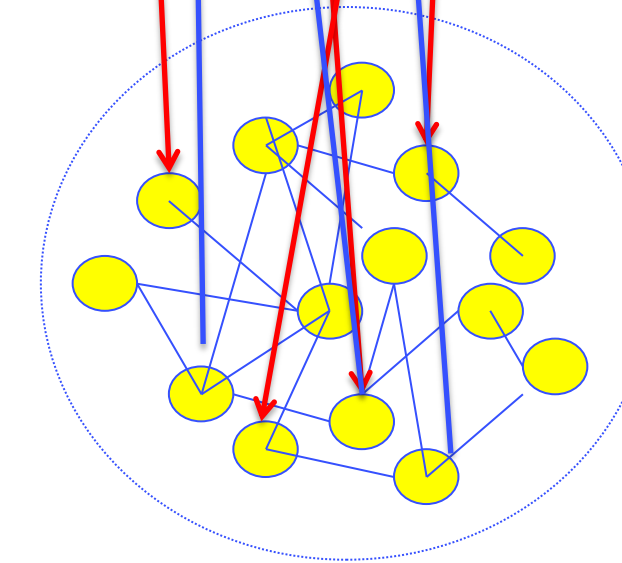
Homogeneous population:

- each neuron receives input from k neurons in the population
- each neuron receives the same (mean) external input (could come from other populations)
- each neuron in the population has roughly the same parameters

Example: 2 populations

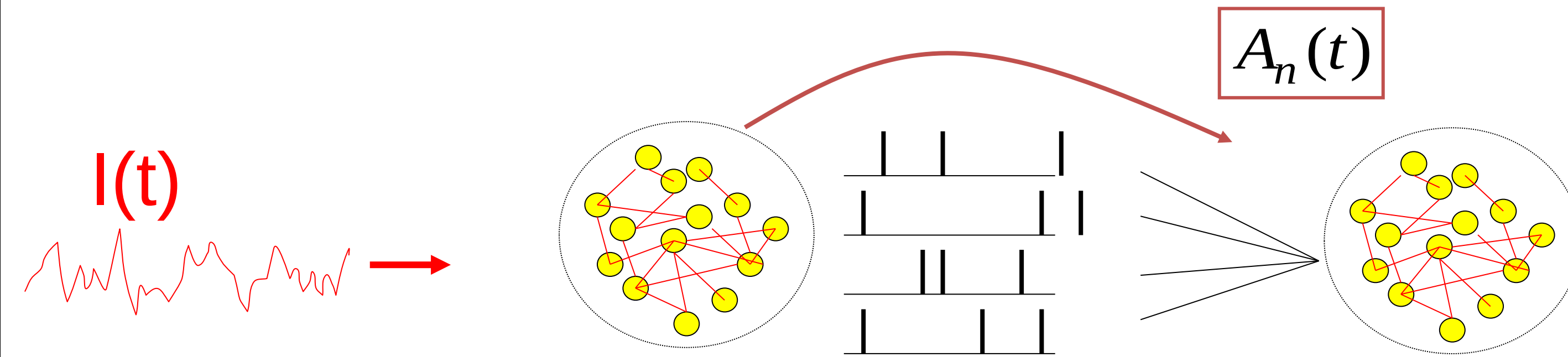


excitation



inhibition

Week 15-part 1: Review: microscopic vs. macroscopic



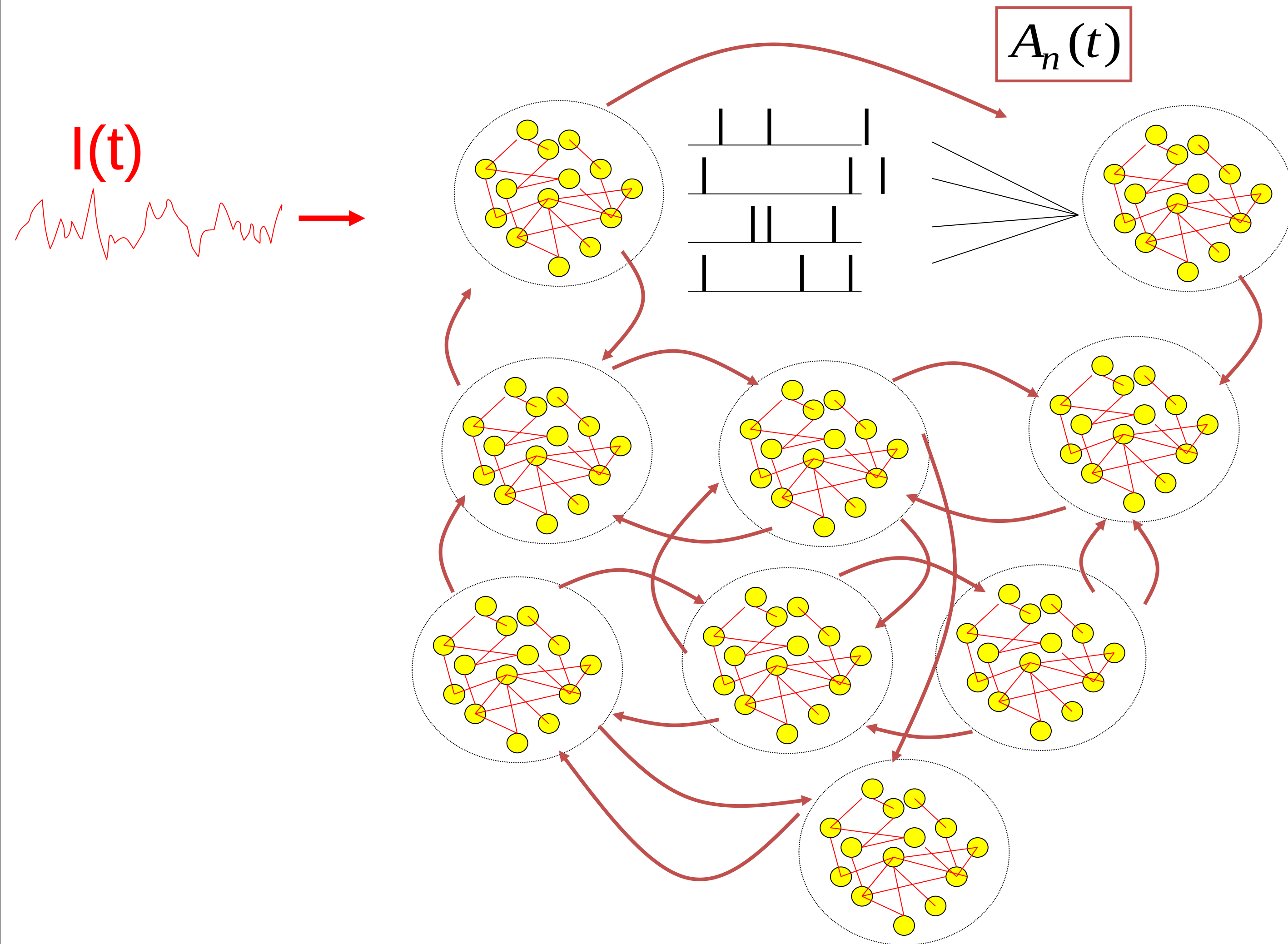
microscopic:

- spikes are generated by individual neurons
- spikes are transmitted from neuron to neuron

macroscopic:

- Population activity $A_n(t)$
- Populations interact *via* $A_n(t)$

Week 15-part 1: Review: coupled populations



Week 15-part 1: Example: coupled populations in barrel cortex

Neuronal Populations

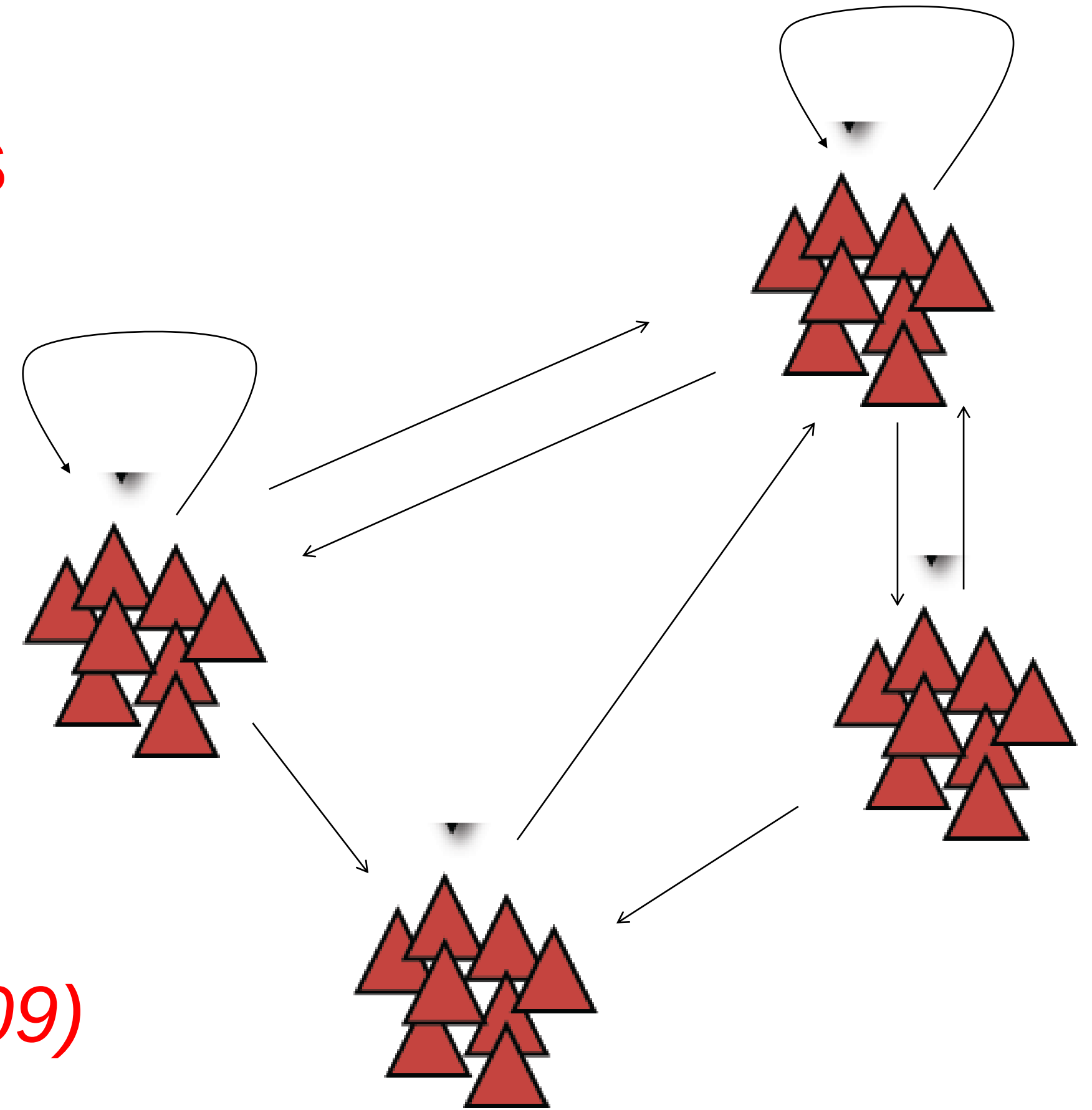
= a homogeneous group of neurons
(more or less)

Populations:

- pyramidal neurons in layer 5 of barrel D1
- pyramidal neurons in layer 2/3 of barrel D1
- inhibitory neurons in layer 2/3 of barrel D1

100-2000 neurons

per group (*Lefort et al., NEURON, 2009*)

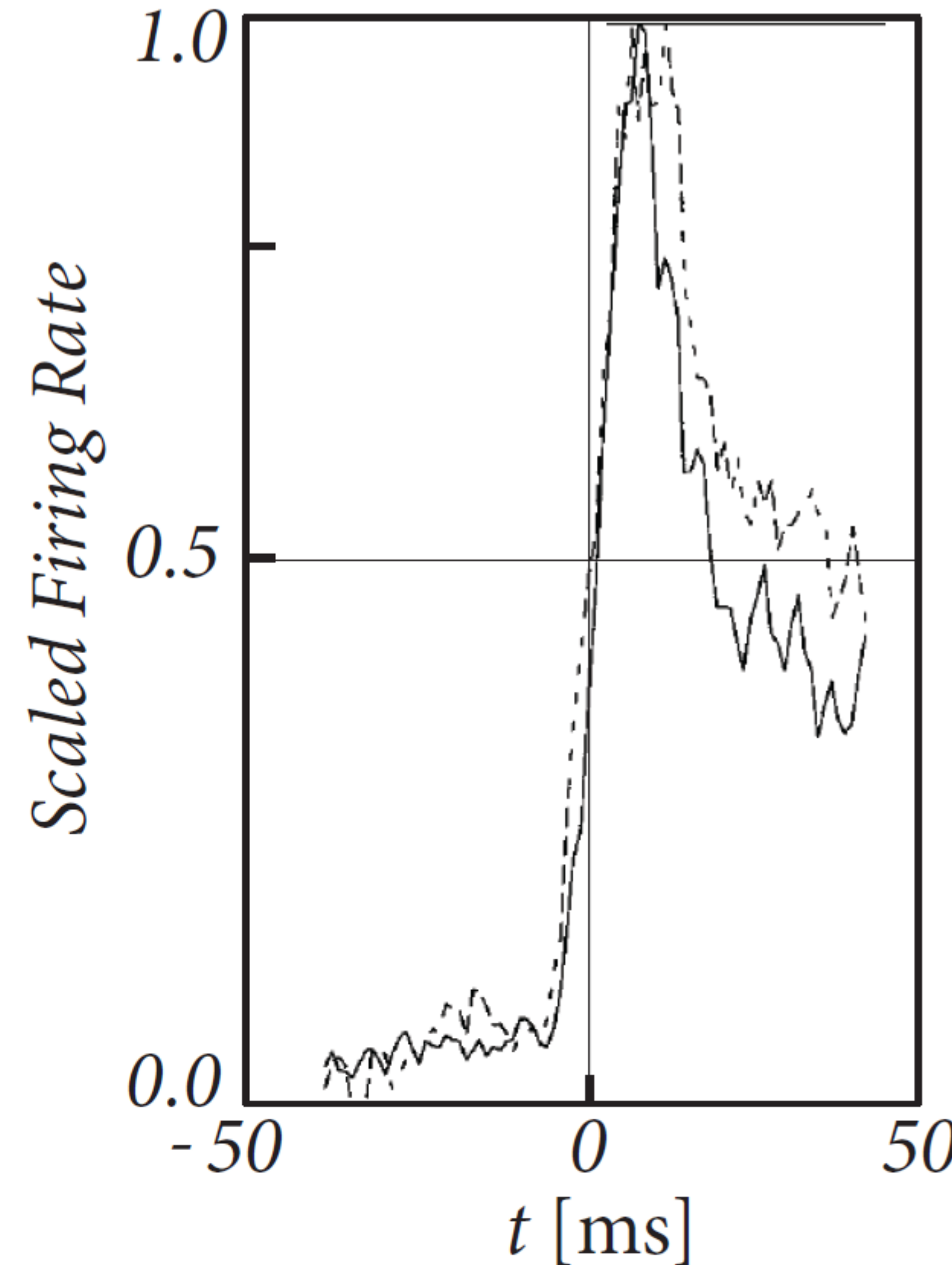
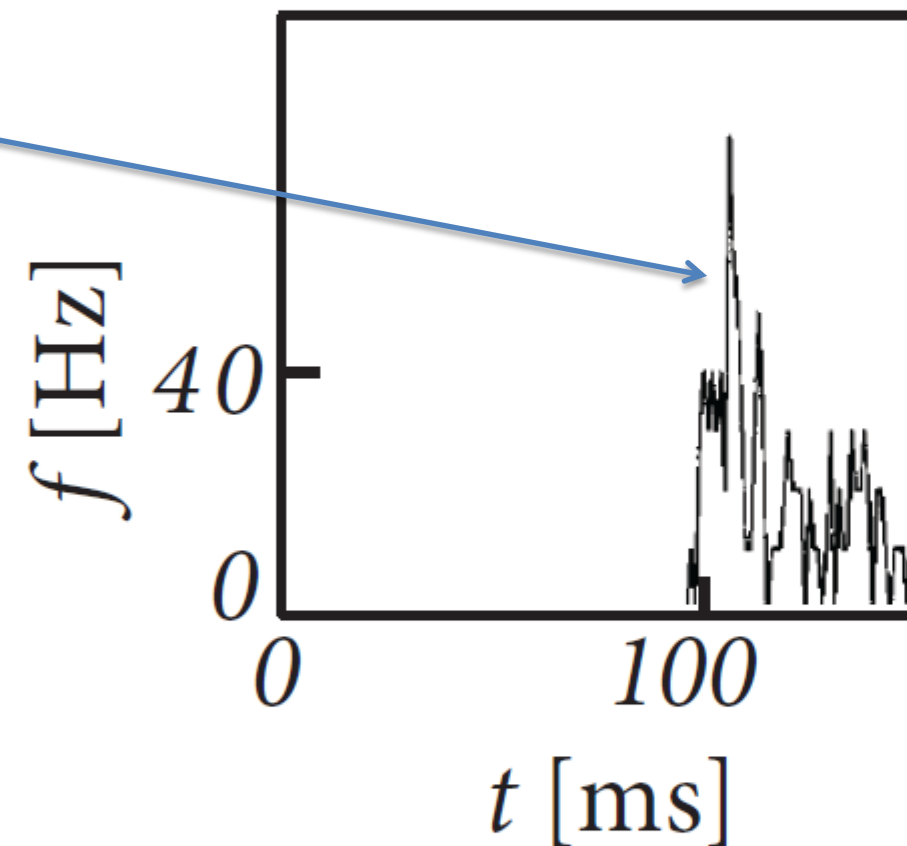
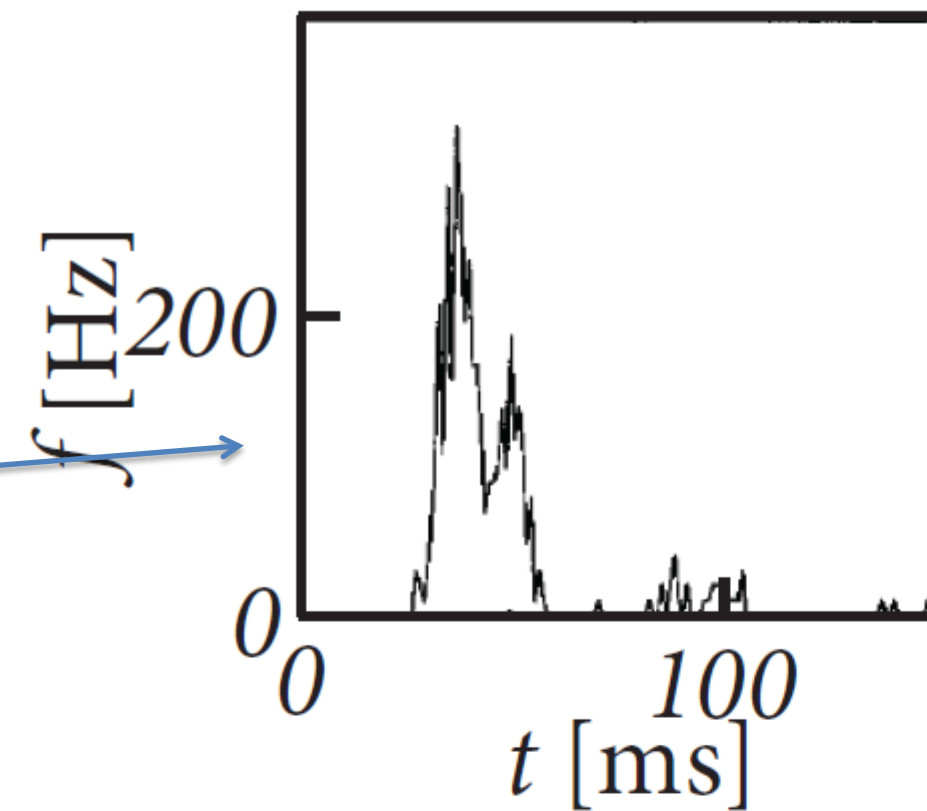


different barrels and layers, different neuron types → different populations

Week 15-part 1: $A(t)$ in experiments: light flash, in vivo

Experimental data: Marsalek et al., 1997

PSTH, single neurons



aligned
PSTH, many
neurons in
V1 (solid)
V4 (dashed)

Fig. 15.3: Transient response of neurons in visual cortex. At $t = 0$ a high-contrast grating is flashed on a gray screen. Top left: A neuron in visual cortex V1 of a behaving monkey responds with a sharp onset after a latency of 27ms, as shown by the PSTH. Bottom left:

A Week 15-part 2: **$A(t)$** in experiments: auditory click stimulus

Experimental data: Sakata and Harris, 2009

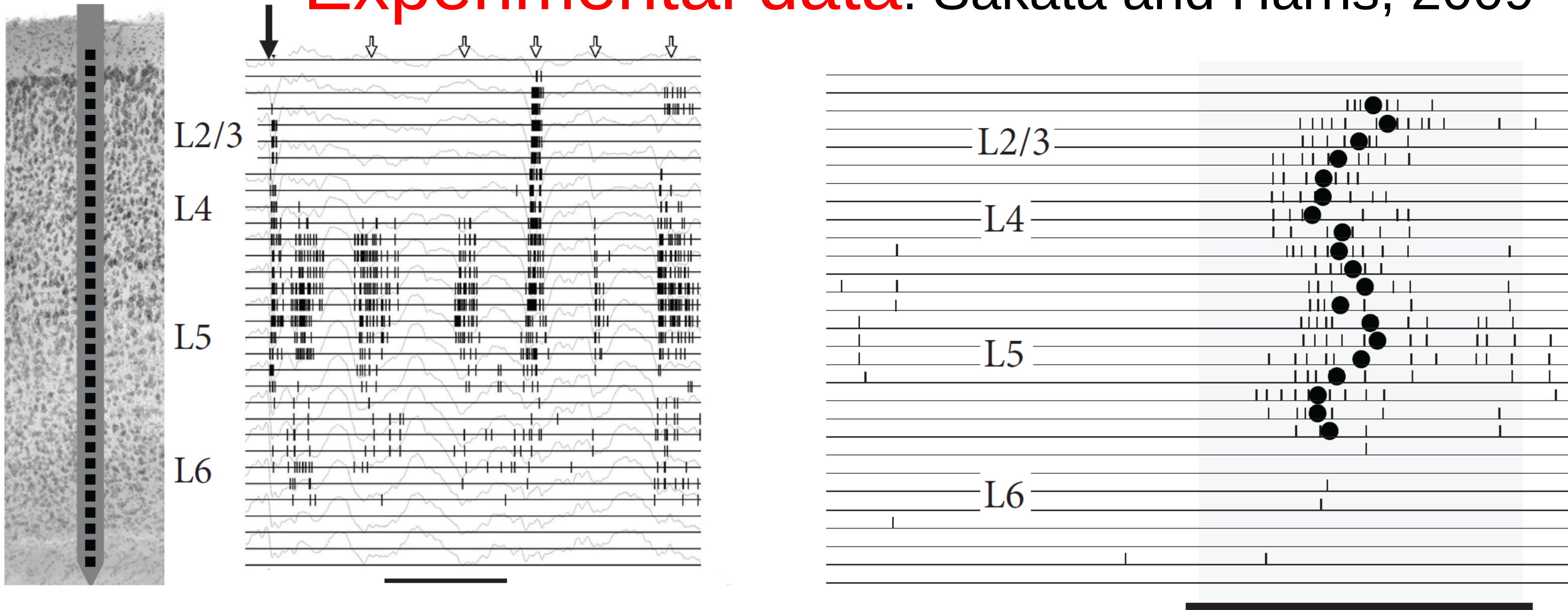
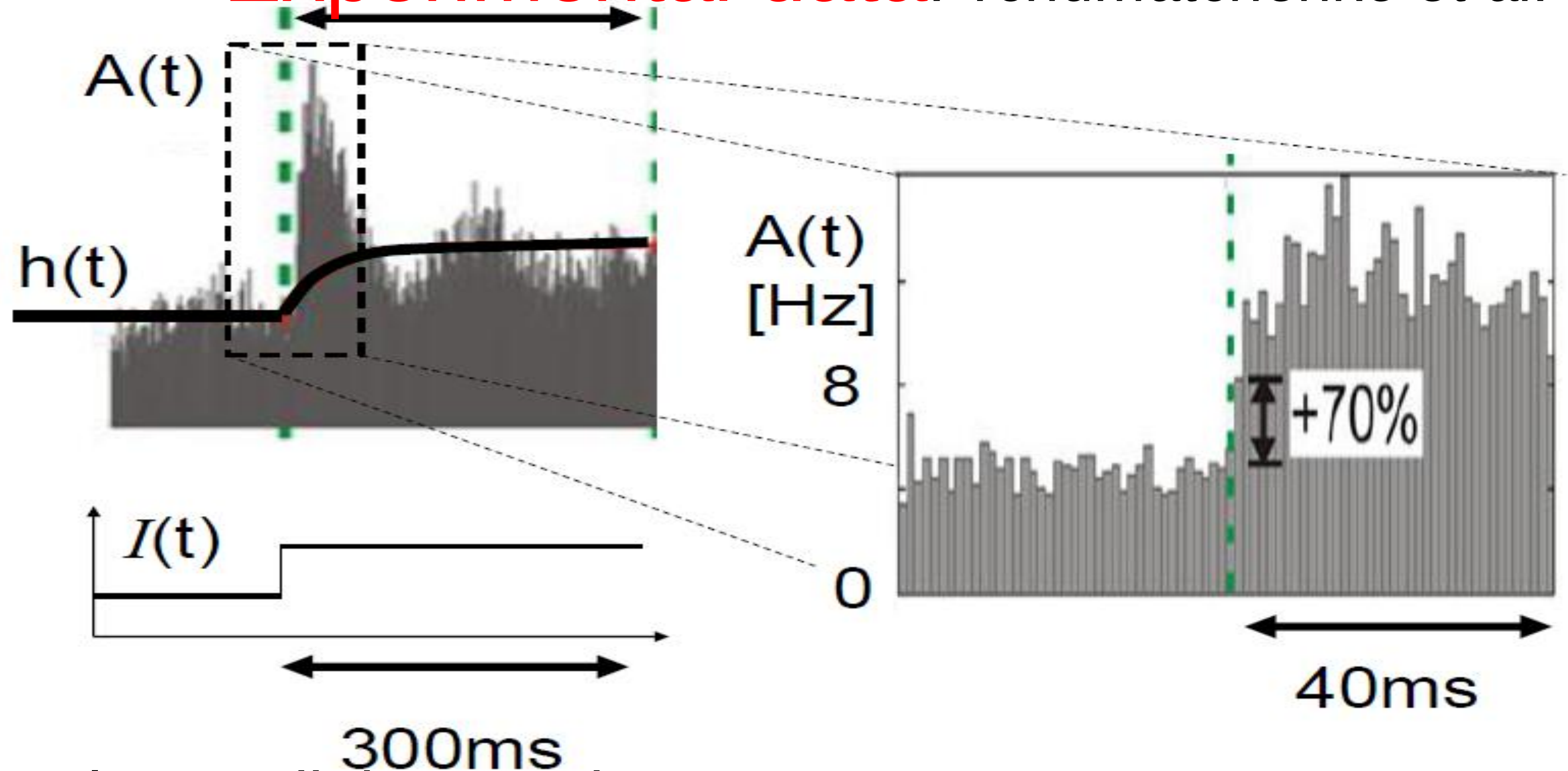


Fig. 15.2: Response of auditory neurons across different layers to short stimuli. **A.** Left: Schematic drawing of an electrode with 32 sites overlaid on top of stained cortical tissue in order to show that the electrode crosses all cortical layers. Right: Spike responses of

Week 15-part 1: $A(t)$ in experiments: step current, (*in vitro*)

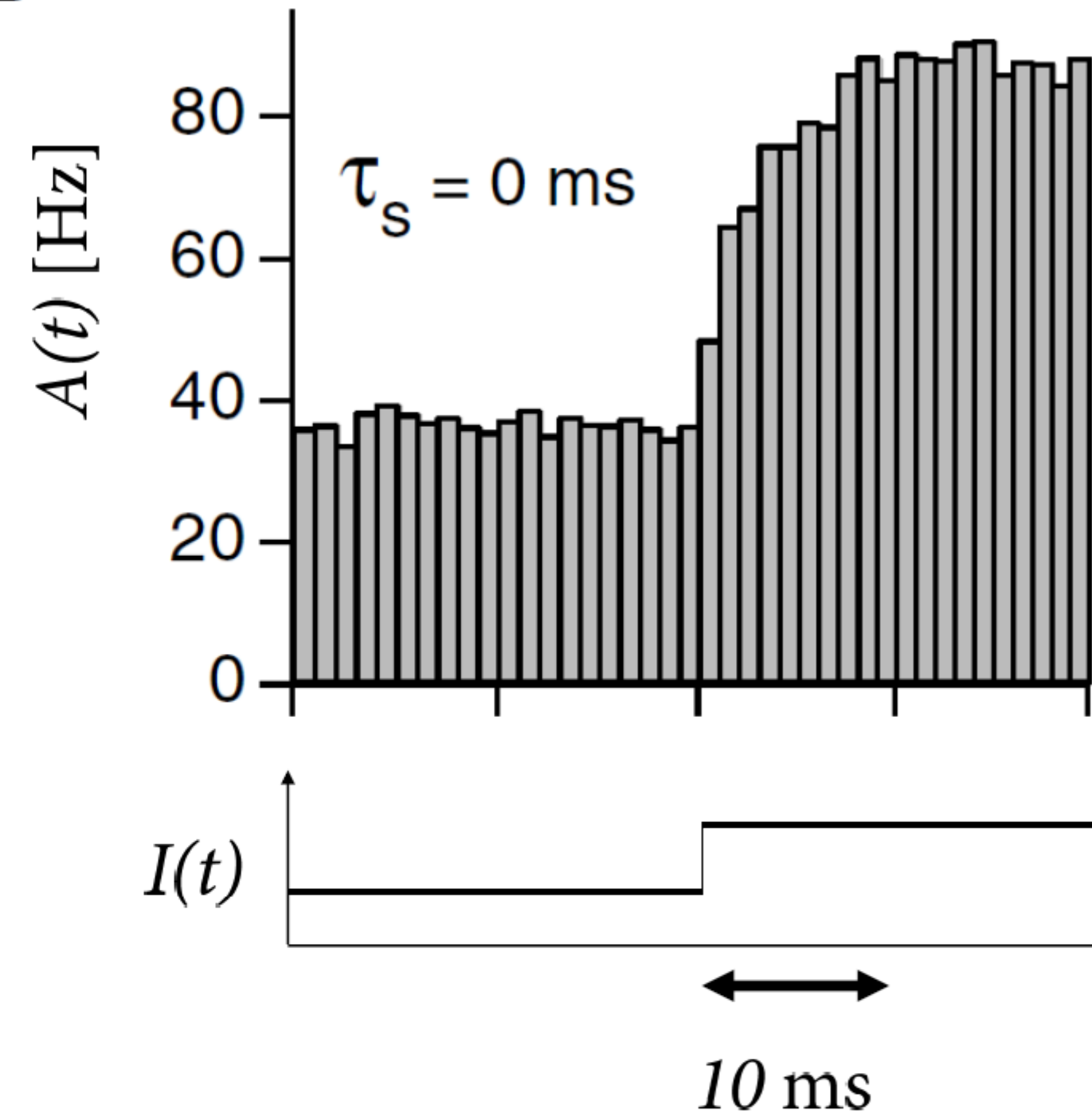
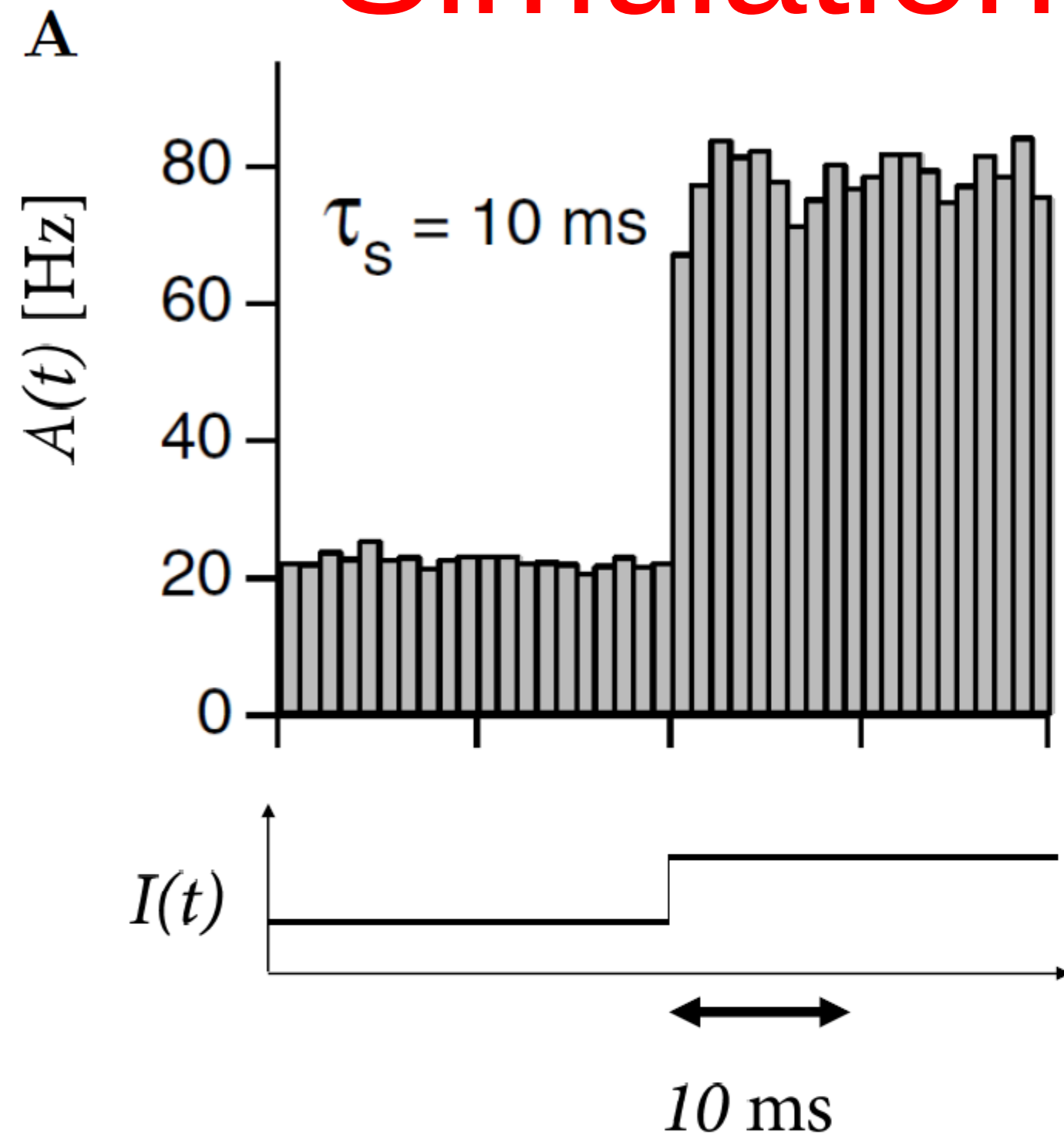
Experimental data: *Tchumatchenko et al. 2011*



See also: *Poliakov et al. 1997*

Week 15-part 1: $A(t)$ in simulations of integrate-and-fire neurons

Simulation data: Brunel et al. 2011

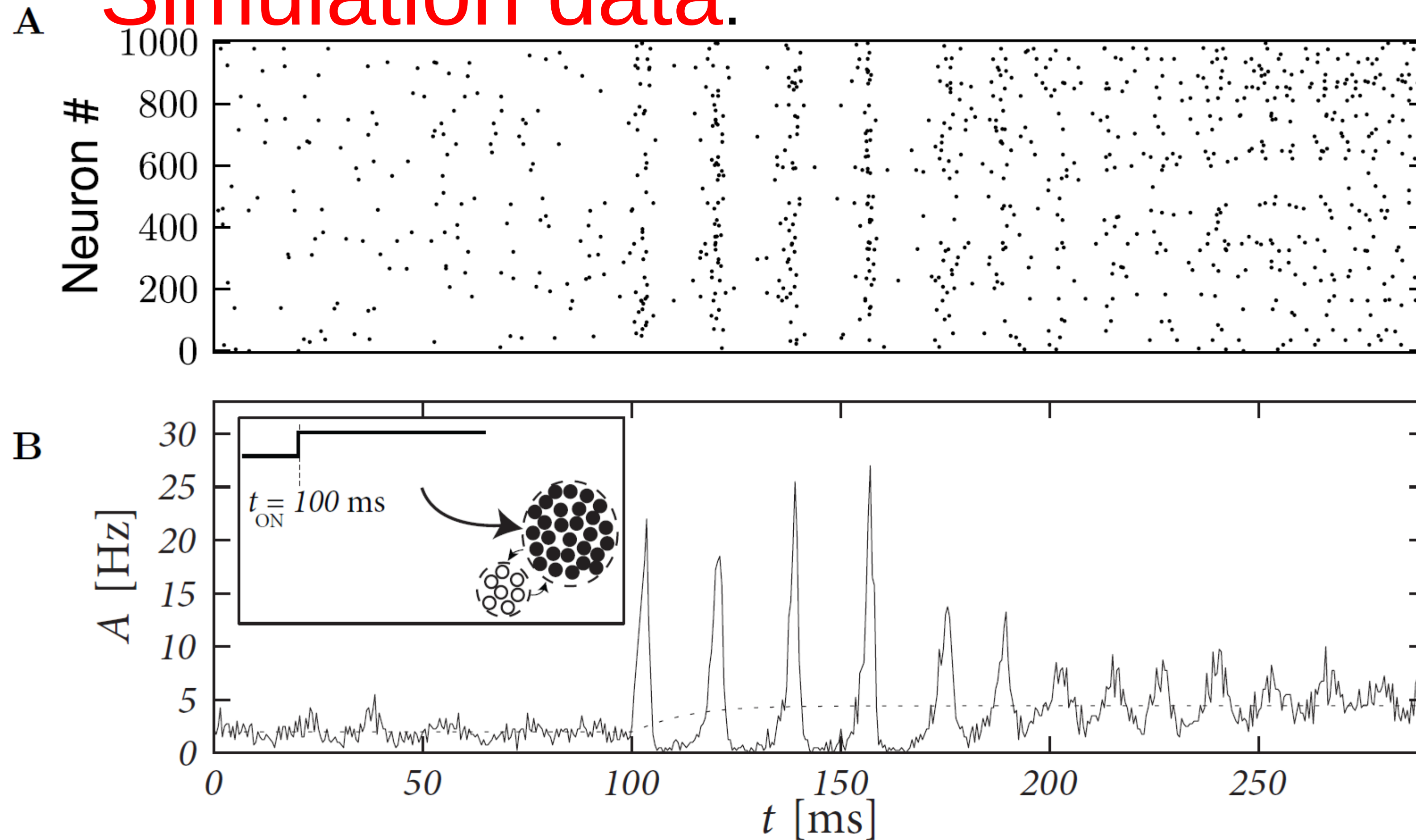


*Image:
Gerstner et al.,
Neuronal
Dynamics,
2014*

Fig. 15.9: Slow (colored) diffusive noise versus white diffusive noise. A population of integrate-and-fire models with a time constant of $\tau_m = 20$ ms was simulated and responses to a step stimulus reported in time bins of 1ms. **A.** Colored noise with a filtering time constant $\tau_s = 10$ ms leads to an abrupt, instantaneous response. **B.** White noise leads to a smoothly increasing fairly slow response. Figures adapted from (Brunel et al., 2001).

Week 15-part 1: $A(t)$ in simulations of integrate-and-fire neurons

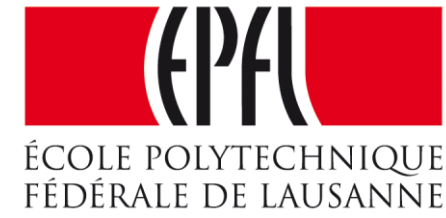
Simulation data:



*Image:
Gerstner et al.,
Neuronal
Dynamics,
2014*

Fig. 15.1: Transients and rate models. All neurons receive the same step current stimulus at time $t_{\text{ON}} = 100\text{ms}$. A randomly connected network of 8000 excitatory and 2000

Week 15 – transients and rate models



Biological Modeling of Neural Networks:

Conclusion of part 1

Week 15 – Fast Transients and Rate models

Wulfram Gerstner

EPFL, Lausanne, Switzerland

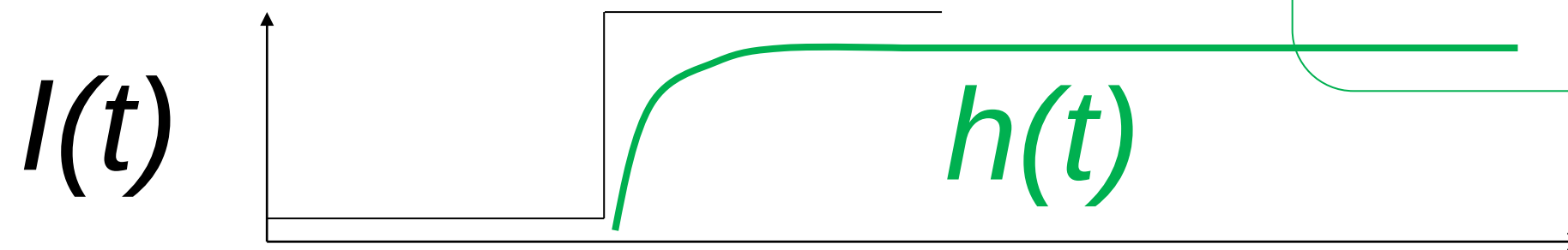
Transients are very fast
-In experiments
-In simulations of spiking neurons

Week 15-part 2: *Rate model* with differential equation

$$A(t) = g(h(t))$$

$$\tau \frac{d}{dt} h(t) = -h(t) + R \cdot I(t)$$

→ Input potential

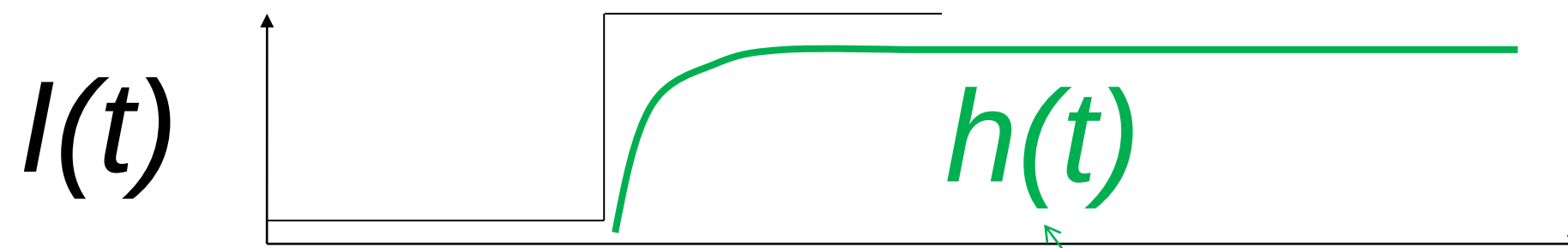
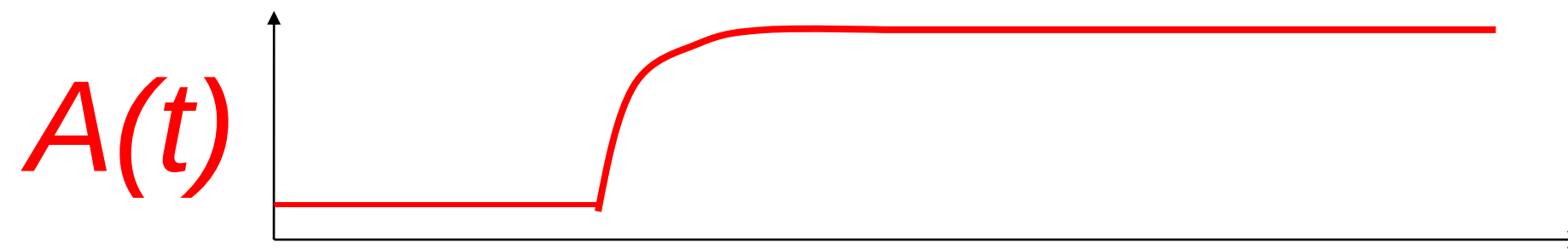


$$h(t) = \int_0^{\infty} \varepsilon \ s \ I(t-s) ds$$

Differential equation can be integrated!, exponential kernel

Week 15-part 2: *Rate model*: step current

$$A(t) = g(h(t)) = g\left(\int \varepsilon(s) I(t-s) ds\right)$$



Input potential

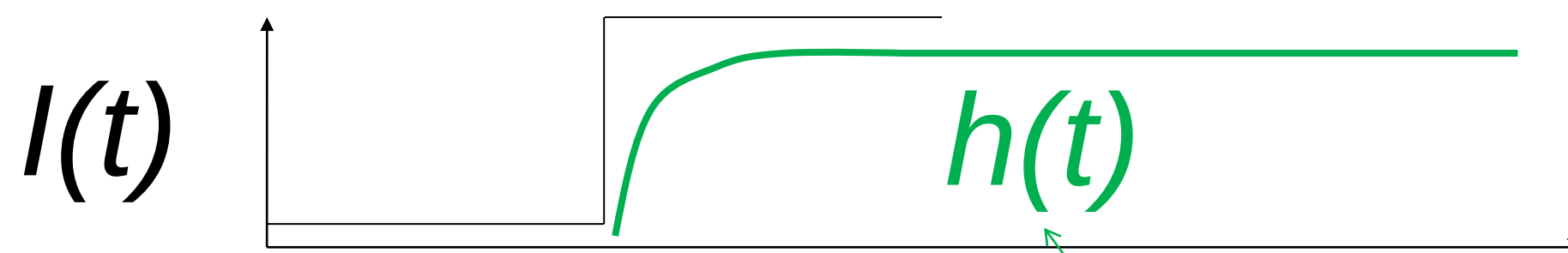
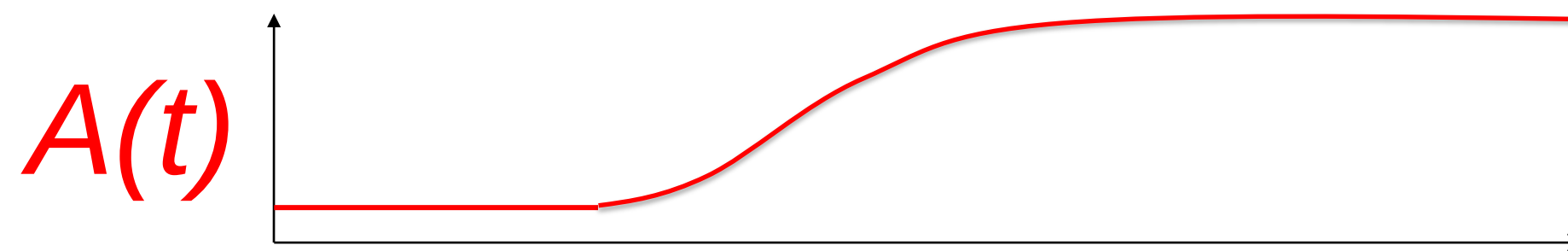
$$h(t) = \int_0^\infty \varepsilon(s) I(t-s) ds + \dots$$

Example LNP rate model: Linear-Nonlinear-Poisson

Spikes are generated by a Poisson process, from rate model

Week 15-part 2: *Wilson-Cowan model*: step current

$$\frac{d}{dt} A(t) = -A(t) + g(h(t))$$



Input potential

$$h(t) = \int_0^\infty \varepsilon s I(t-s) ds + \dots$$

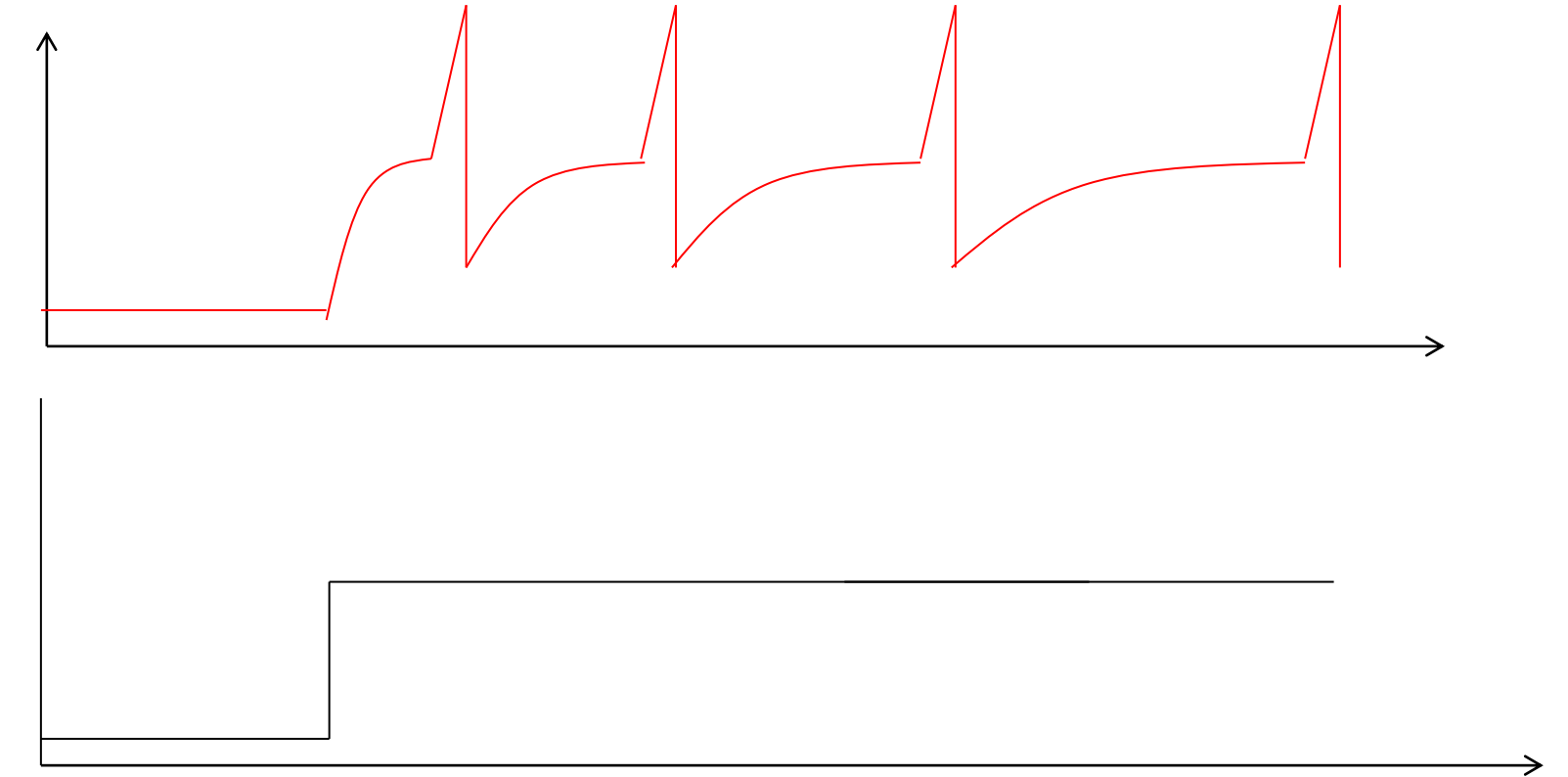
Week 15-part 2: Aim: from single neuron to population

Single neurons

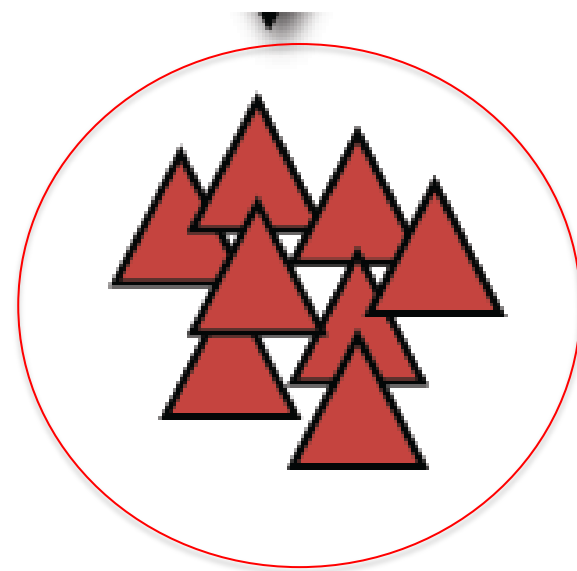
- model exists
- parameters extracted from data
- works well for step current
- works well for time-varying current

$u(t)$

$I(t)$



Population of Neurons?

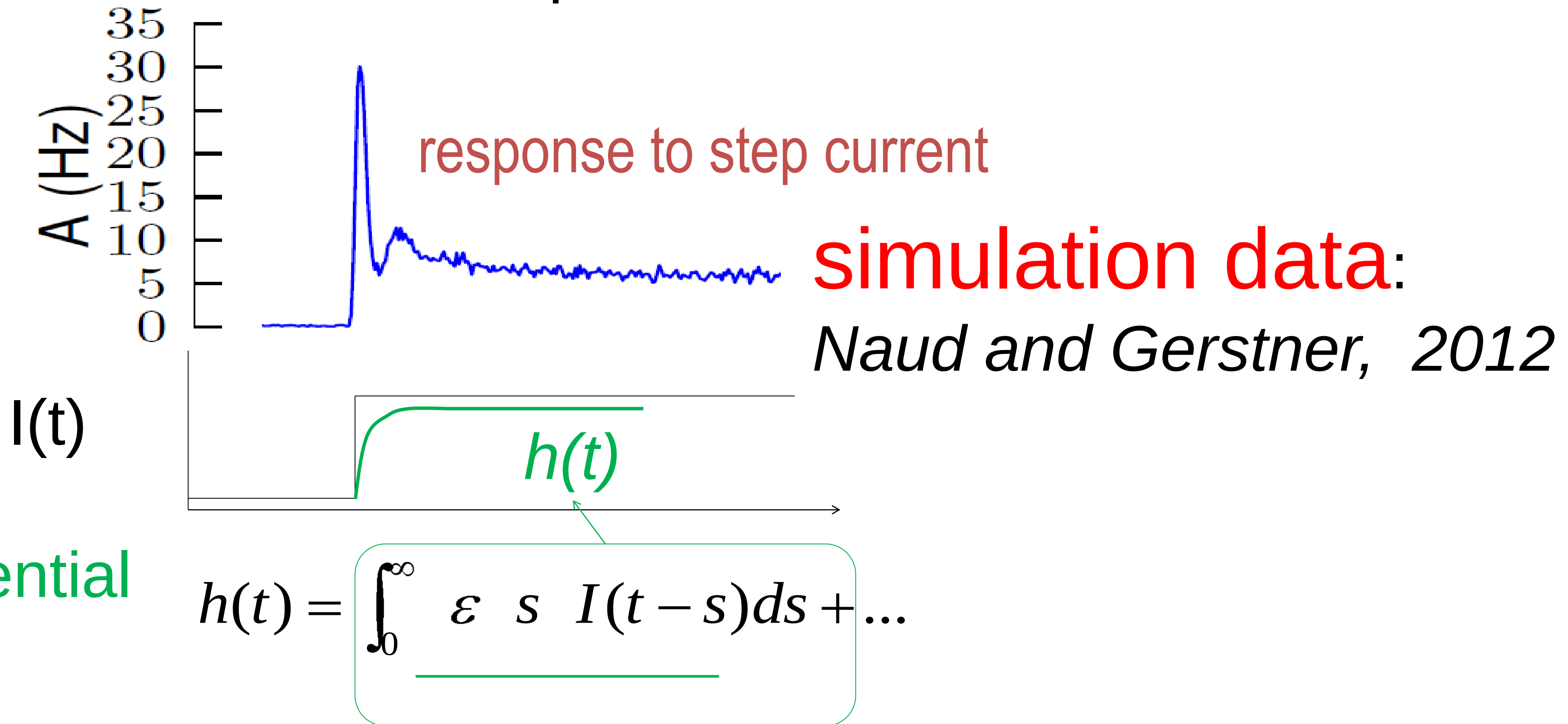


Population equation for $A(t)$

- existing models
- integral equation

Week 15-part 2: $A(t)$ in simulations : step current

25000 identical model neurons (GLM/SRM with escape noise)
parameters extracted from experimental data



Week 15-part 2: rate model

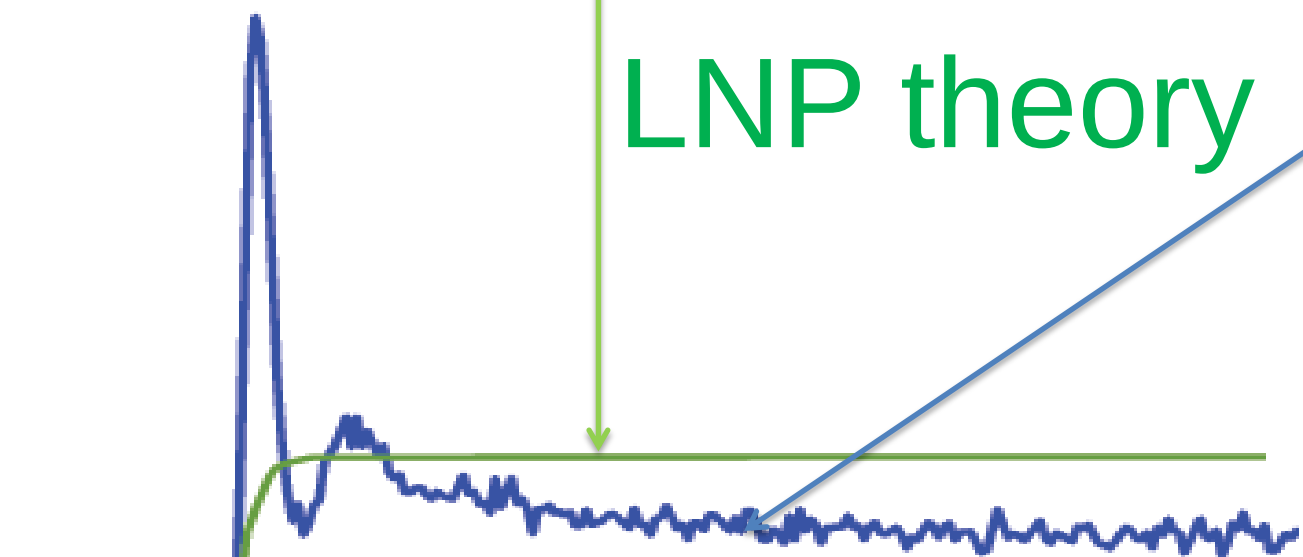
1) LNP

$I(t)$



$$A(t) = f(h(t)) \sim e^{h(t)}$$

$A(t)$



LNP theory

simulation

Simple rate model:

- too slow during the transient
- no adaptation

Week 15-part: rate model with adaptation

1) LNP

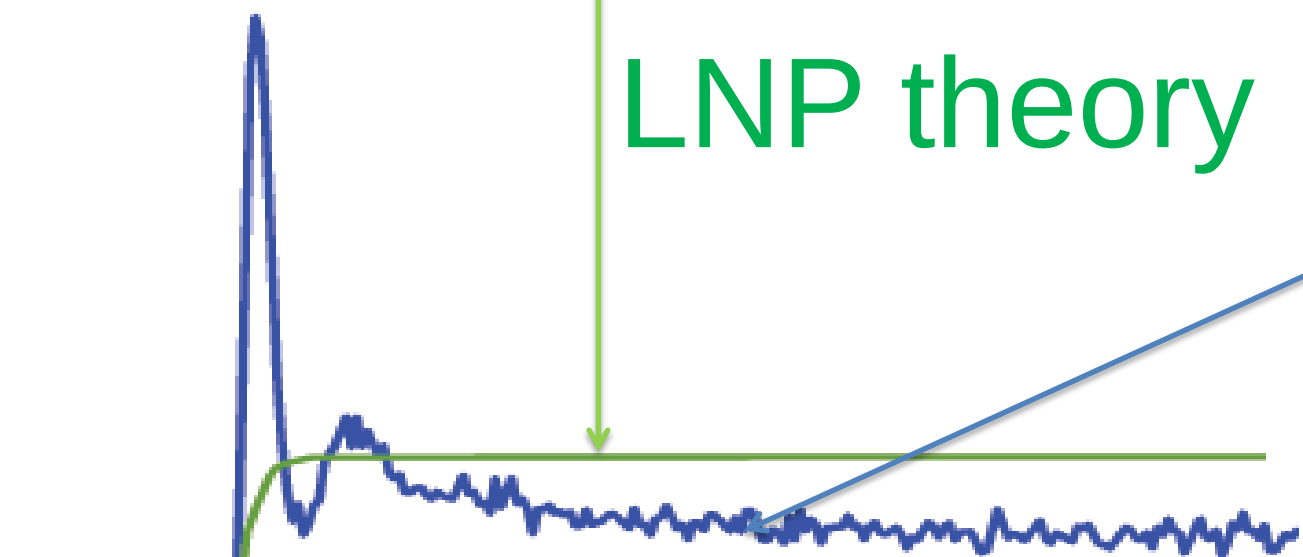
$I(t)$



$$A(t) = f(h(t)) \sim e^{h(t)}$$

LNP theory

$A(t)$

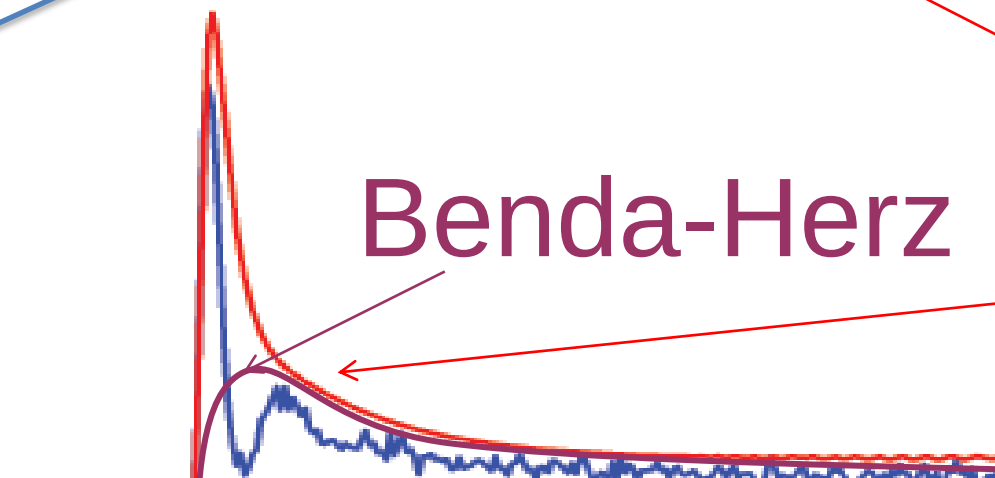


2) Rate adapt.



$$e^{h(t) - [g * A](t)}$$

Benda-Herz



Naud and Gerstner 2012

simulation

optimal filter
(rate
adaptation)

Week 15-part 2: rate model

Simple rate model:

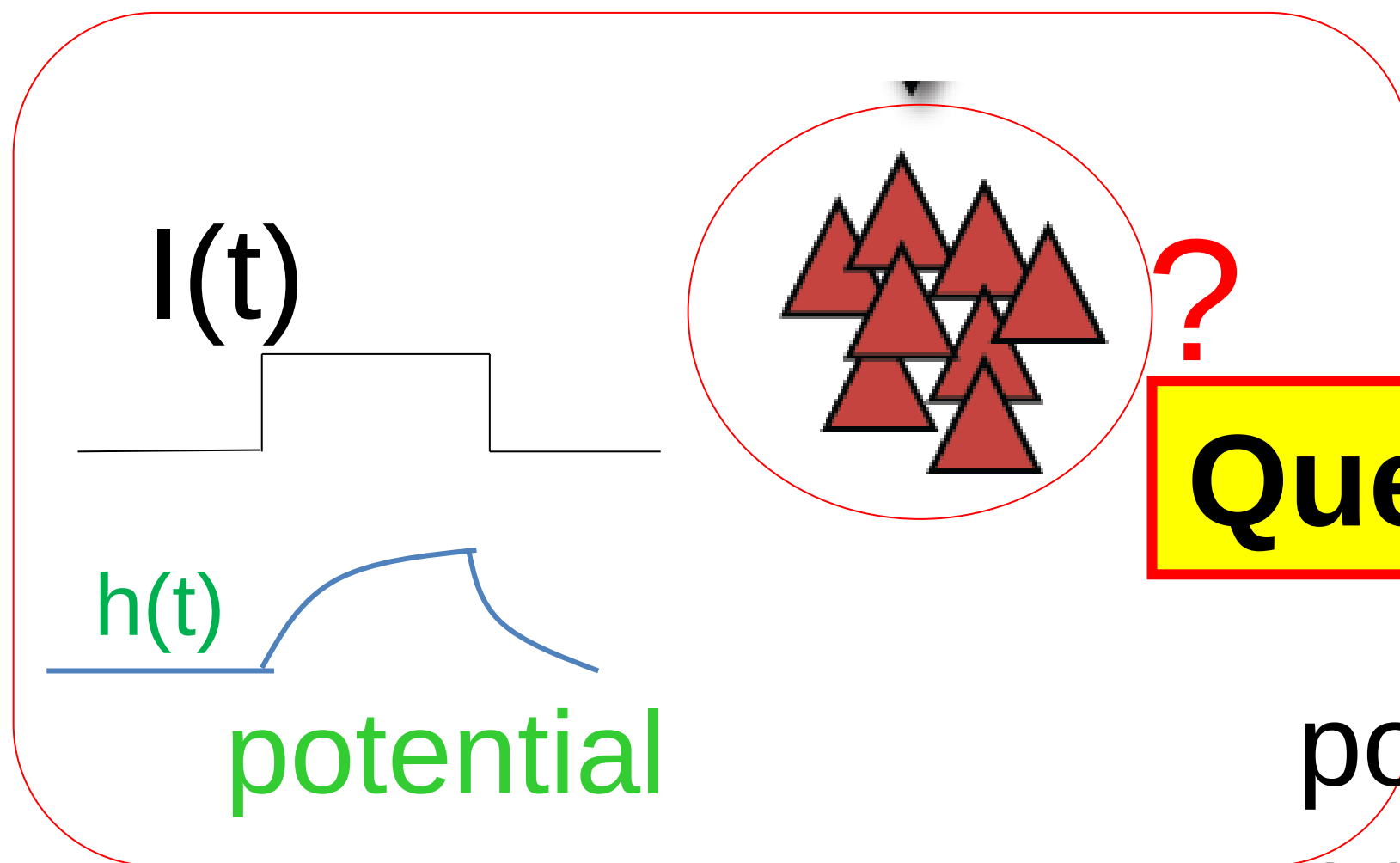
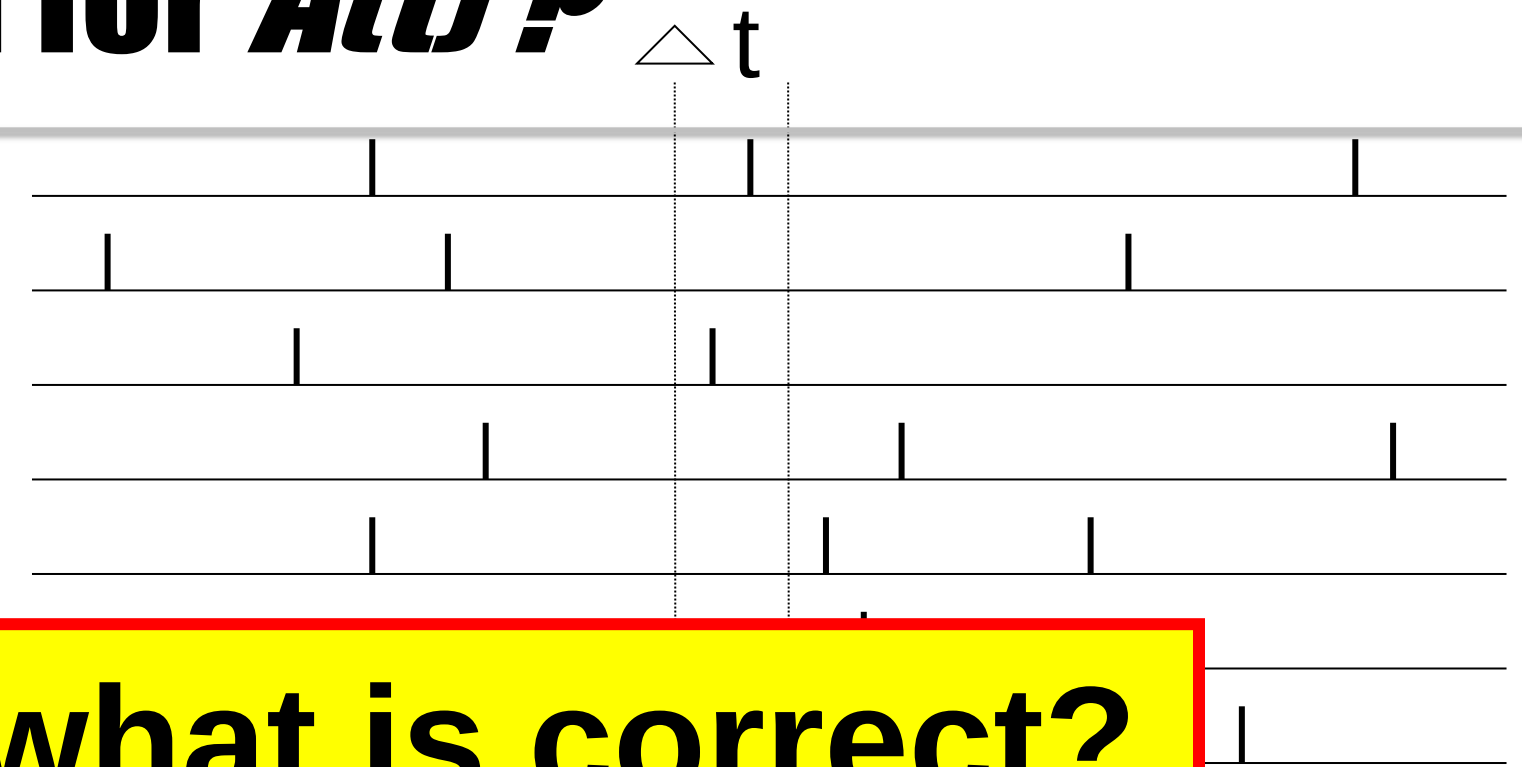
- too slow during the transient
- no adaptation

Adaptive rate model:

- too slow during the transient
- final adaptation phase correct

Rate models can get the slow dynamics and stationary state correct, but are wrong during transient

Week 15-part 2: population equation for $A(t)$?



Question: – what is correct?

$$A(t) = \frac{n(t; t + \Delta t)}{N \Delta t}$$

$A(t)$ ~~Wilson-Cowan~~ $\tau \frac{d}{dt} A(t) = -A(t) + f(h(t))$

$A(t)$ ~~LNP~~ ~~Ostojic-Brunel~~ $A(t) = f(h(t)) = f\left(\int \varepsilon(s) I(t-s) ds\right)$

$A(t)$ ~~current~~ $A(t) = g(I(t))$

Poll!!!

$A(t)$ ~~Benda-Herz~~ $A(t) = g(I(t) - \int G(s) A(t-s) ds)$

Improved rate models

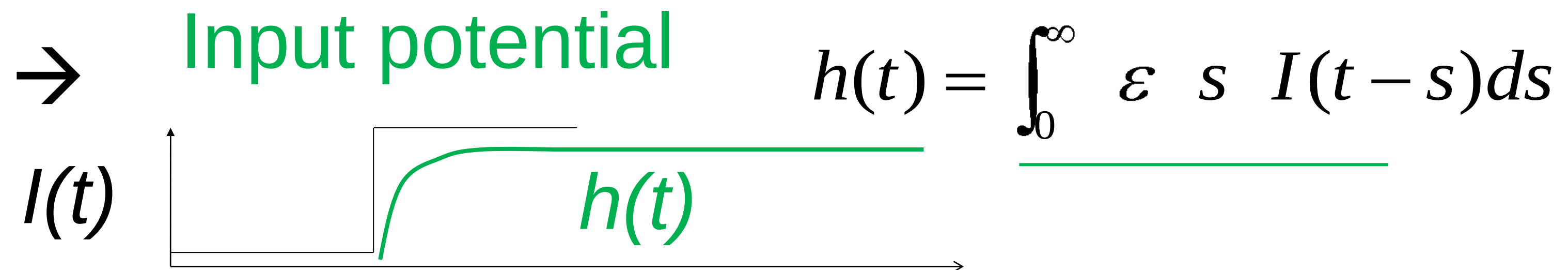
- a) rate model with effective time constant
→ next slides
- b) integral equation approach
→ chapter 14

Week 15-part 2: *Rate model* with effective time constant

$$A(t) = F(h(t))$$

$$\tau_{eff}(t) \sim F' / A_0(t)$$

$$\tau_{eff}(t) \frac{d}{dt} h(t) = -h(t) + R \cdot I(t) \quad \text{Ostoja-Brunel, 2011}$$



→ Shorter effective time constant during high activity

Week 15-part 2: *Rate model* with effective time constant

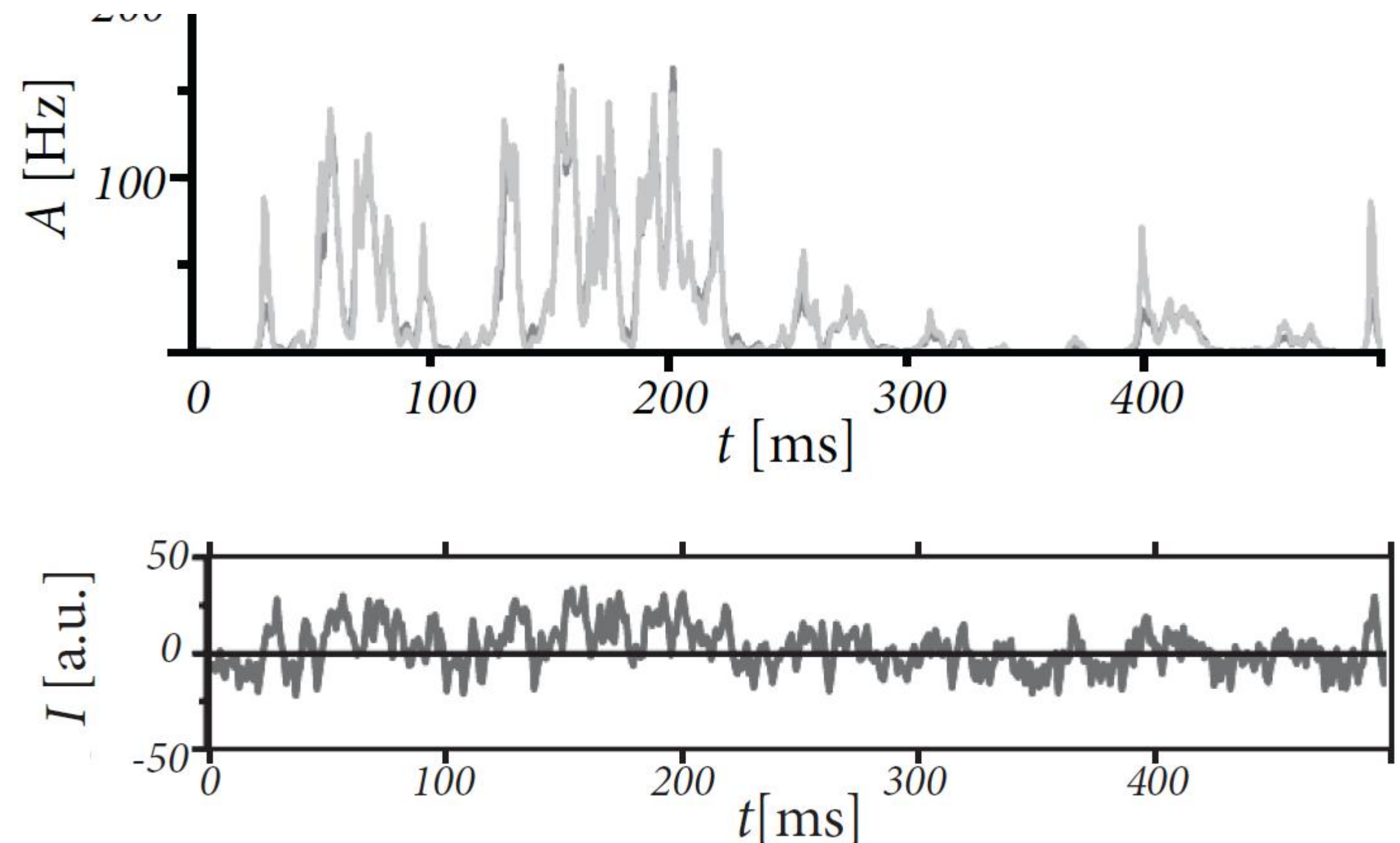
$$A(t) = F(h(t))$$

$$\tau_{\text{eff}}(t) \frac{d}{dt} h(t) = -h(t) + I$$

Ostojic-Brunel, 2011

$$\tau_{\text{eff}}(t) \sim F' / A_0(t)$$

*Image: Gerstner et al.,
Neuronal Dynamics, CUP 2014*



- Shorter effective time constant during high activity
- Theory fits simulation very well

Rate models

- are useful, because they are simple
- slow dynamics and stationary state correct
- simple rate models have wrong transients
- improved rate models/population activity models exist

The end

*Reading: Chapter 15 of
NEURONAL DYNAMICS,
Gerstner et al., Cambridge Univ. Press (2014)*