Nonlinear Integrate-and-Fire Model

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 1 and Week 4:
Nonlinear Integrate-and-fire Model

Wulfram Gerstner
EPFL, Lausanne, Switzerland

Nonlinear Integrate-and-fire (NLIF)
- Definition
  - quadratic and expon. IF
- Extracting NLIF model from data
  - exponential Integrate-and-fire
- Extracting NLIF from detailed model
  - from two to one dimension
- Quality of NLIF?

Neuronal Dynamics – Review: Nonlinear Integrate-and Fire

LIF (Leaky integrate-and-fire)

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

NLIF (nonlinear integrate-and-fire)

$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

If firing:

$$u \rightarrow u_{reset}$$

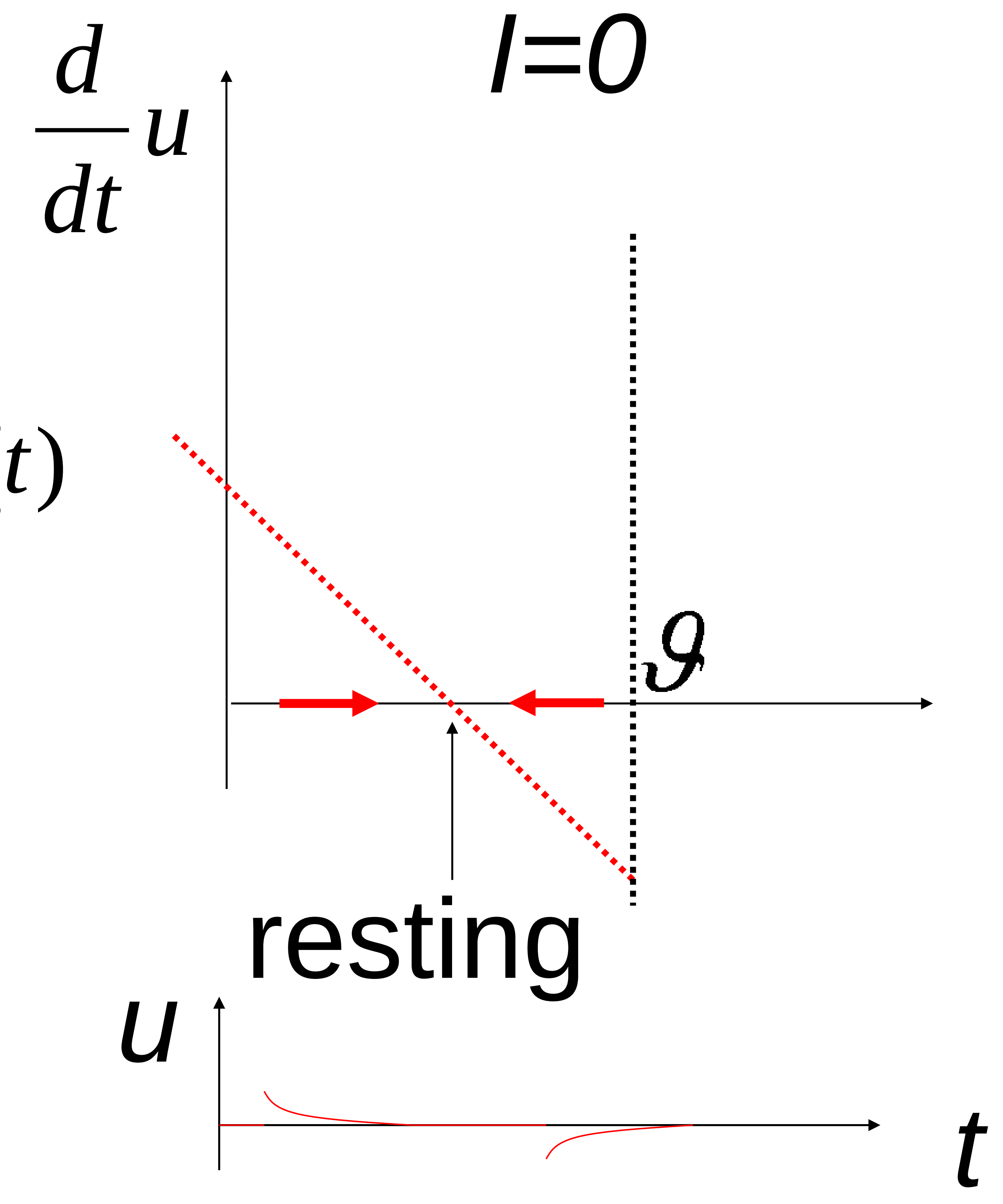
Neuronal Dynamics – 1.4. Leaky Integrate-and Fire revisited

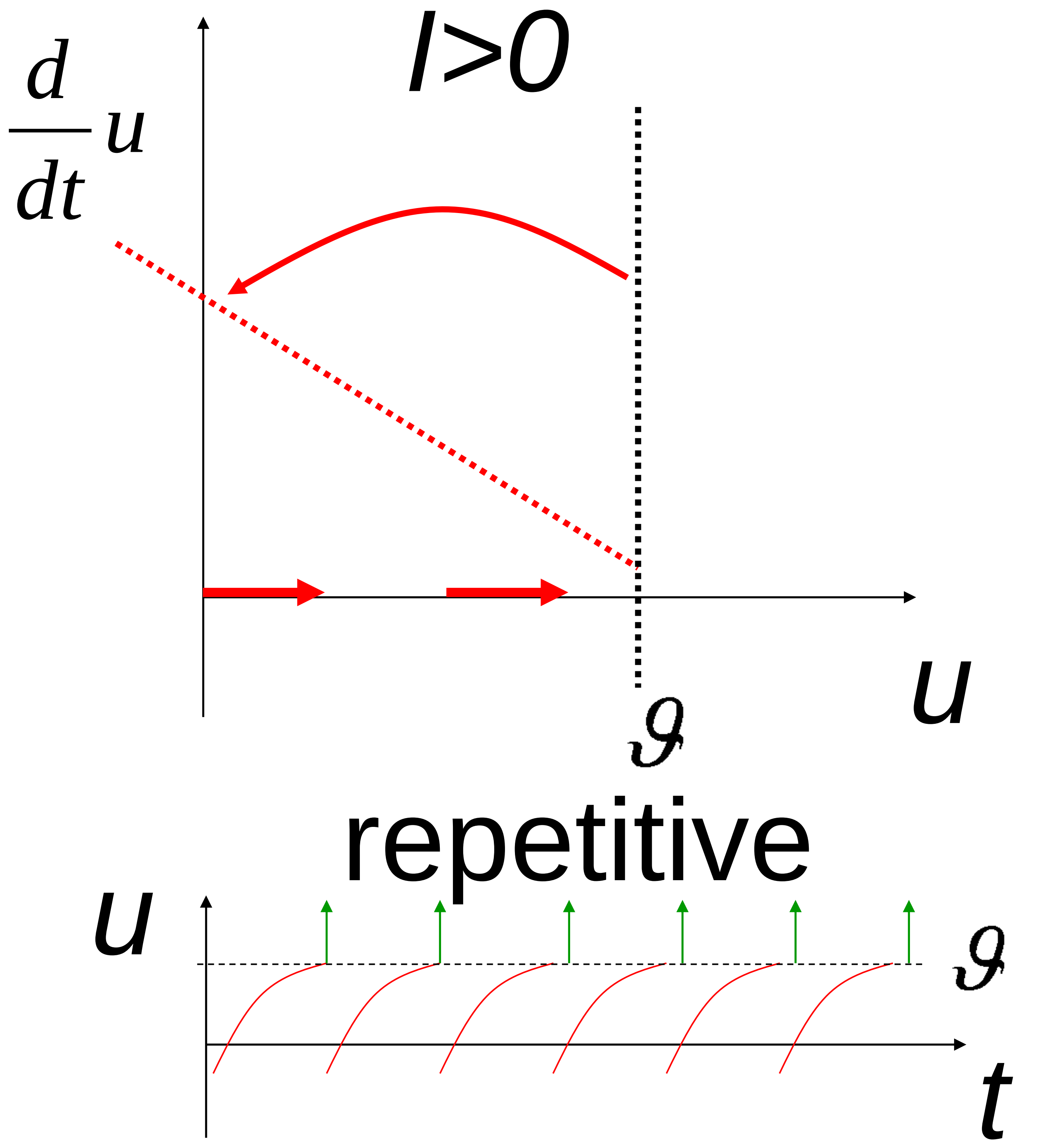
LIF

$$\tau \cdot \frac{d}{dt}u = -(u - u_{rest}) + RI(t)$$

If firing:

$$u \rightarrow u_r$$





Neuronal Dynamics – 1.4. Nonlinear Integrate-and Fire

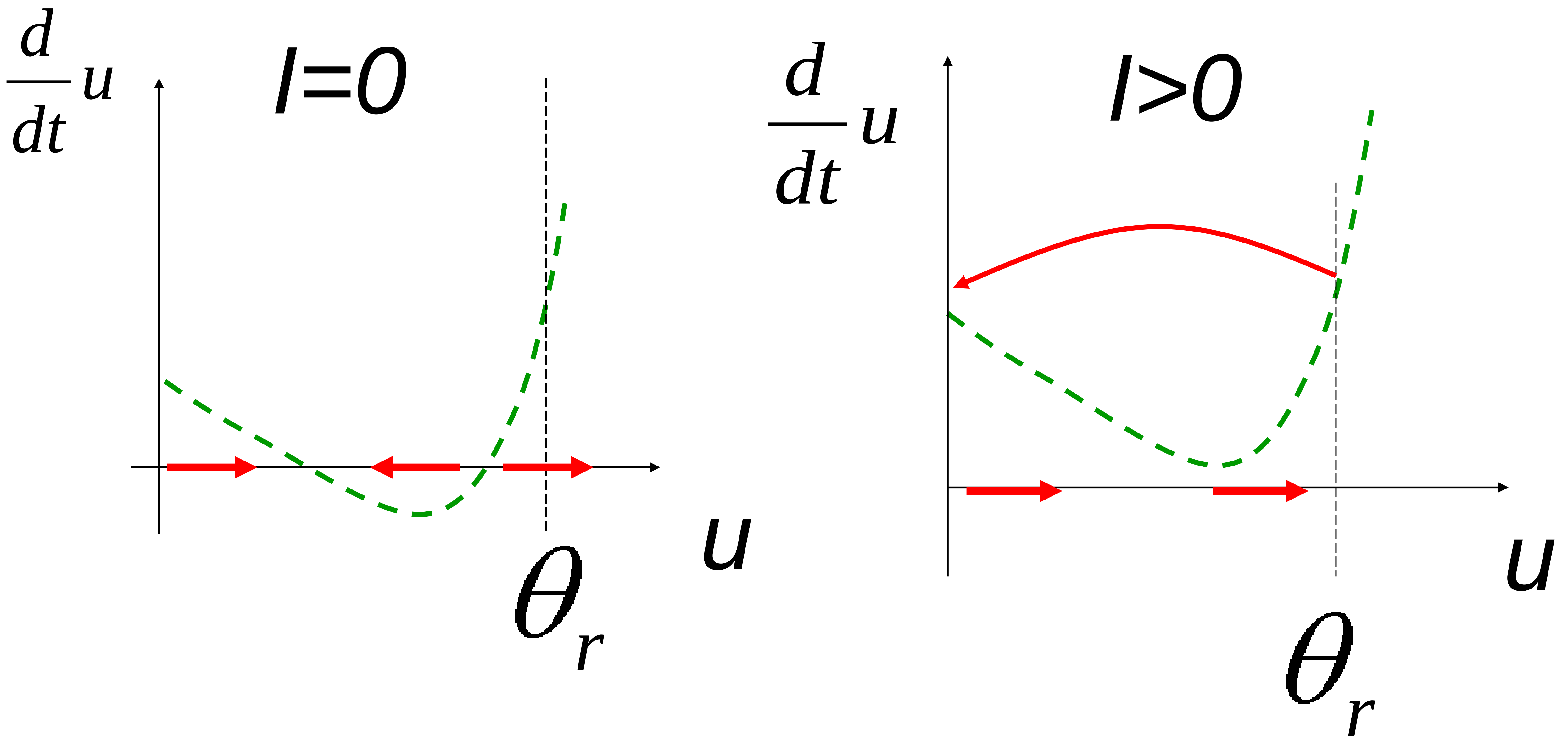
Nonlinear Integrate-and-Fire

NLIF

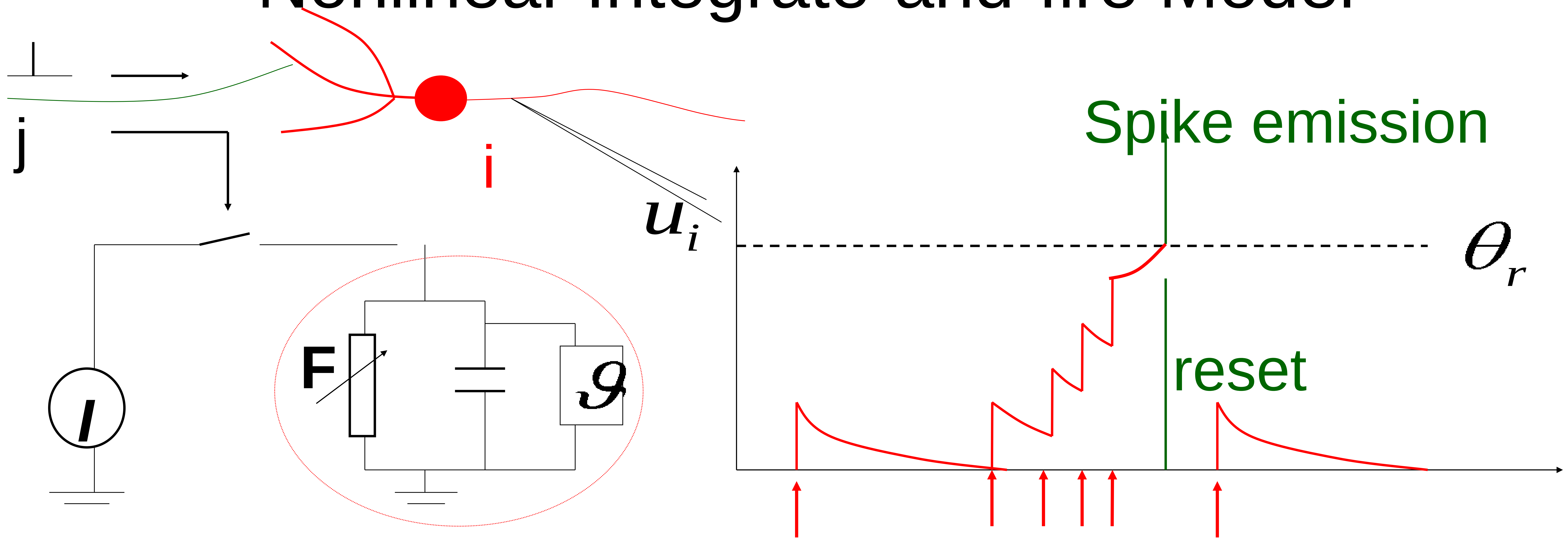
$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

firing: *if* $u(t) = \theta_r$ *then*

$$u \to u_r$$



Nonlinear Integrate-and-fire Model

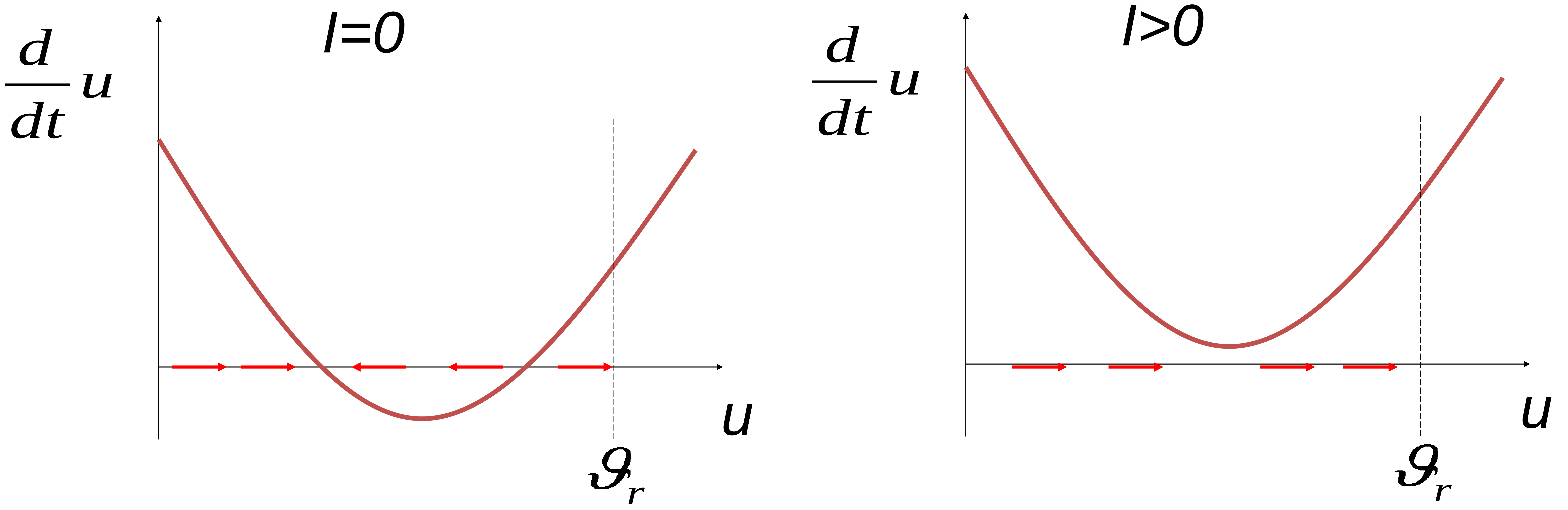


$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

NONlinear

if $u(t) = \theta_r$ *then* Fire+reset threshold

Nonlinear Integrate-and-fire Model



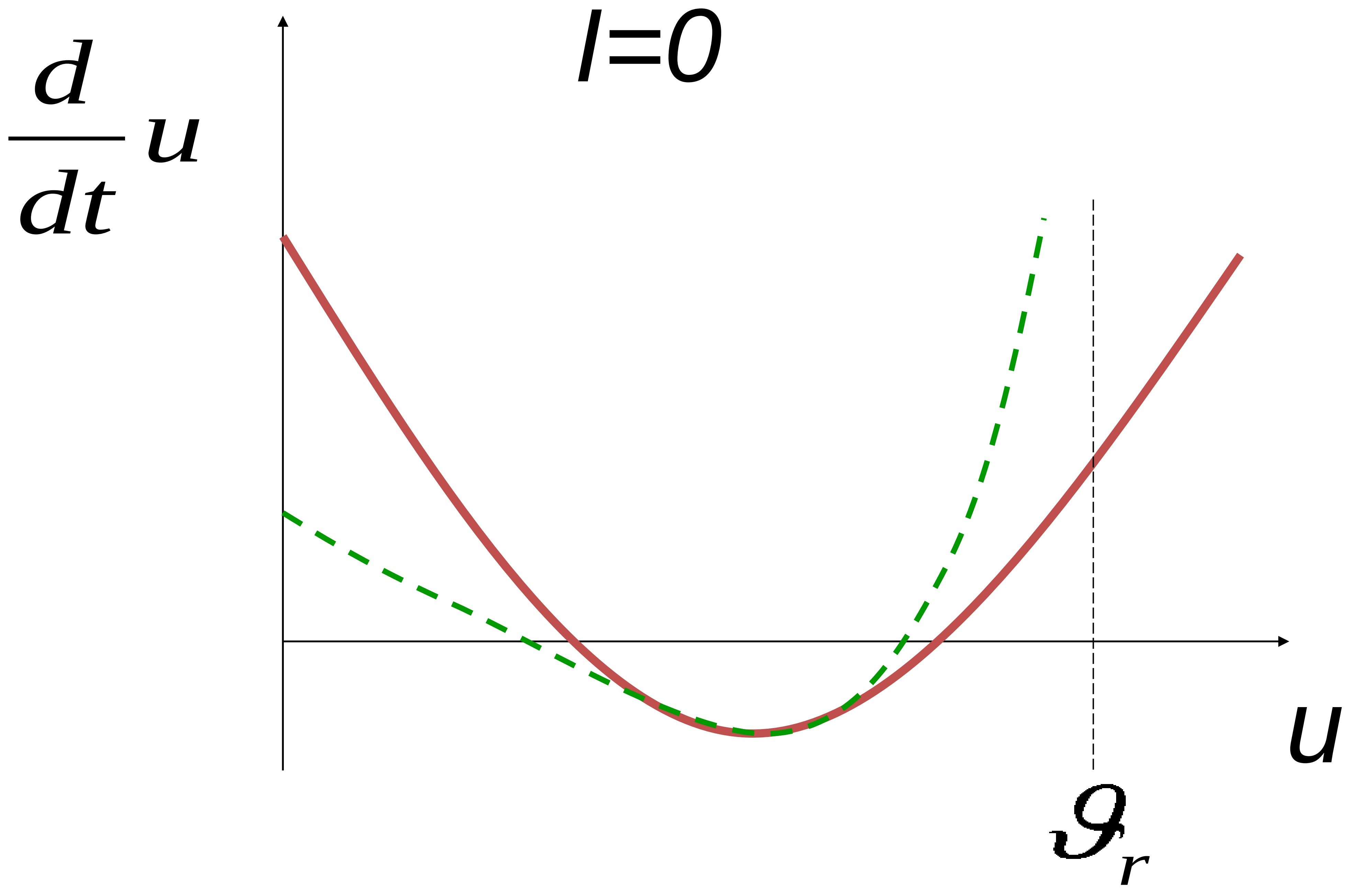
$$\tau \cdot \frac{d}{dt}u = F(u) + RI(t)$$ NONlinear

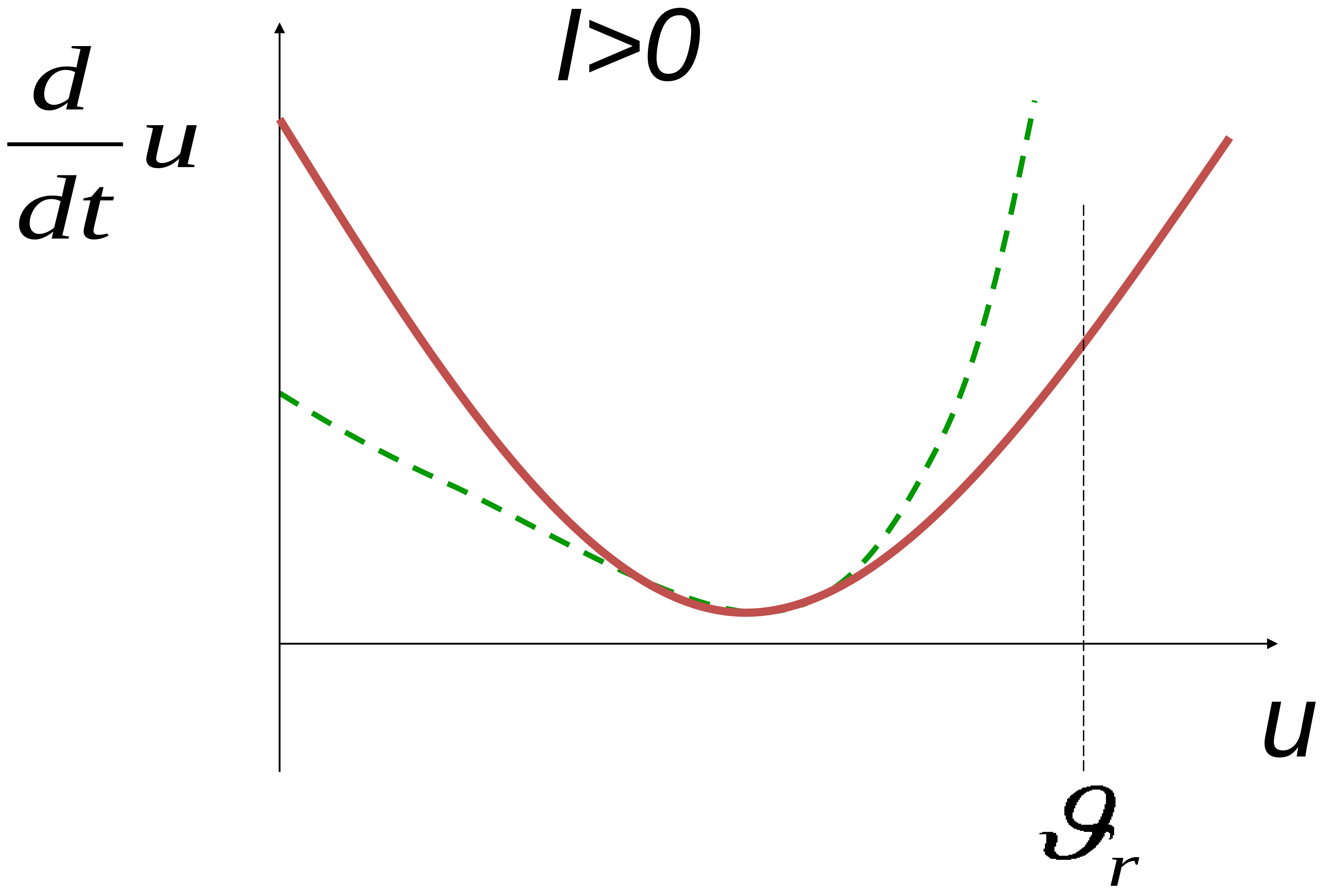
if $u(t) = \theta_r$ *then* Fire+reset threshold

Quadratic I&F:

$$F(u) = c_2(u - c_1)^2 + c_0$$

Nonlinear Integrate-and-fire Model





$$\tau \cdot \frac{d}{dt}u = F(u) + RI(t)$$

if $u(t) = \theta_r$ *then* Fire+reset

Quadratic I&F:

$$F(u) = c_2(u - c_1)^2 + c_0$$

exponential I&F:

$$F(u) = -(u - u_{rest}) + c_0 \exp(u - \vartheta)$$

Nonlinear Integrate-and-Fire Model

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  - exponential Integrate-and-fire
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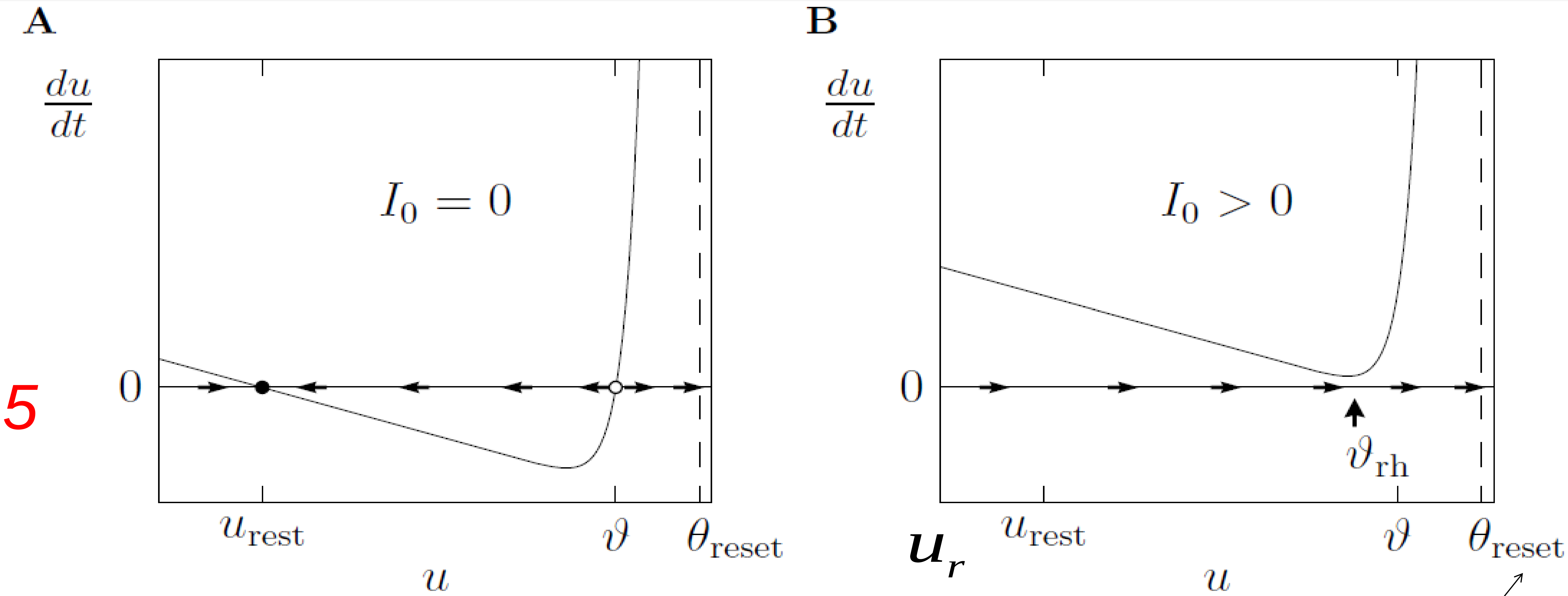
  - from two to one dimension

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

See:
week 1,
lecture 1.5



$$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

What is a good choice of **f** ?

If $u = \theta_{reset}$

then reset to

$$u = u_r$$

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(1) $$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

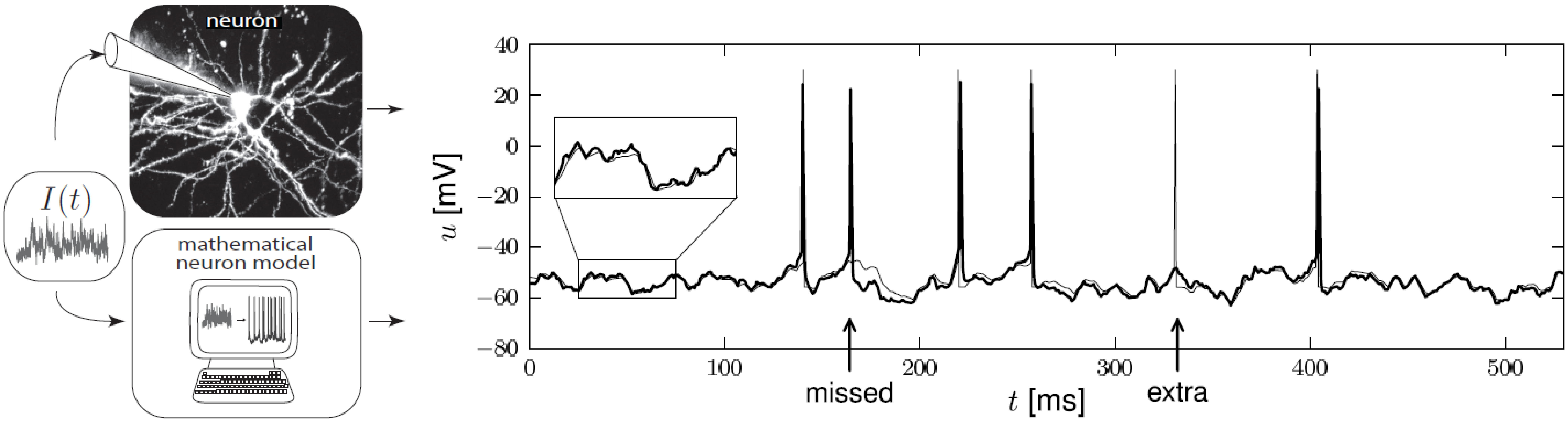
(2) $$If \quad u = \theta_{reset} \quad then \quad reset\,to\; u = u_r$$

What is a good choice of ***f*** ?

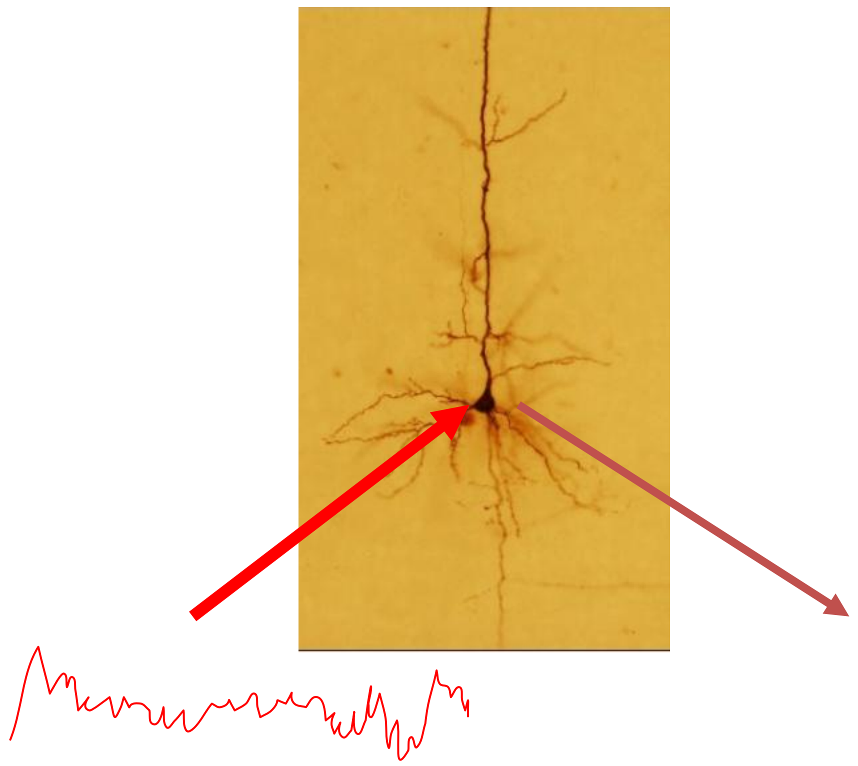
(i) Extract *f* from data

(ii) Extract *f* from more complex models

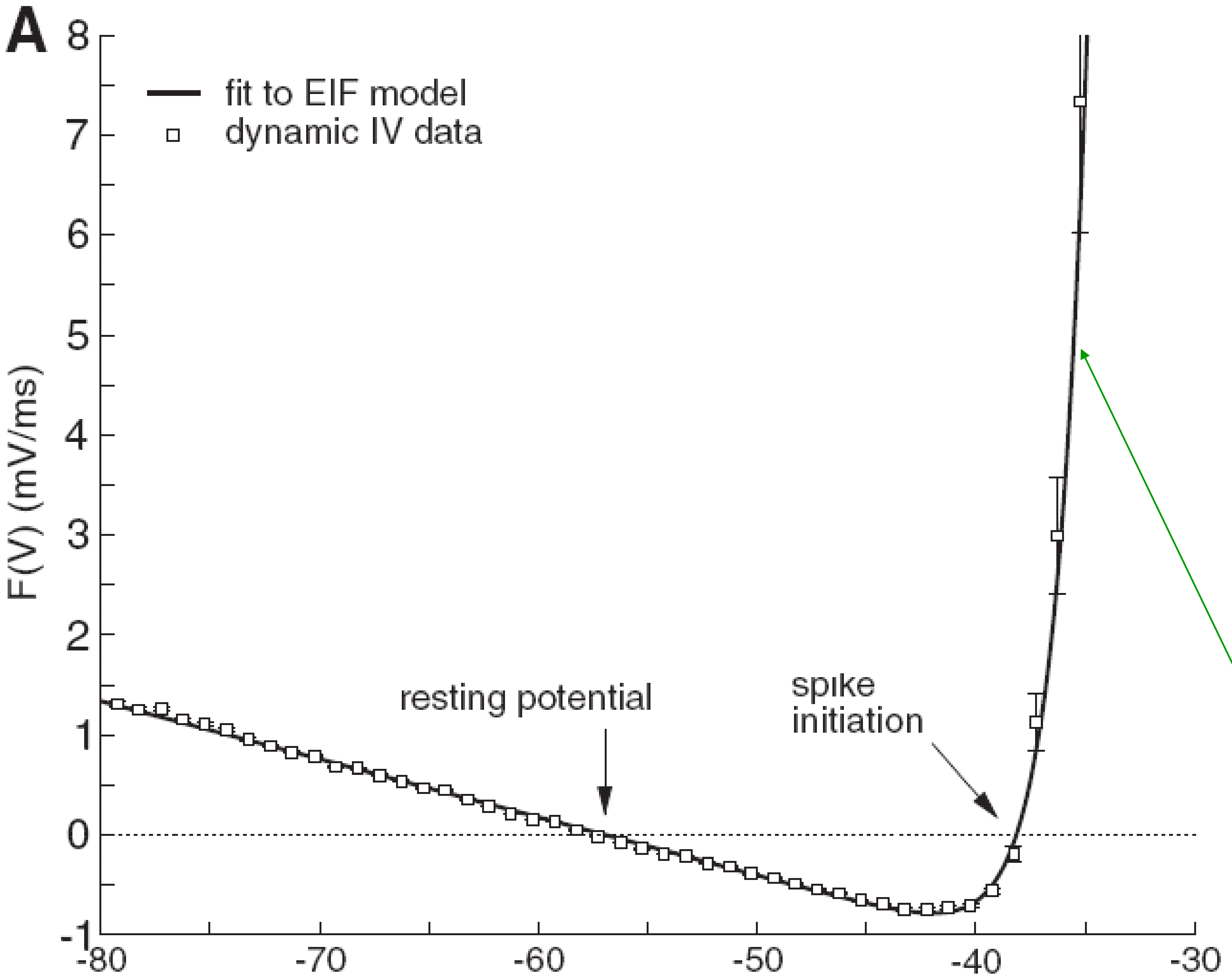
Neuronal Dynamics – 1.5. Inject current – record voltage



Neuronal Dynamics – Inject current – record voltage



I(t)



$$F(u) = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right)$$

u [mV]

$$\frac{du}{dt} - \frac{1}{C} I(t) = F(u) \frac{1}{\tau}$$

Badel et al., J. Neurophysiology 2008

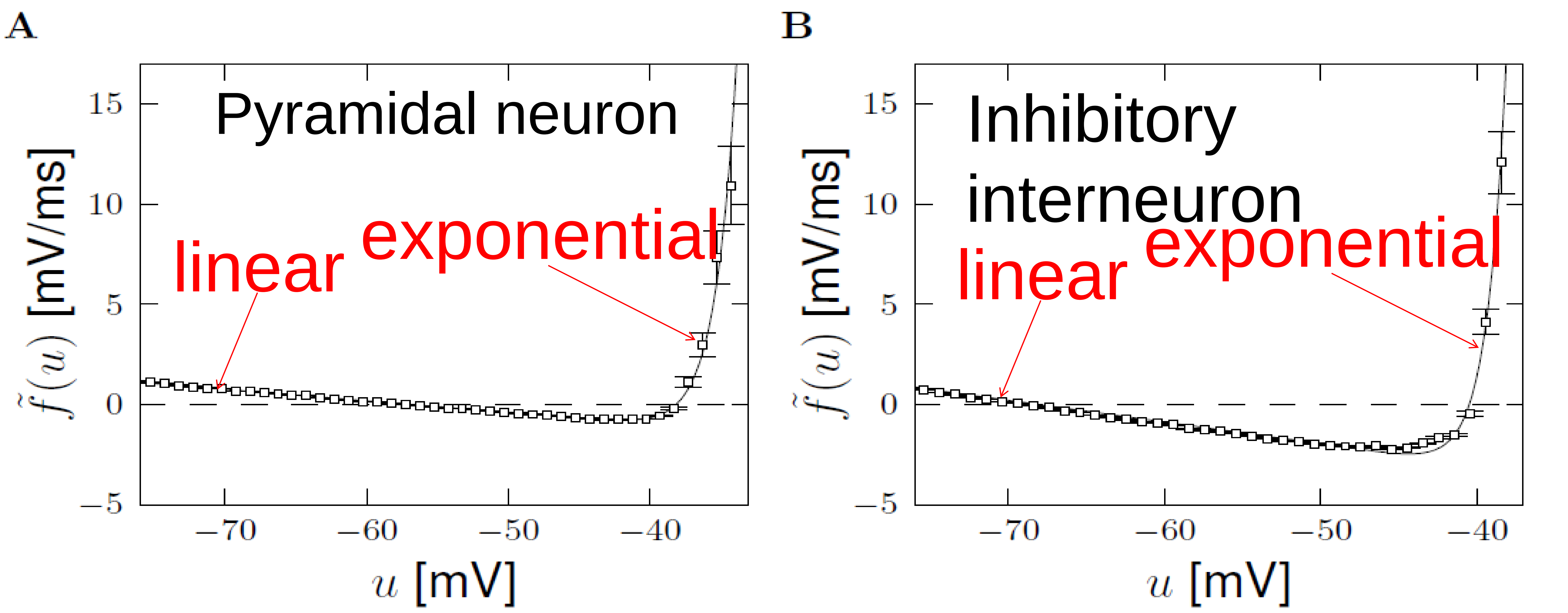
(i) Extract *f* from data *Badel et al. (2008)*

$$\tau \frac{du}{dt} = f(u) + RI(t)$$

$$\tilde{f}(u) = \frac{f(u)}{\tau}$$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right)$$

Exp. Integrate-and-Fire, *Fourcaud et al. 2003*



Badel et al. (2008)

$$(1)\quad \tau \frac{du}{dt} = f(u) + R\,I(t)$$

$$(2)\quad If \;\; u = \theta_{reset} \;\; then \;\; reset\, to\; u = u_r$$

Best choice of ***f*** : linear + exponential

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u - \vartheta}{\Delta})$$

BUT: **Limitations – need to add**

-Adaptation on slower time scales

-Possibility for a diversity of firing patterns

-Increased threshold ϑ after each spike

-Noise

Week 4 – part 5: Nonlinear Integrate-and-Fire Model

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Neuronal Dynamics – 4.5. Further reduction to 1 dimension

After reduction of HH to two dimensions:

2-dimensional equation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

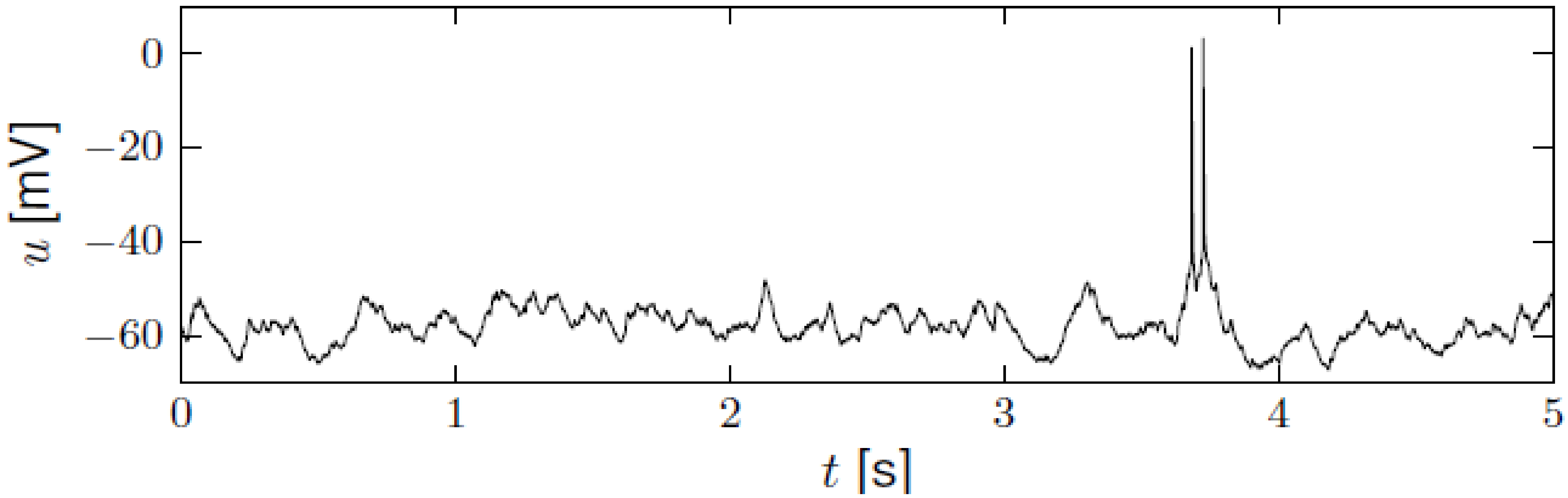
slow!

Separation of time scales

-*w* is nearly constant (most of the time)

Spontaneous activity *in vivo*

awake mouse, cortex, freely whisking,



-spikes are rare events

Crochet et al., 2011

-membrane potential fluctuates around 'rest'

Aims of Modeling: - predict spike initation times
- predict subthreshold voltage

Neuronal Dynamics – 4.5. Further reduction to 1 dimension

stimulus

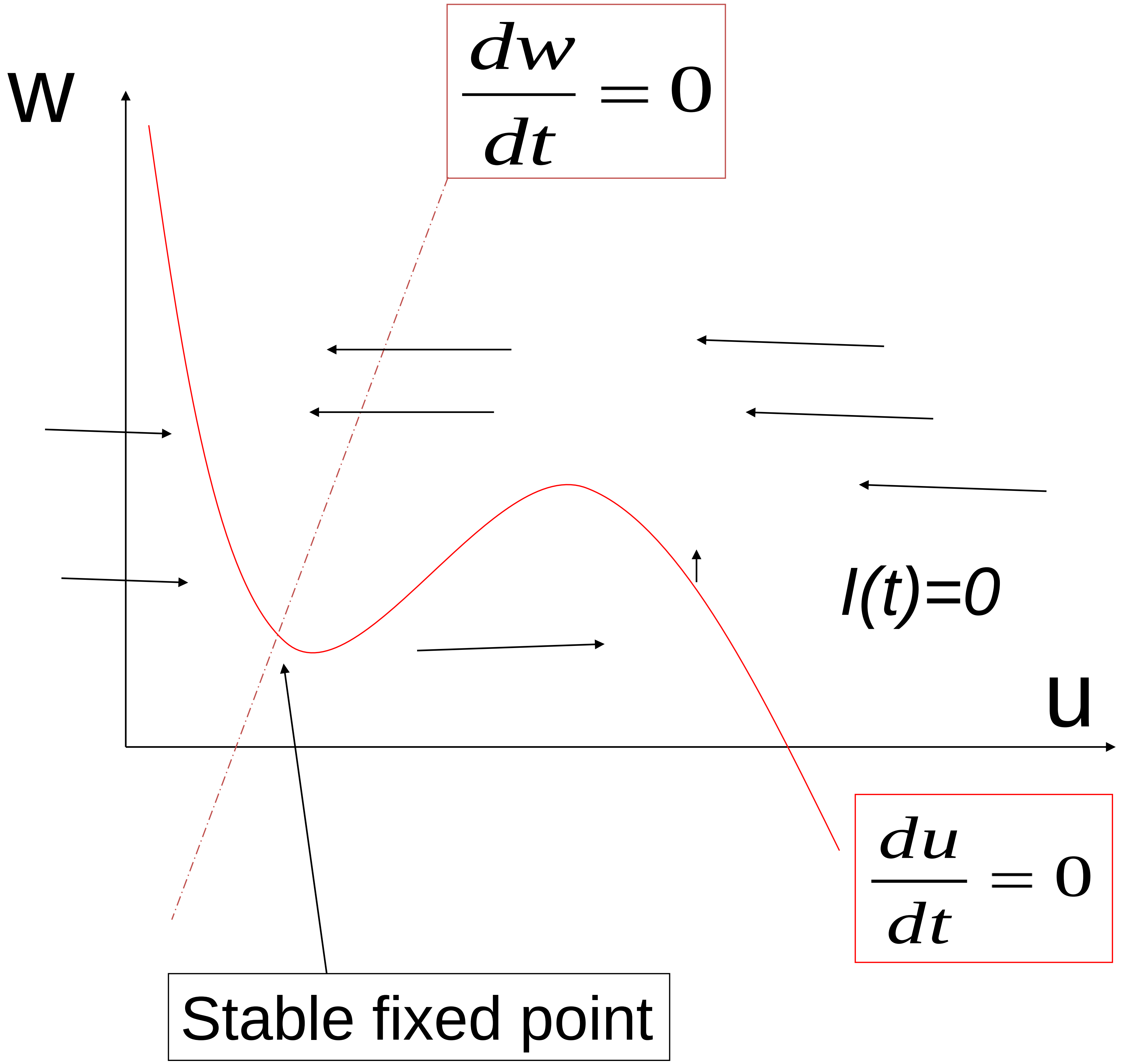
$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Separation of time scales

$$\tau_w >> \tau_u$$

→ Flux nearly horizontal



Neuronal Dynamics – 4.5. Further reduction to 1 dimension

Hodgkin-Huxley reduced to 2dim

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

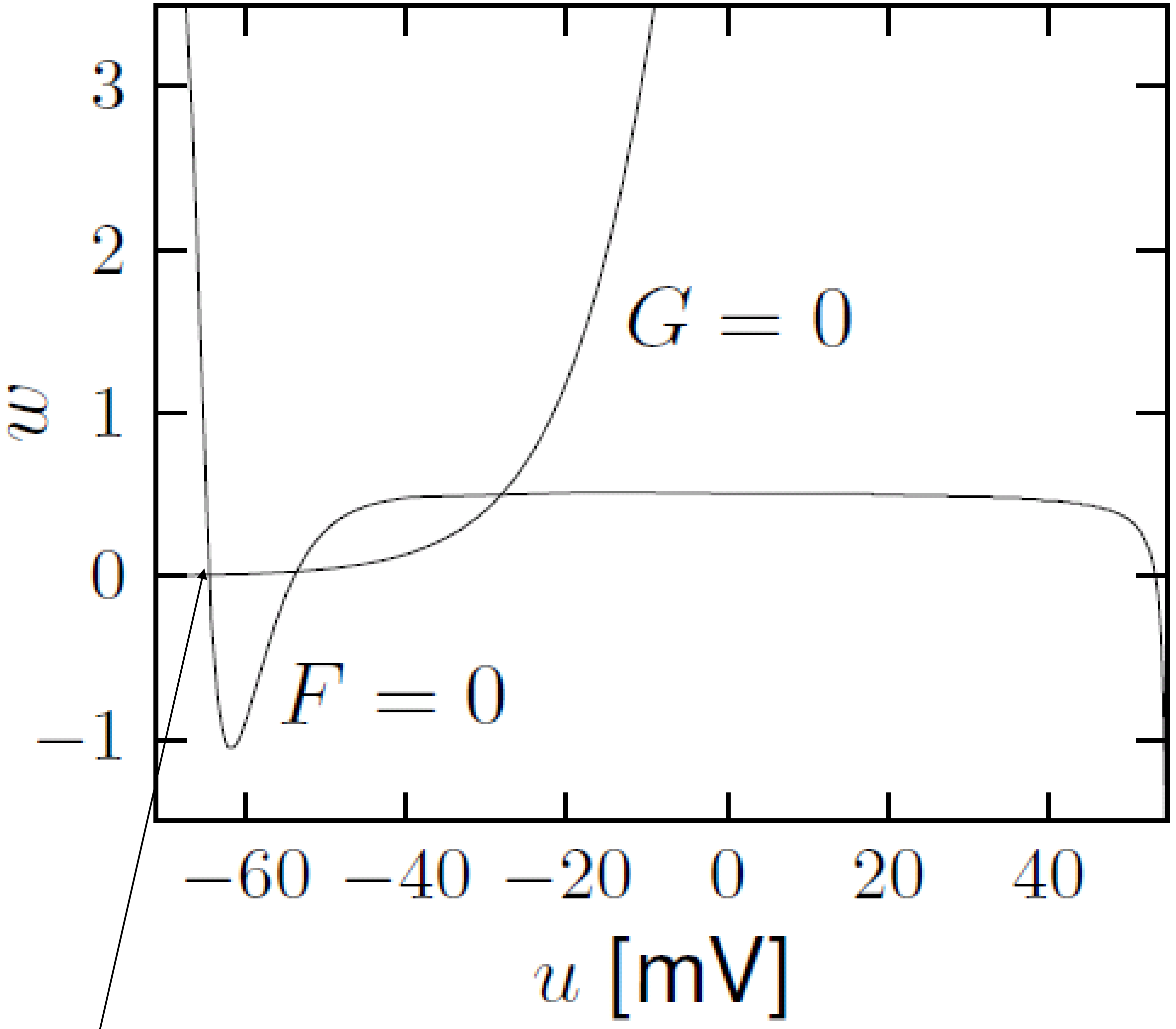
Separation of time scales

$$\tau_w >> \tau_u$$

$$\tau_w \frac{dw}{dt} \approx 0 \rightarrow \; w \approx w_{rest}$$

$$\tau \frac{du}{dt} = F(u, w_{rest}) + RI(t)$$

$$\frac{dw}{dt} = 0$$



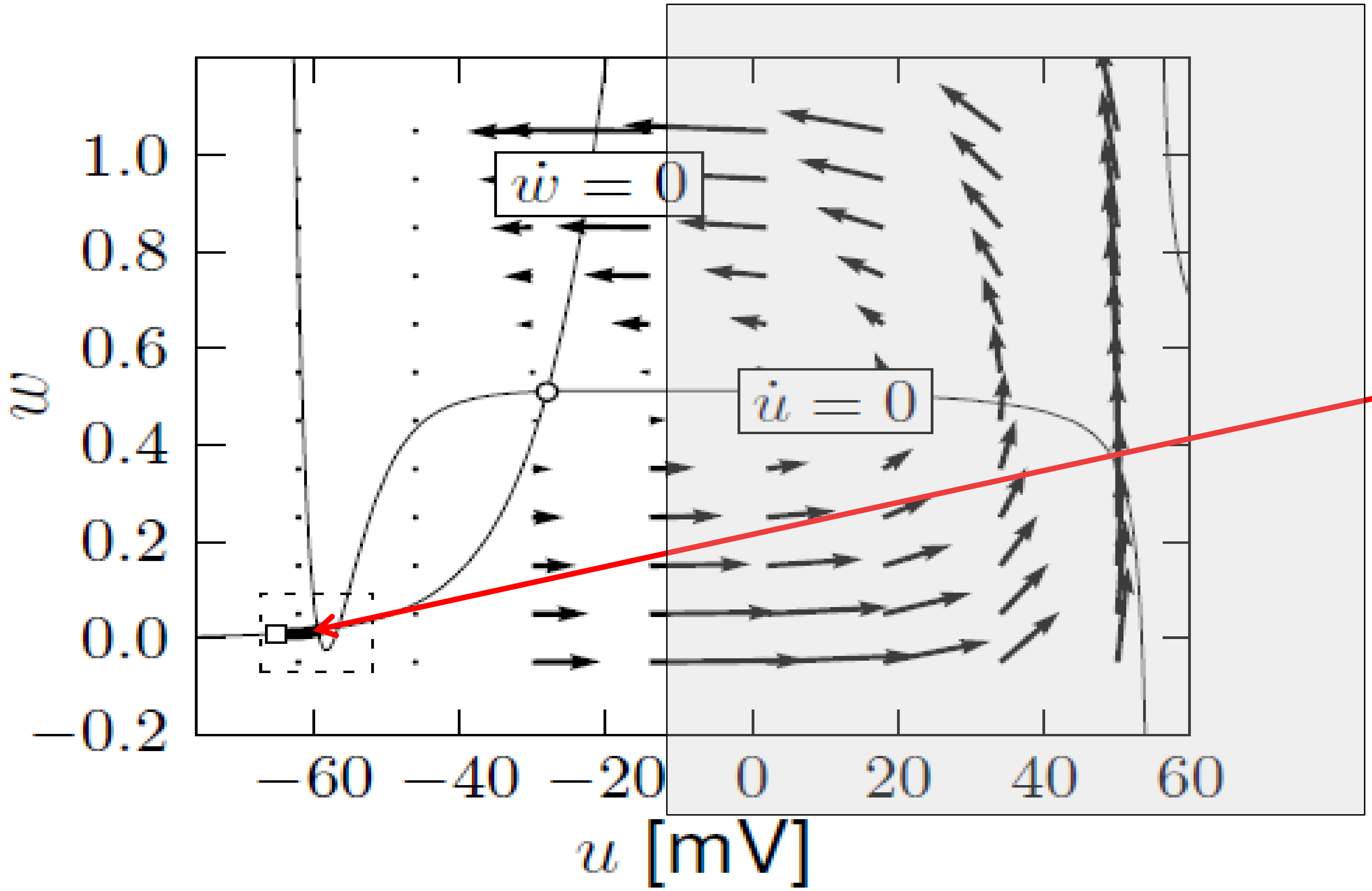
$$\frac{du}{dt} = 0$$

Stable fixed point

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(i) Extract *f* from more complex models

$$\tau \frac{du}{dt} = f(u) + R\,I(t)$$



A. detect spike and reset

resting state

Separation of time scales:
Arrows are nearly horizontal

Spike initiation, from rest

$$w \approx w_{rest}$$

See week 3:
2dim version of
Hodgkin-Huxley

$$\tau \frac{du}{dt} = F(u,w) + R\,I(t)$$

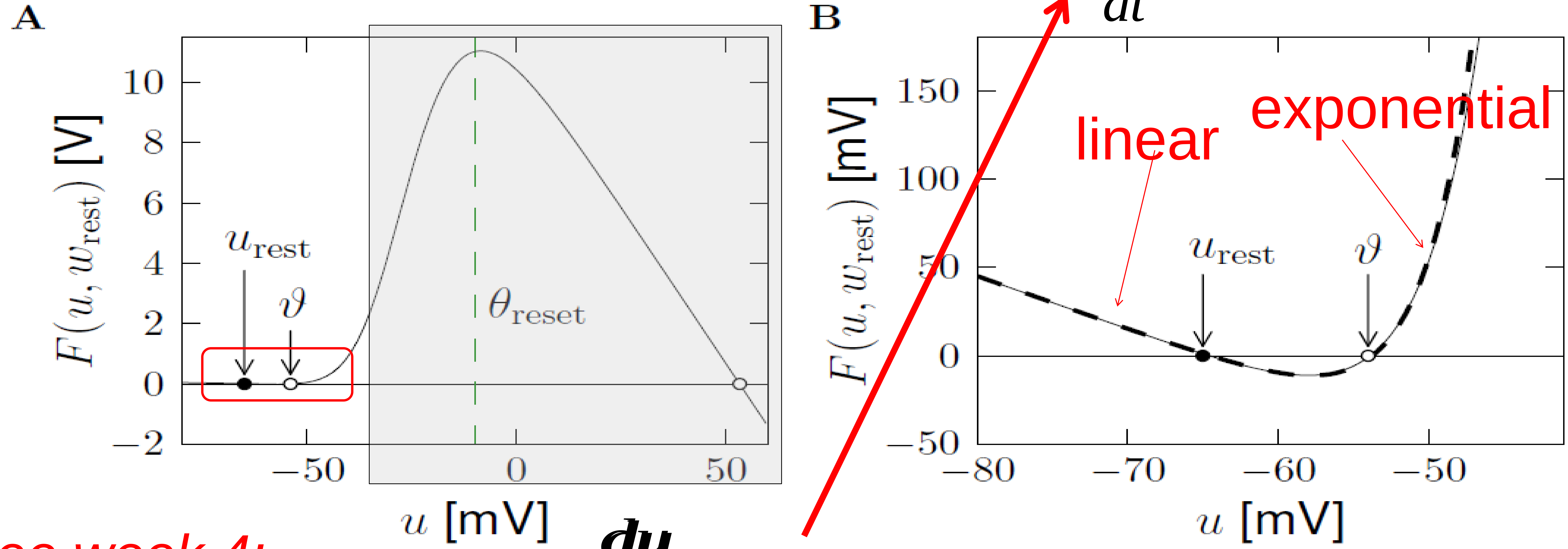
$$\tau_w \frac{dw}{dt} = G(u,w)$$

B. Assume *w*=*w*$_{rest}$

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(i) Extract f from more complex models

$$\tau \frac{du}{dt} = f(u) + R\,I(t)$$



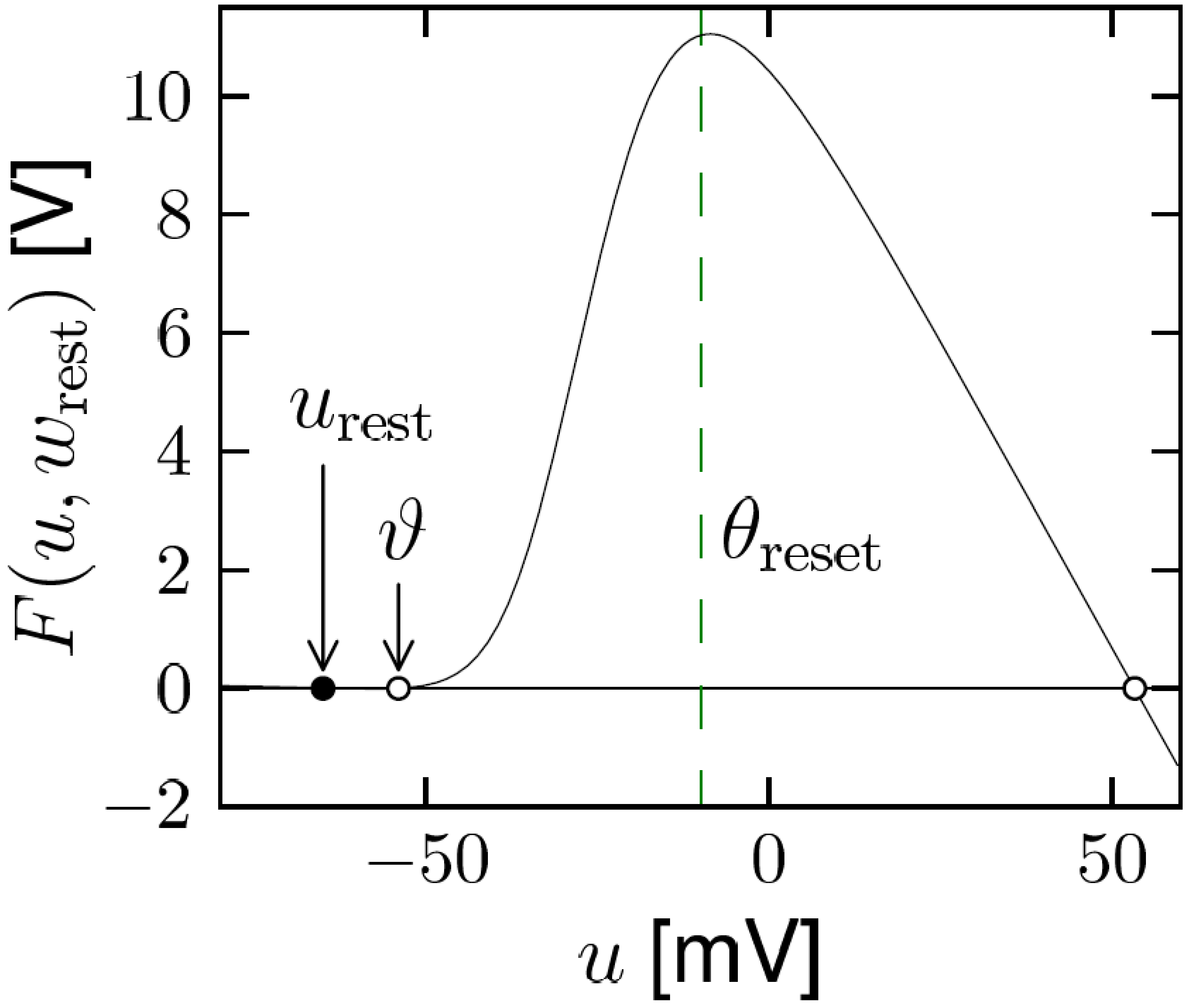
See week 4:
2dim version of
Hodgkin-Huxley

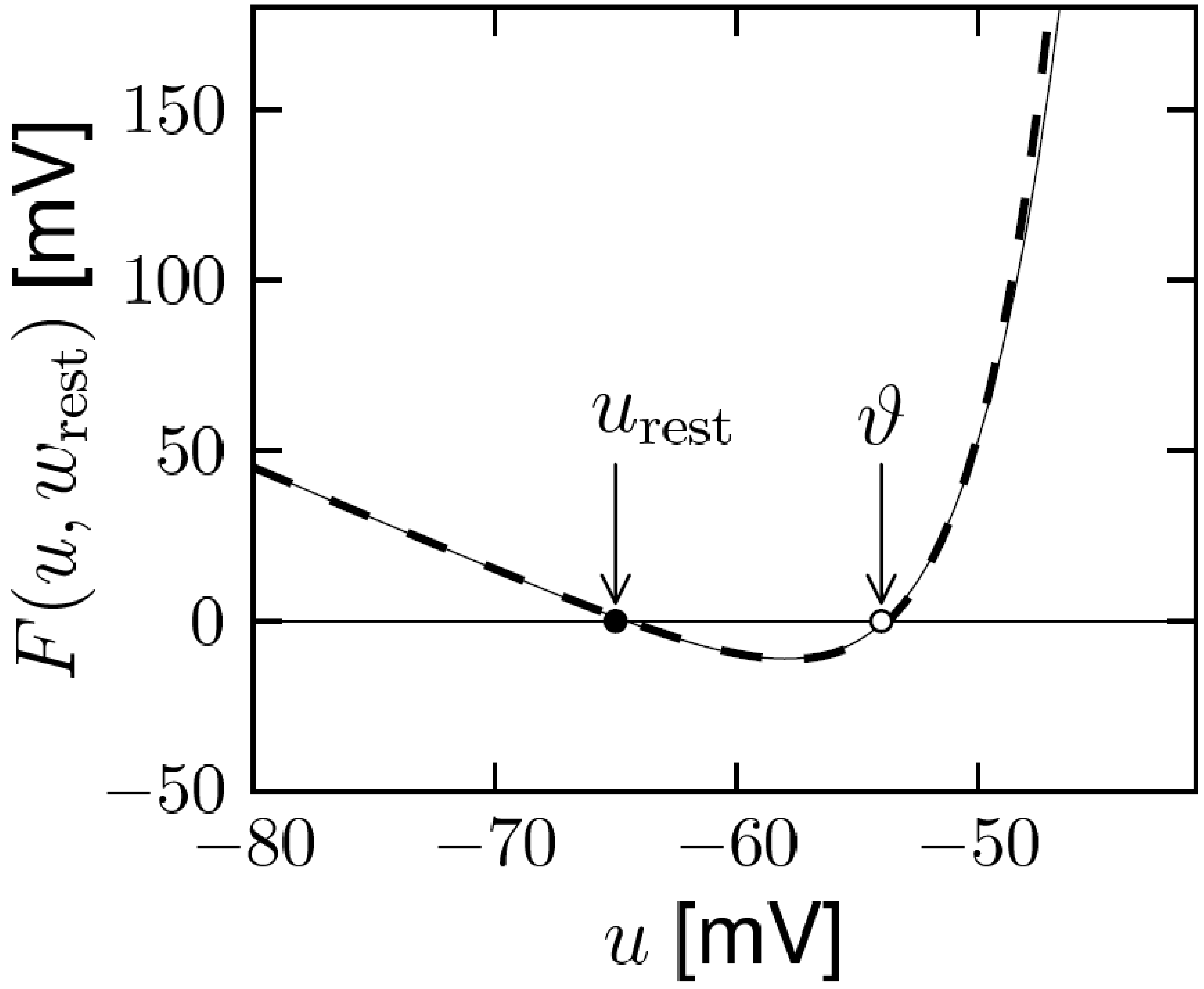
$$\tau \frac{du}{dt} = F(u, w_{rest}) + R\,I(t)$$

Separation of time scales

$$\tau_w \frac{dw}{dt} = G(u, w) \longrightarrow w \approx w_{rest}$$

Neuronal Dynamics – 4.5. Nonlinear Integrate-and-Fire Model





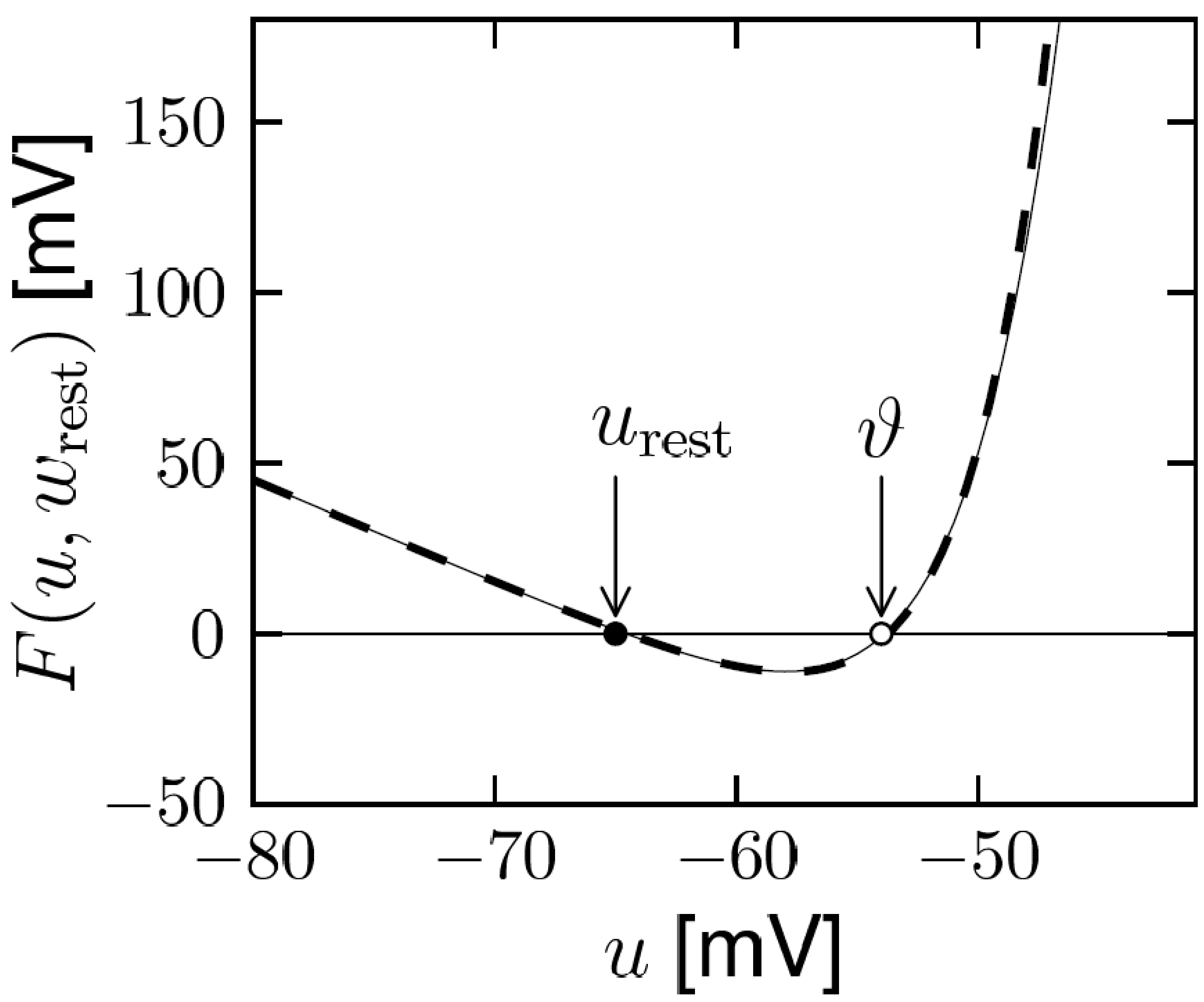
$$\tau \frac{du}{dt} = F(u, w_{rest}) + RI(t) = f(u) + RI(t)$$

→ Nonlinear I&F (see week 1!)

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

Exponential integrate-and-fire model (EIF)

$$f(u) = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right)$$



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$$\tau \frac{du}{dt} = F(u, w_{rest}) + RI(t) = f(u) + RI(t)$$

→ Nonlinear I&F (see week 1!)

Exponential integrate-and-fire model (EIF)

$$f(u) = -(u - u_{rest}) + \Delta \exp(\frac{u-\vartheta}{\Delta})$$

linear

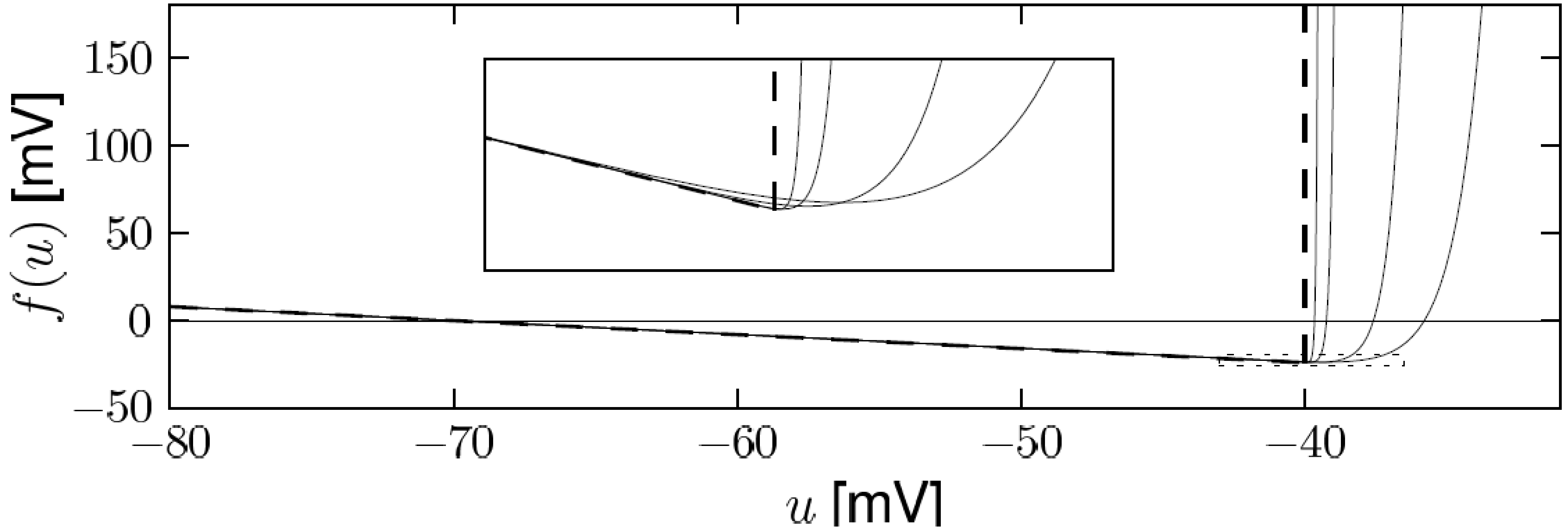


Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

Direct derivation from Hodgkin-Huxley

$$C\frac{du}{dt} = -g_{Na}\, m^3\, h(u - E_{Na}) - g_K n^4\,(u - E_K) - g_l(u - E_l) + I(t)$$

$$C\frac{du}{dt} = -g_{Na}[m_0(u)]^3 h_{rest}(u - E_{Na}) - g_K[n_{rest}]^4(u - E_K) - g_l(u - E_l) + I(t)$$

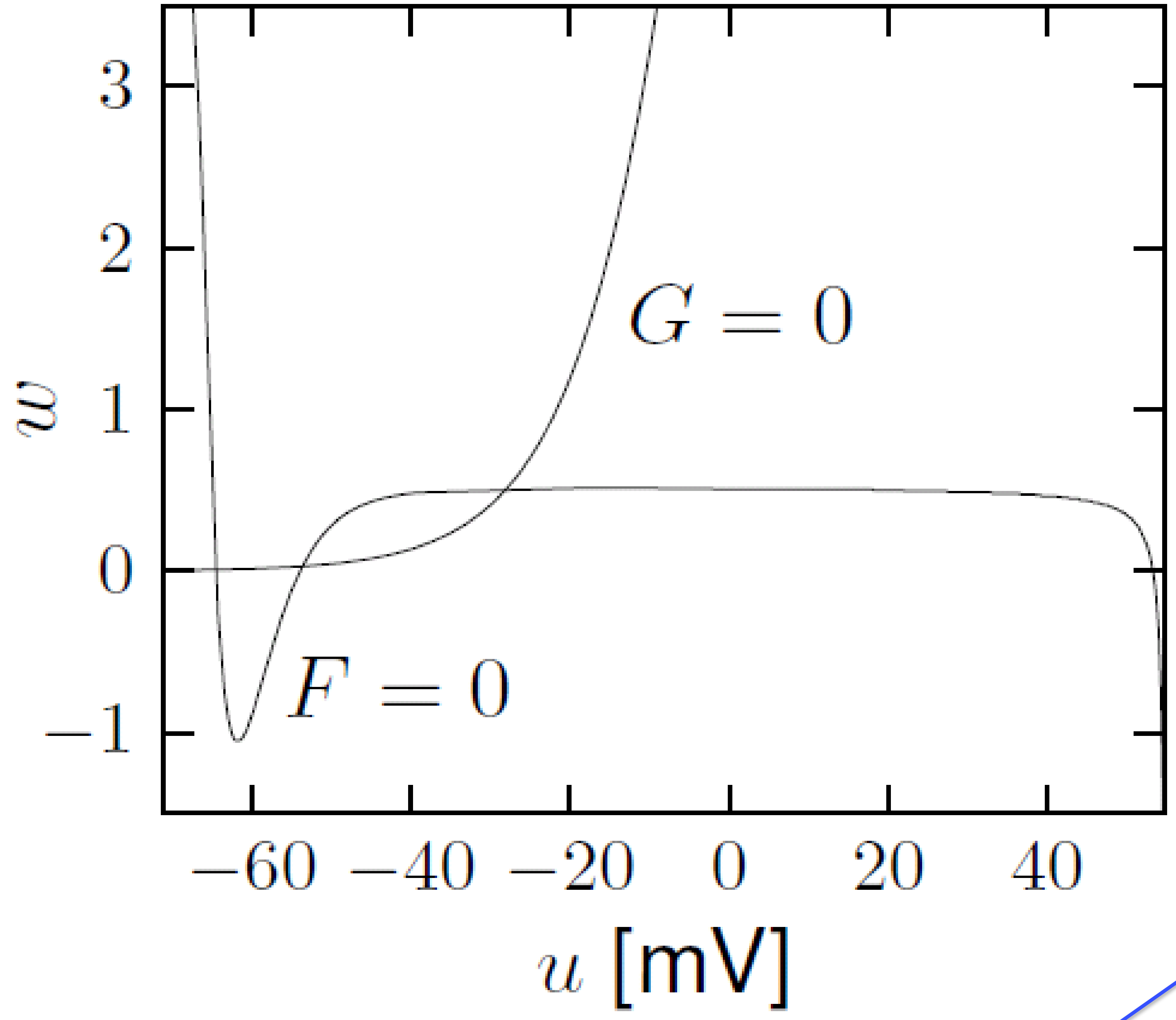
Fourcaud-Trocme et al, J. Neurosci. 2003

$$f(u) = -(u - u_{rest}) + \Delta \exp(\tfrac{u-\vartheta}{\Delta})$$

$$\tau\frac{du}{dt} = F(u, h_{rest}, n_{rest}) + RI(t) = f(u) + RI(t)$$

gives expon. I&F

Neuronal Dynamics – 4.5. Nonlinear Integrate-and-Fire Model



Relevant during spike and downswing of AP

2-dimensional equation

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Separation of time scales

-w is constant (if not firing)

$$\tau \frac{du}{dt} = f(u) + RI(t)$$

threshold+reset for firing

2-dimensional equation

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Separation of time scales

-*w* is constant (if not firing)

$$\tau \frac{du}{dt} = f(u) + RI(t)$$

Linear plus exponential

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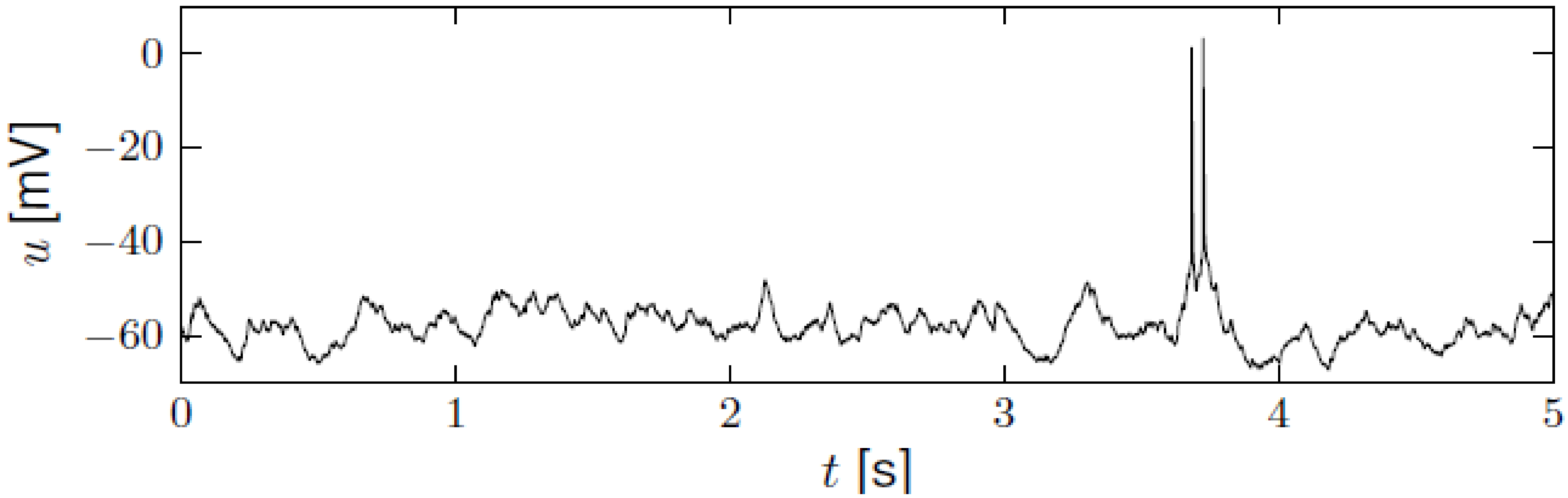
- **Quality of NLIF?**

Spontaneous activity *in vivo*

awake mouse, cortex, freely whisking,



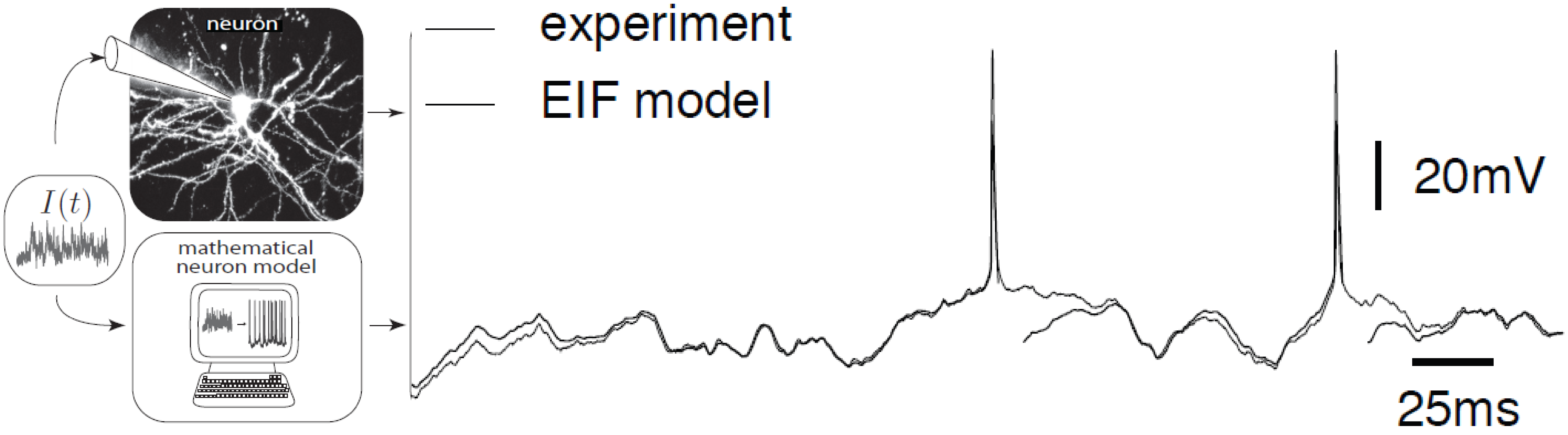
-spikes are rare events

Crochet et al., 2011

-membrane potential fluctuates around ‘rest’

Aims of Modeling: - predict spike initation times
- predict subthreshold voltage

Neuronal Dynamics – 4.5.How good are integrate-and-fire models?



Badel et al., 2008

Aims: - predict spike initation times
- predict subthreshold voltage

Add adaptation and refractoriness (week 7)

Neuronal Dynamics – Quiz 4.7.

A. Exponential integrate-and-fire model.

The model can be derived

[ ] from a 2-dimensional model, assuming that the auxiliary variable w is constant.

[ ] from the HH model, assuming that the gating variables h and n are constant.

[ ] from the HH model, assuming that the gating variables m is constant.

[ ] from the HH model, assuming that the gating variables m is instantaneous.

B. Reset.

[ ] In a 2-dimensional model, the auxiliary variable w is necessary to implement a reset of the voltage after a spike

[ ] In a nonlinear integrate-and-fire model, the auxiliary variable w is necessary to implement a reset of the voltage after a spike

[ ] In a nonlinear integrate-and-fire model, a reset of the voltage after a spike is implemented algorithmically/explicitly

Neuronal Dynamics – Nonlinear Integrate-and-Fire

Reading: W. Gerstner, W.M. Kistler, R. Naud and L. Paninski, *Neuronal Dynamics: from single neurons to networks and models of cognition.* Chapter 4*: Introduction*. Cambridge Univ. Press, 2014
OR W. Gerstner and W.M. Kistler, *Spiking Neuron Models*, Ch.3. Cambridge 2002
OR J. Rinzel and G.B. Ermentrout, (1989). Analysis of neuronal excitability and oscillations. In Koch, C. Segev, I., editors, *Methods in neuronal modeling*. MIT Press, Cambridge, MA.

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-Badel, L., Lefort, S., Berger, T., Petersen, C., Gerstner, W., and Richardson, M. (2008). Biological Cybernetics, 99(4-5):361-370.
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