Week 9 – part 1 : Models and data

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

9.1 What is a good neuron model?
- Models and data

9.2 AdEx model
- Firing patterns and adaptation

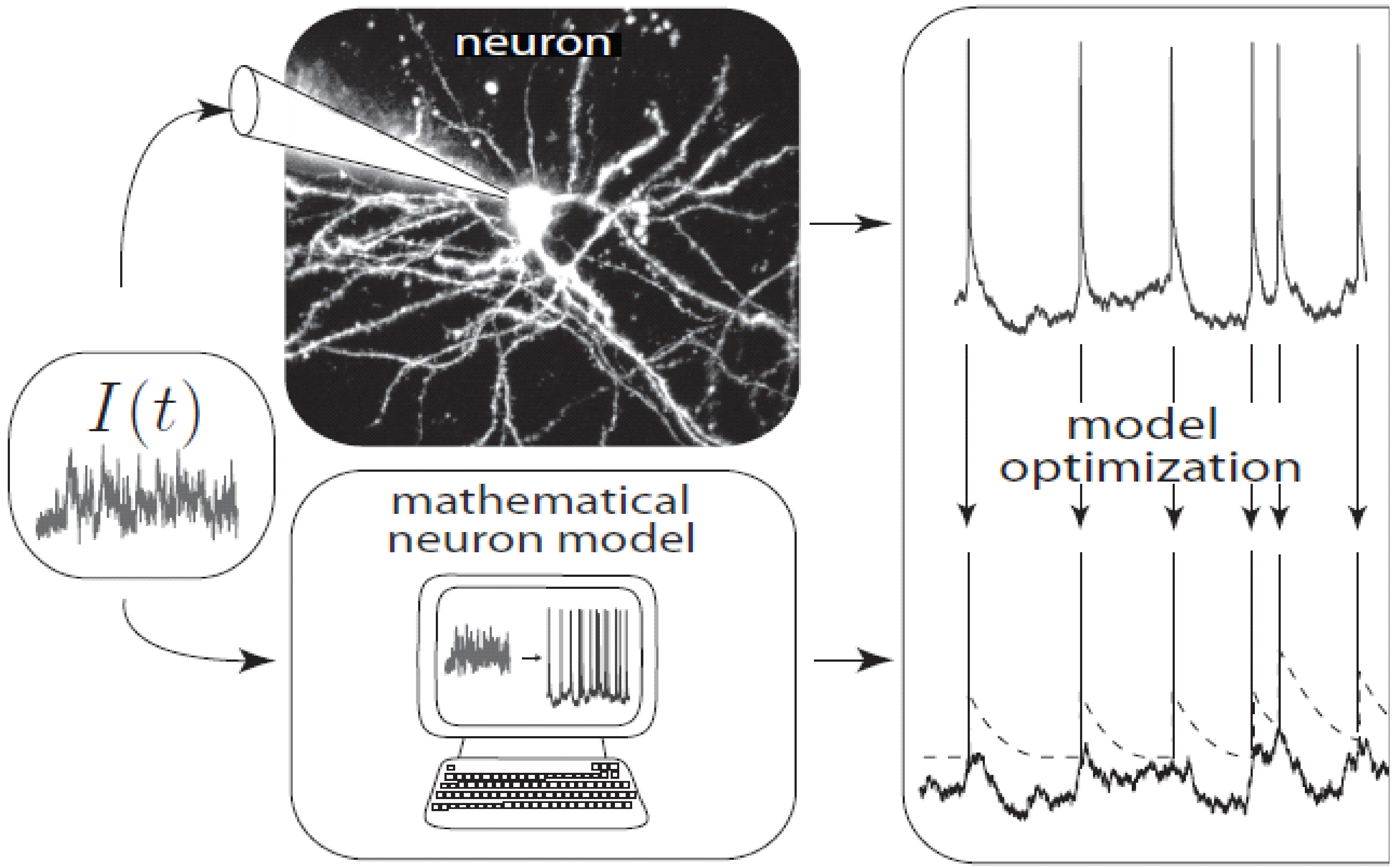
9.3 Spike Response Model (SRM)
- Integral formulation

9.4 Generalized Linear Model
- Adding noise to the SRM

9.5 Parameter Estimation
- Quadratic and convex optimization

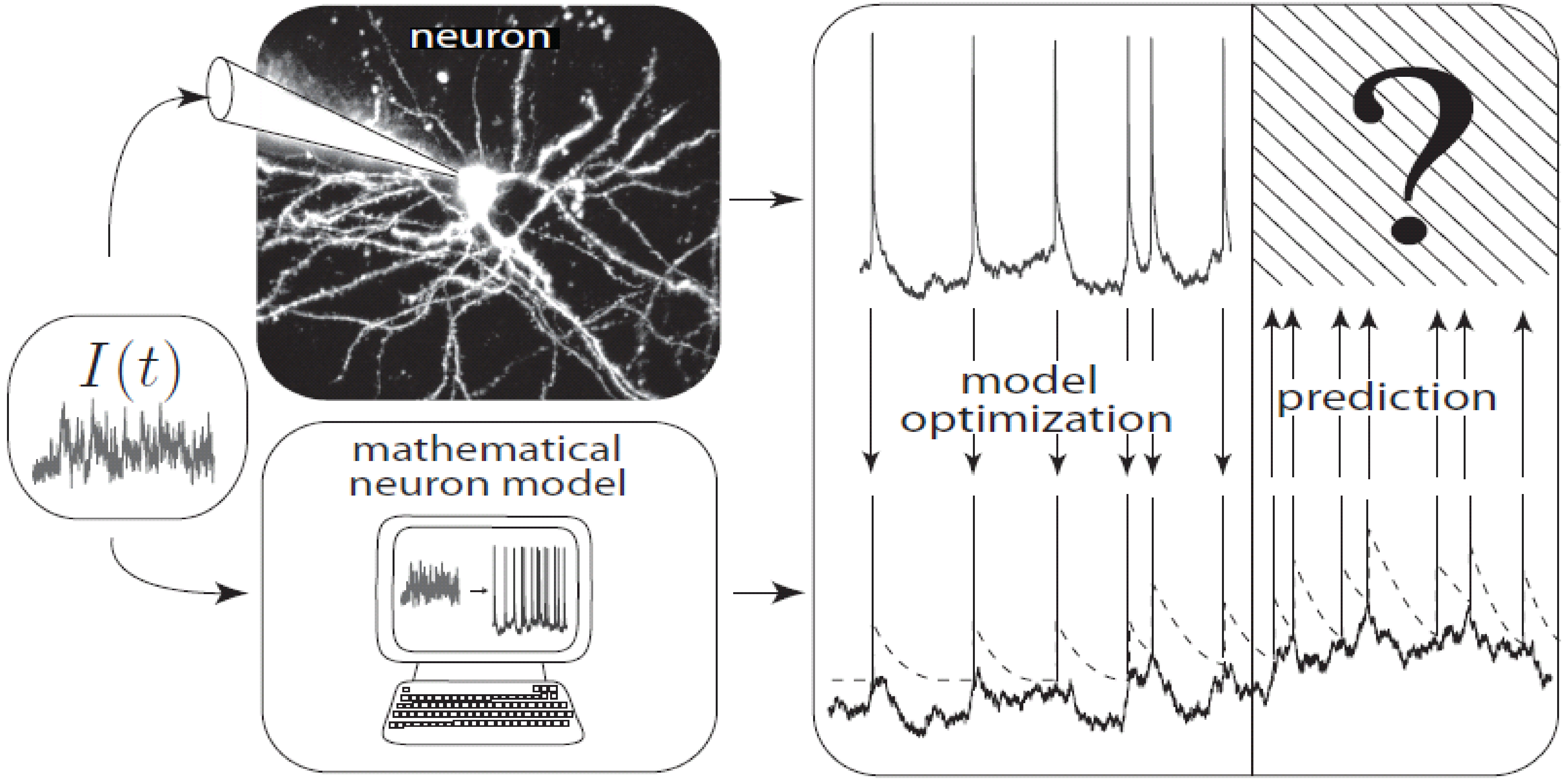
9.6. Modeling in vitro data
- how long lasts the effect of a spike?

Neuronal Dynamics – 9.1 Neuron Models and Data



-What is a good neuron model?
-Estimate parameters of models?

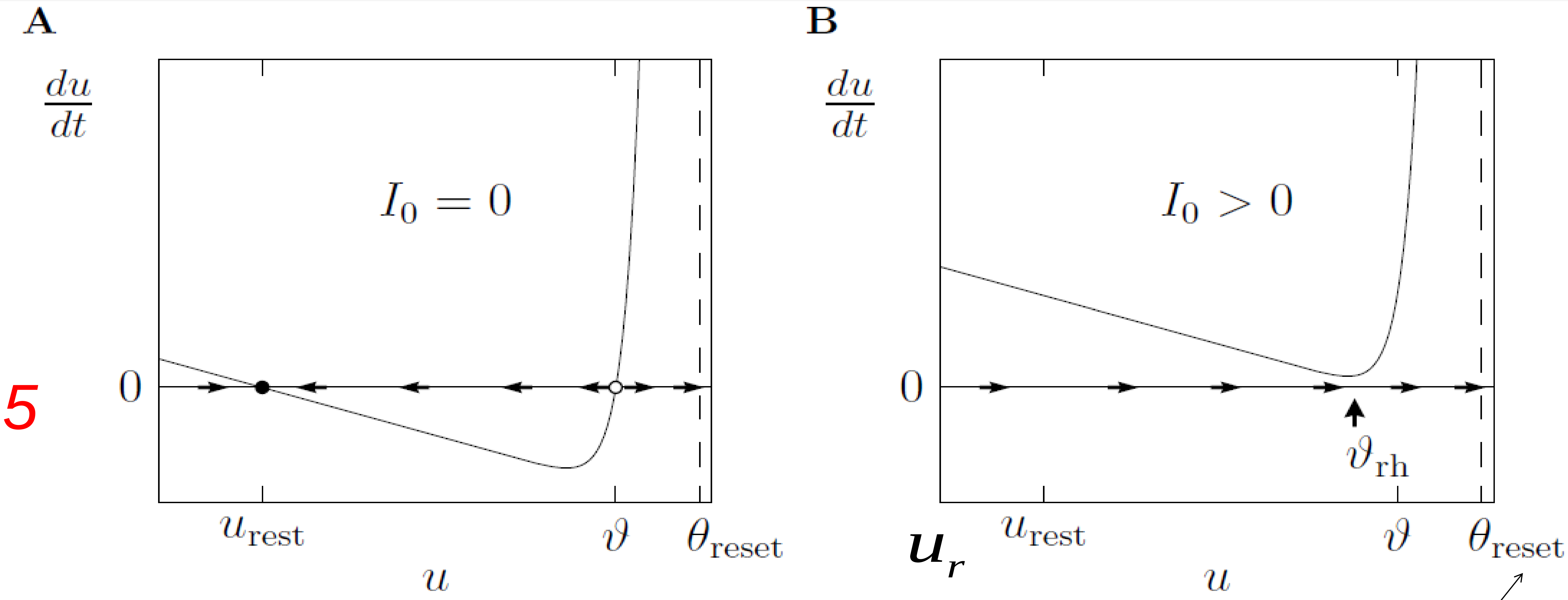
Neuronal Dynamics – 9.1 What is a good neuron model?



A) Predict spike times
B) Predict subthreshold voltage
C) Easy to interpret (not a ‘black box’)
D) Flexible enough to account for a variety of phenomena
E) Systematic procedure to ‘optimize’ parameters

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

See:
week 1, lecture 1.5



u_r

$$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

What is a good choice of **f** ?

If $u = \theta_{reset}$

then reset to $u = u_r$

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(1) $$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

(2) $$If \;\; u = \theta_{reset} \;\; then \;\; reset\, to\; u = u_r$$

What is a good choice of ***f*** ?

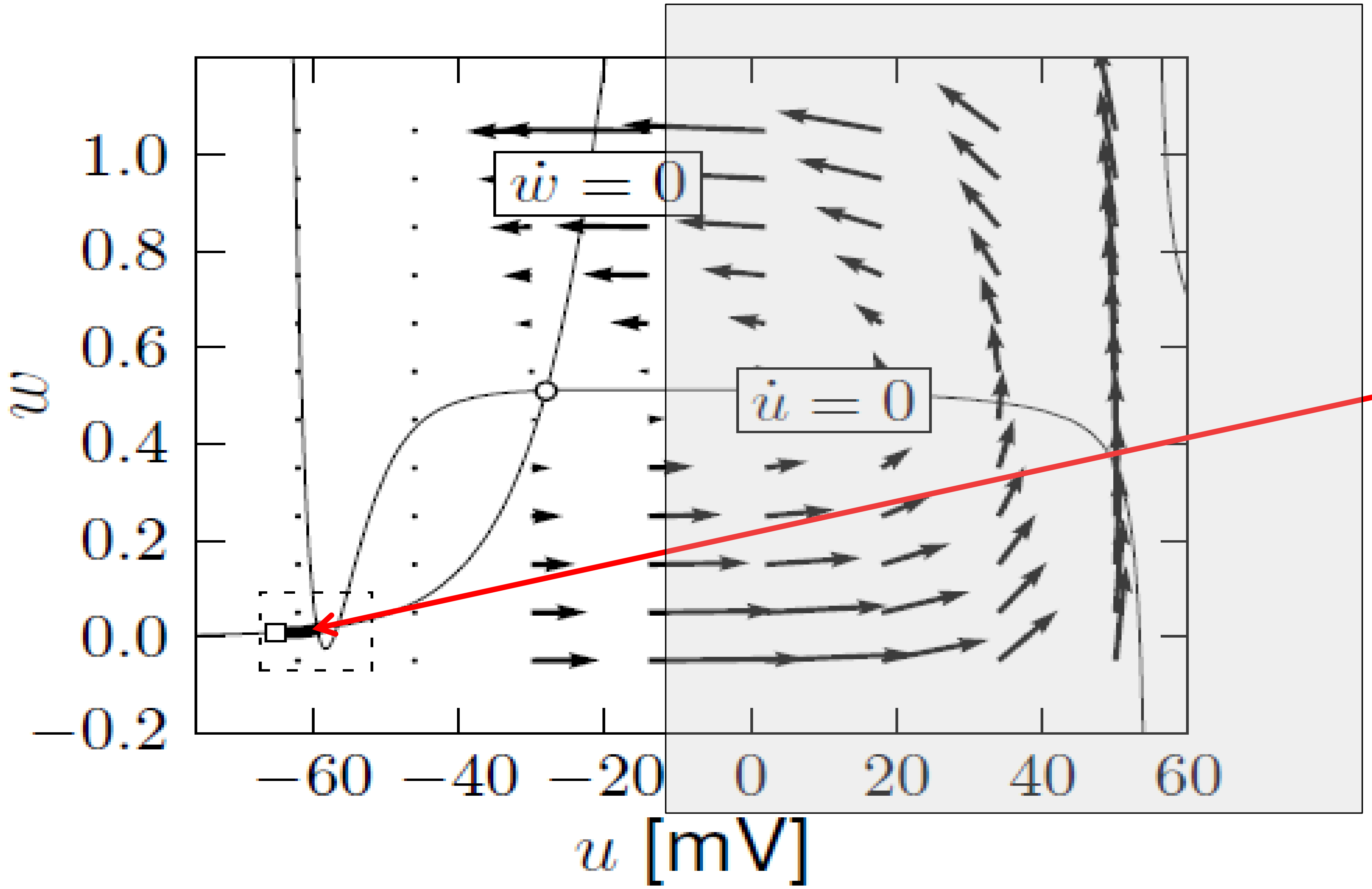
(i) Extract *f* from more complex models

(ii) Extract *f* from data

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(i) Extract f from more complex models

$$\tau \frac{du}{dt} = f(u) + R\,I(t)$$



A. detect spike and reset

resting state

Separation of time scales:
Arrows are nearly horizontal

Spike initiation, from rest

$$w \approx w_{rest}$$

See week 3:
2dim version of
Hodgkin-Huxley

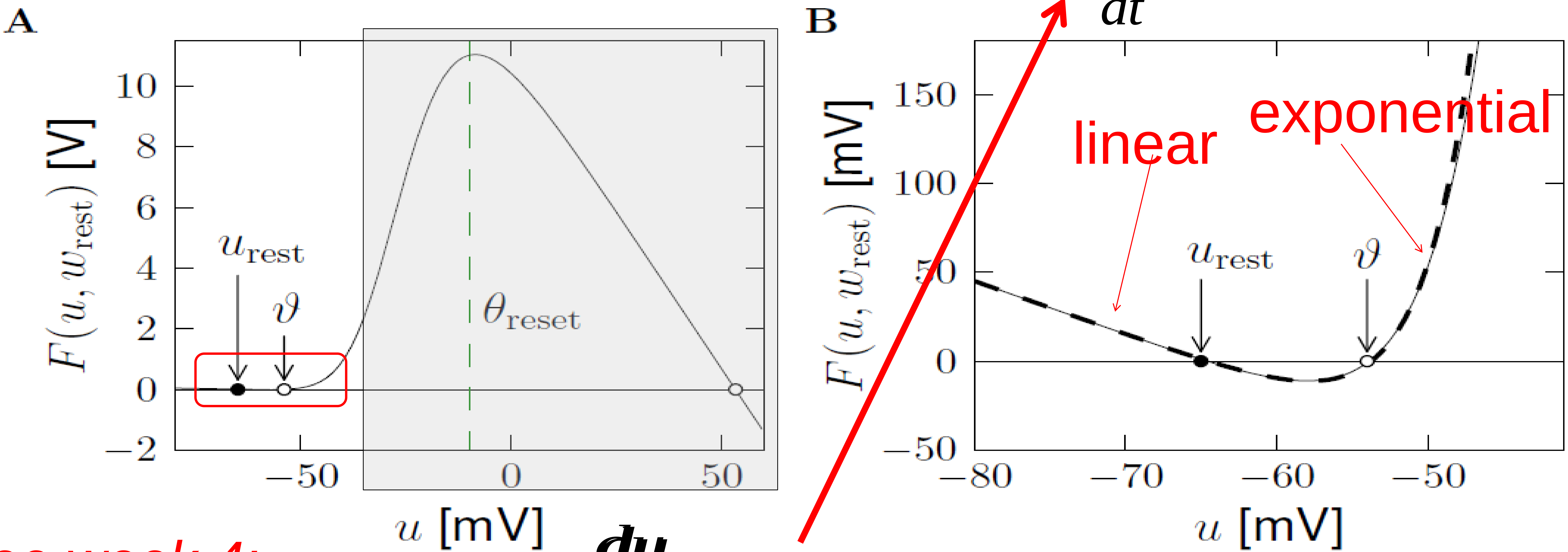
$$\tau \frac{du}{dt} = F(u,w) + R\,I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

B. Assume $w = w_{rest}$

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(i) Extract f from more complex models $\tau \frac{du}{dt} = f(u) + RI(t)$



See week 4:
2dim version of
Hodgkin-Huxley

$$\tau \frac{du}{dt} = F(u, w_{rest}) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u, w) \longrightarrow w \approx w_{rest}$$

Separation of time scales

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

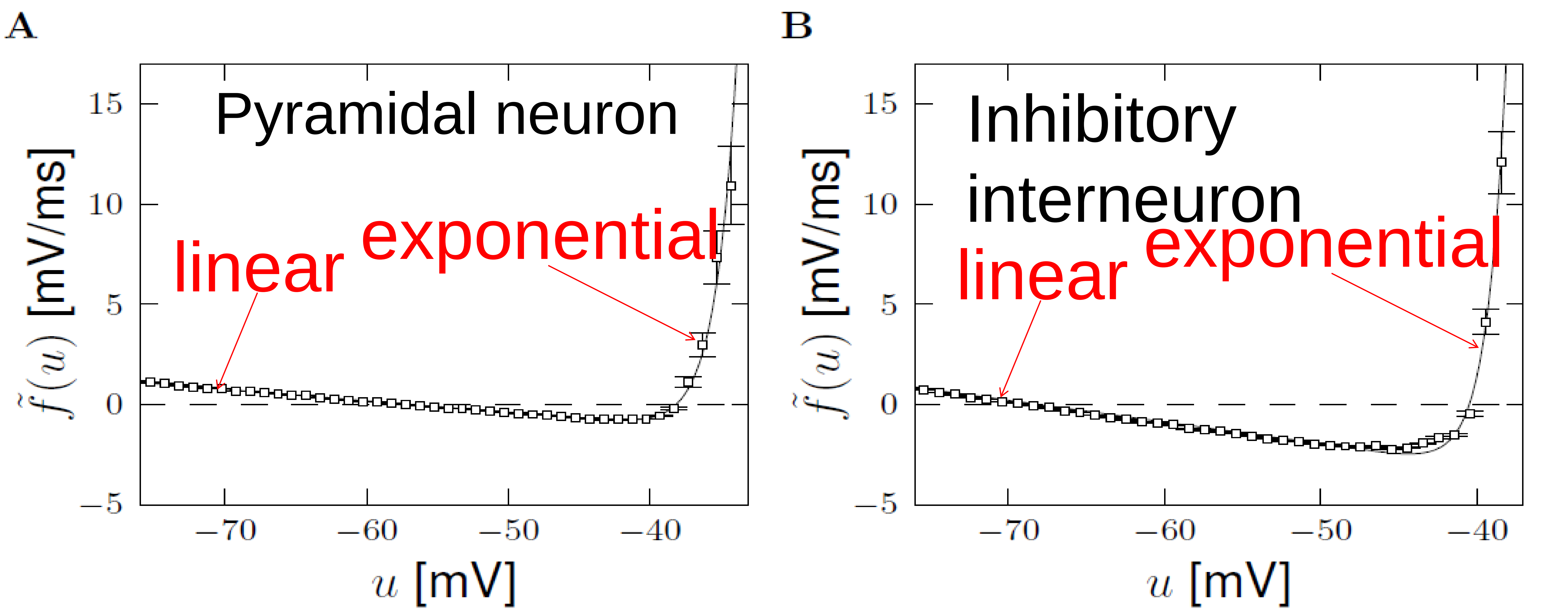
(ii) Extract *f* from data *Badel et al. (2008)*

$$\tau \frac{du}{dt} = f(u) + RI(t)$$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right)$$

$$\tilde{f}(u) = \frac{f(u)}{\tau}$$

Exp. Integrate-and-Fire, *Fourcaud et al. 2003*



Badel et al. (2008)

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

$$(1) \quad \tau \frac{du}{dt} = f(u) + R\,I(t)$$

$$(2) \quad If \quad u = \theta_{reset} \quad then \quad reset\,to\; u = u_r$$

Best choice of ***f*** : linear + exponential

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u - \vartheta}{\Delta})$$

BUT: **Limitations – need to add**

-Adaptation on slower time scales

-Possibility for a diversity of firing patterns

-Increased threshold ϑ after each spike

-Noise

Week 9 – part 2 : Adaptive Expontential Integrate-and-Fire Model

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

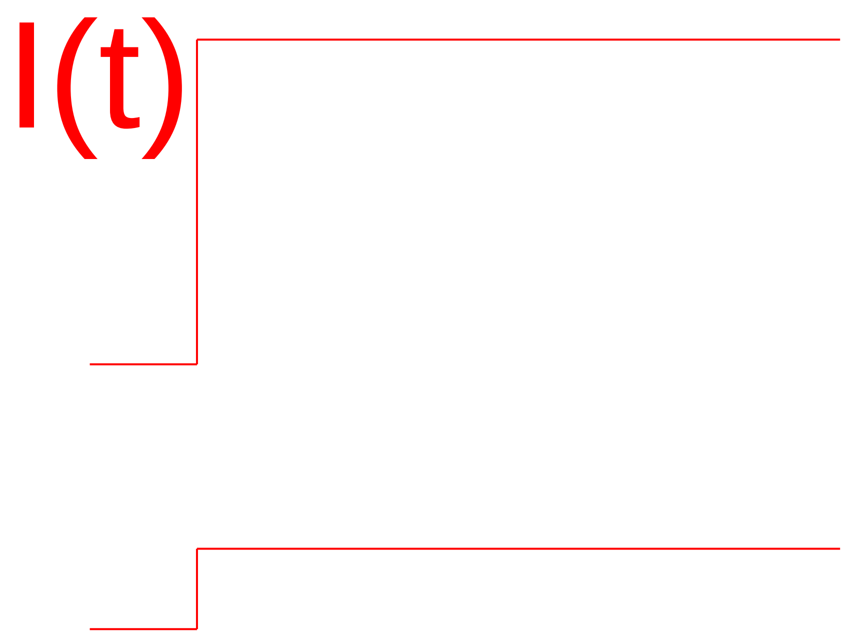
Biological Modeling of Neural Networks:

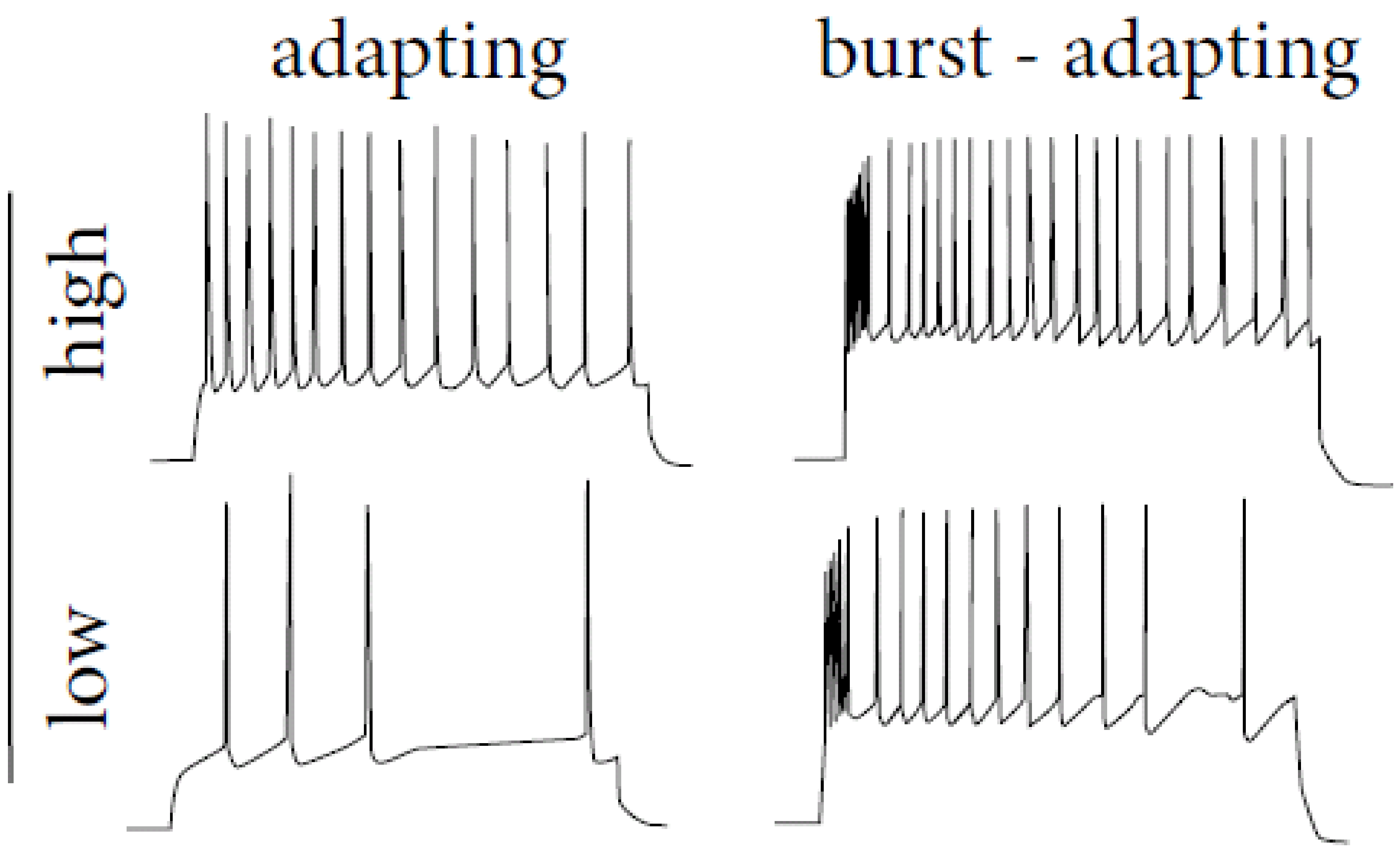
Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Step current input – neurons show adaptation





Data:
Markram et al.
(2004)

1-dimensional (nonlinear) integrate-and-fire model cannot do this!

Add adaptation variables:

Blackboard !

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) - R\sum_k w_k$$

$$\tau_k \frac{dw_k}{dt} = a_k (u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)$$

Exponential I&F
+ **1 adaptation** var.
= AdEx

SPIKE AND RESET

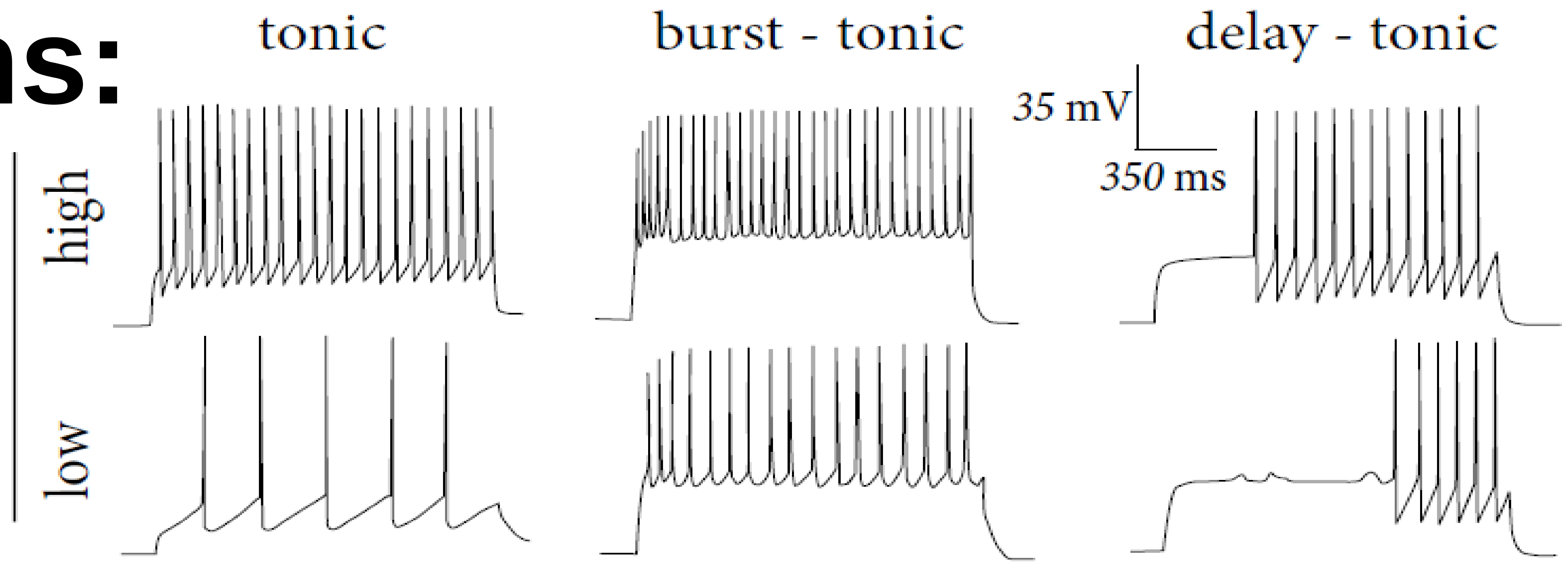
after each spike w_k jumps by an amount b_k

$If \;\; u = \theta_{reset} \;\; then \;\; reset \, to \; u = u_r$

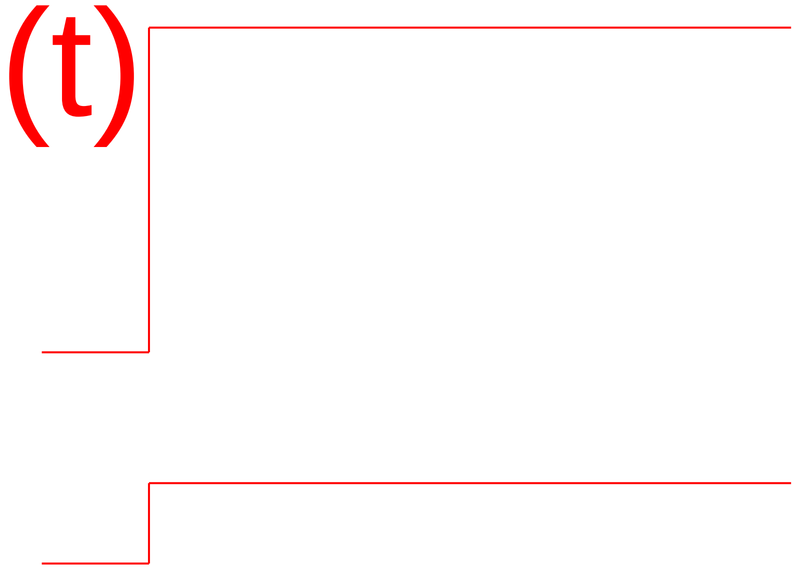
AdEx model, Brette&Gerstner (2005):

Firing patterns:

Response to
Step currents,
Exper. Data,
Markram et al.
(2004)

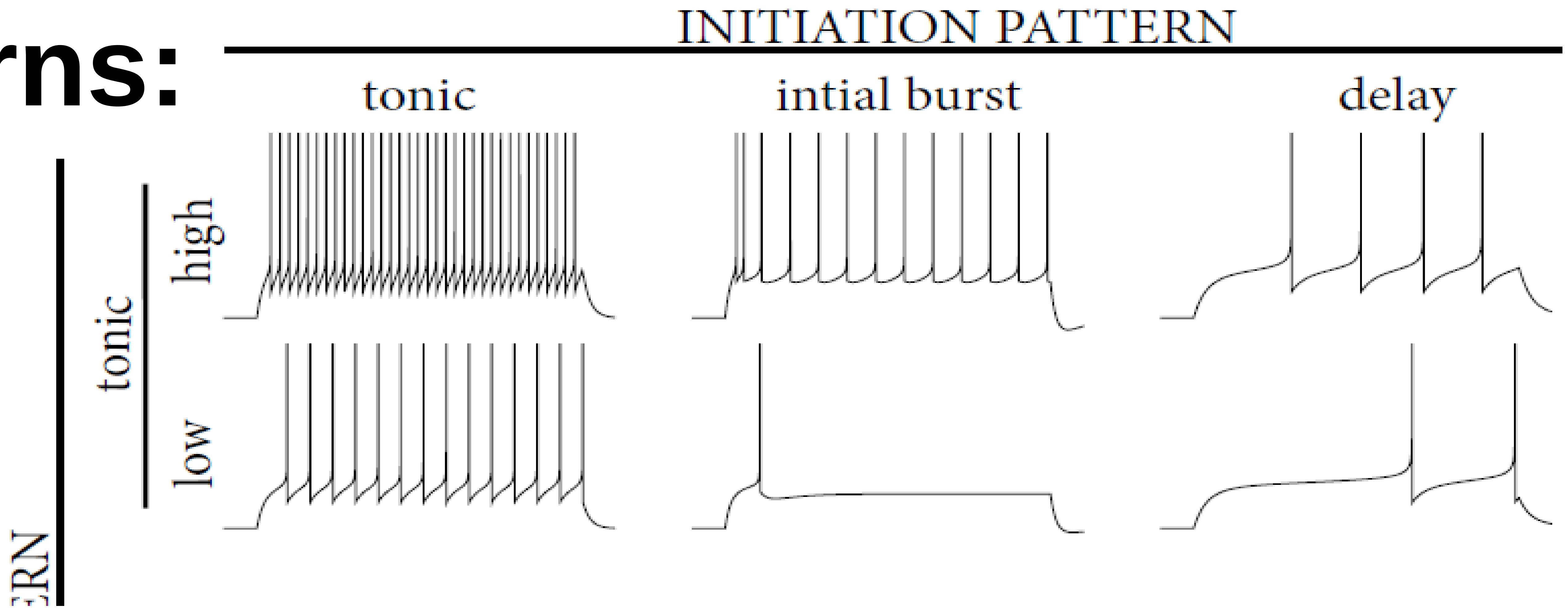


I(t)



Firing patterns:

Response to
Step currents,
AdEx Model,
Naud&Gerstner



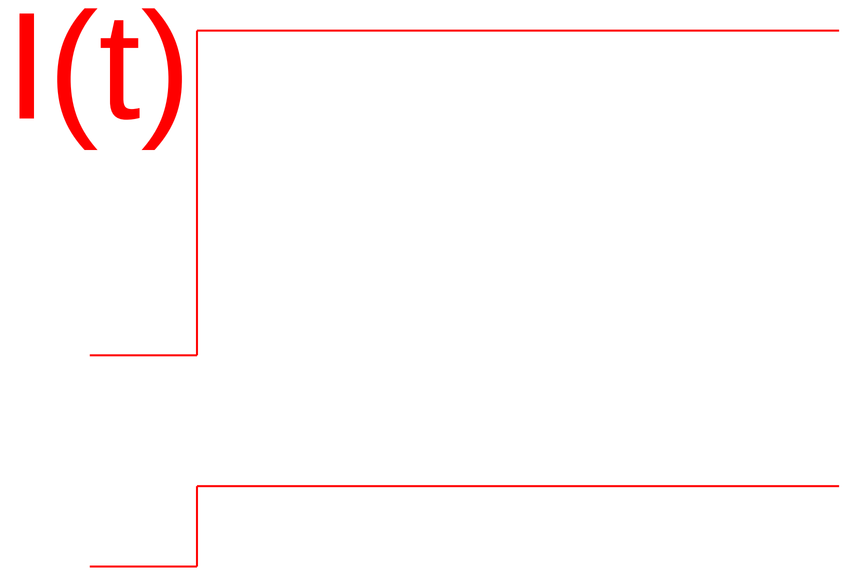


Image:
Neuronal Dynamics,
Gerstner et al.
Cambridge (2002)

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\ \tau_w \sum_f \delta(t - t^f)$$

AdEx model

Phase plane analysis!

Can we understand the different firing patterns?

$$\tau \frac{du}{dt} = f(u) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w$$

-linear + exponential
-adaptation variable

→ Various firing patterns

Neuronal Dynamics – Quiz 9.1. Nullclines of AdEx

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u - \vartheta}{\Delta}) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w$$

A - What is the qualitative shape of the w-nullcline?

[ ] constant
[ ] linear, slope a
[ ] linear, slope 1
[ ] linear + quadratic
[ ] linear + exponential

B - What is the qualitative shape of the u-nullcline?

[ ] linear, slope 1
[ ] linear, slope 1/R
[ ] linear + quadratic
[ ] linear w. slope 1/R+ exponential

2 minutes
Restart at 9:38

Week 9 – part 2b : Firing Patterns

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

9.1 What is a good neuron model?
- Models and data

9.2 AdEx model
- Firing patterns and adaptation

9.3 Spike Response Model (SRM)
- Integral formulation

9.4 Generalized Linear Model
- Adding noise to the SRM

9.5 Parameter Estimation
- Quadratic and convex optimization

9.6. Modeling in vitro data
- how long lasts the effect of a spike?

AdEx model

after each spike
u is reset to **u**$_r$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\,(u - u_{rest}) - w + b\,\tau_w \sum_f \delta(t - t^f)$$

after each spike
w jumps by an amount ***b***

parameter ***a*** **– slope of *w*-nullcline**

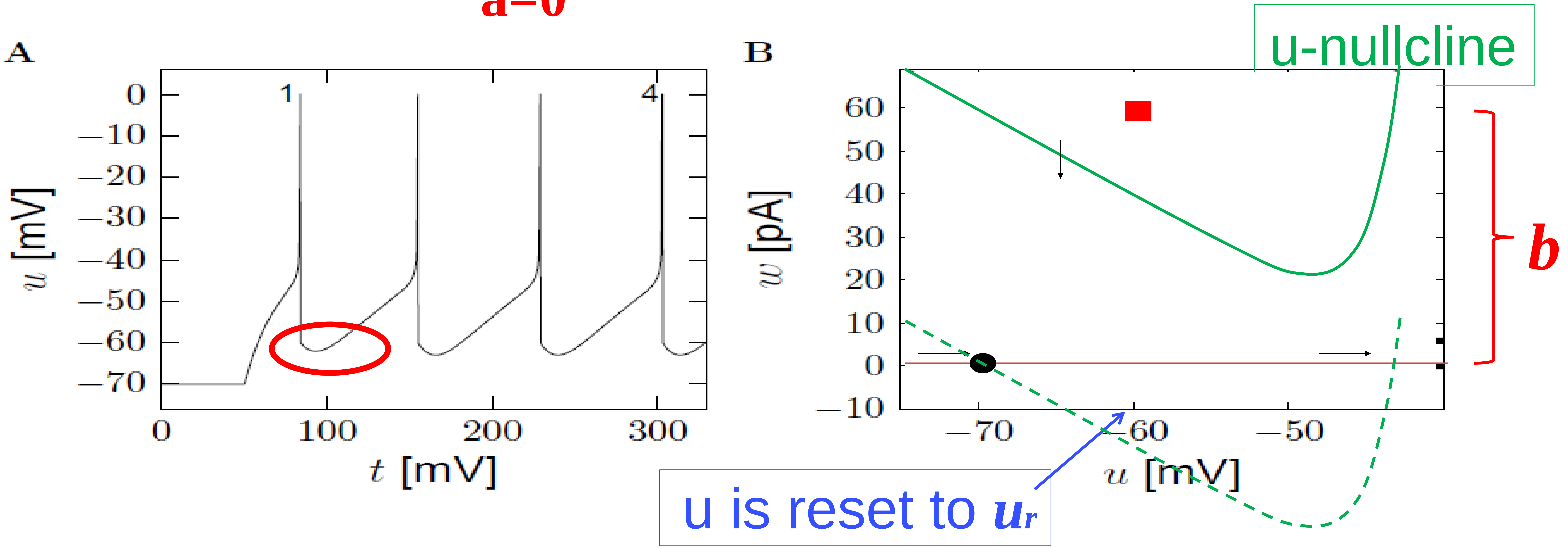
Can we understand the different firing patterns?

AdEx model – phase plane analysis: large *b*

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u-\vartheta}{\Delta}) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\ \tau_w \sum_f \delta(t - t^f)$$

a=0

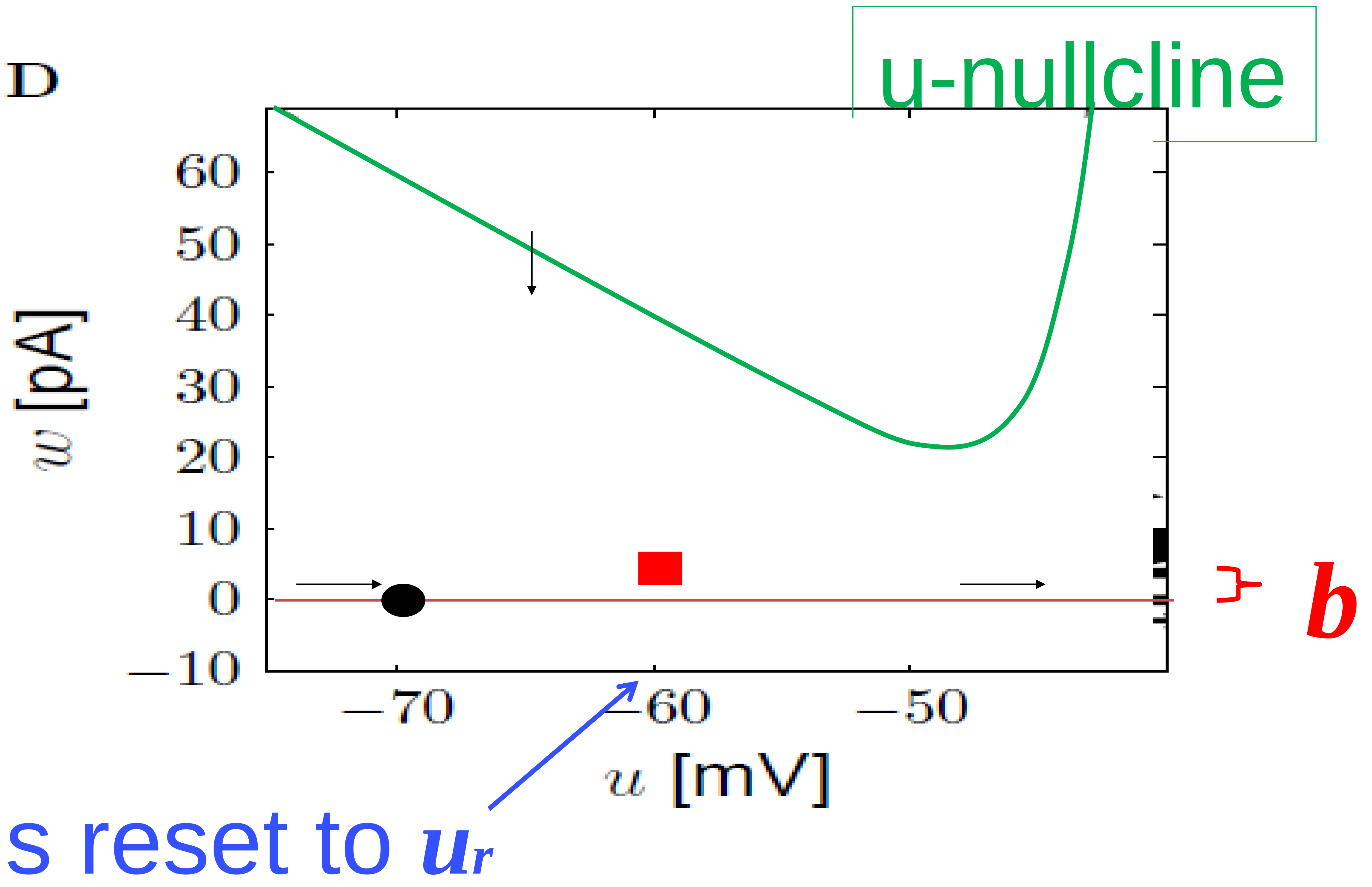


AdEx model – phase plane analysis: small *b*

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\,(u - u_{rest}) - w + b\,\tau_w \sum_f \delta(t - t^f)$$

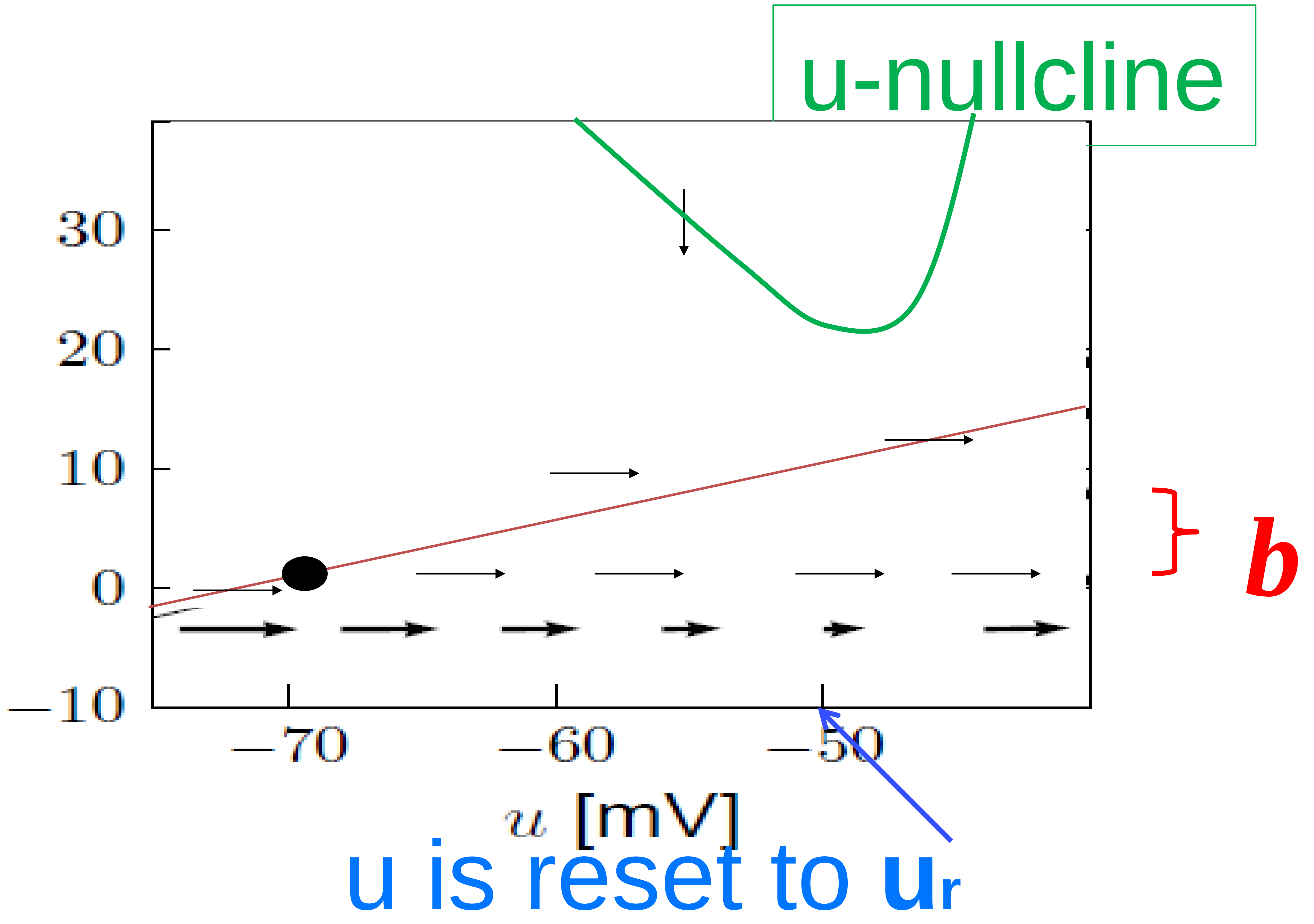
adaptation



AdEx model – phase plane analysis: a>0

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\ \tau_w \sum_f \delta(t - t^f)$$



after each spike u is reset to u_r

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\ \tau_w \sum_f \delta(t - t^f)$$

after each spike

w jumps by an amount **b**

parameter **a – slope of w nullcline**

Firing patterns arise from different parameters!

See Naud et al. (2008), see also Izikhevich (2003)

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(1) $$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

(2) $$\text{If} \quad u = \theta_{reset} \quad \text{then} \quad \text{reset to } u = u_r$$

Best choice of ***f*** : linear + exponential

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right)$$

BUT: **Limitations – need to add**

✓ -Adaptation on slower time scales

✓ -Possibility for a diversity of firing patterns

-Increased threshold ϑ after each spike

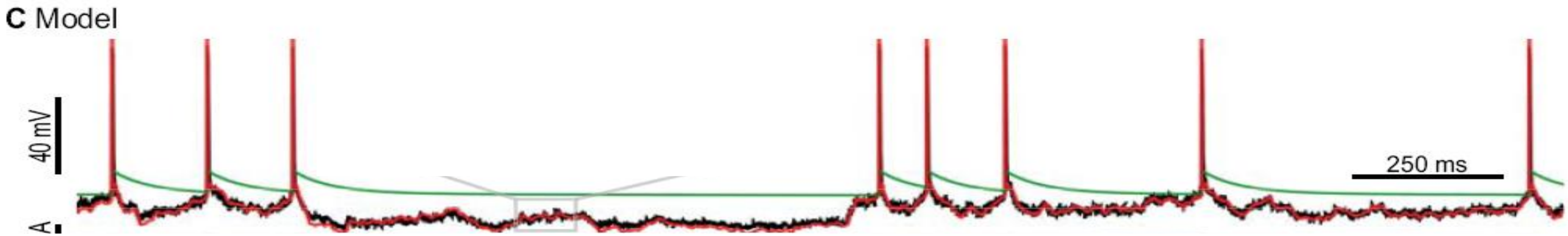
-Noise

Add dynamic threshold:

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - R\sum_k w_k + RI(t)$$

Threshold increases after each spike

$$\vartheta = \theta_0 + \sum_f \theta_1(t - t^f)$$



$$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

$$If \quad u = \theta_{reset} \quad then \quad reset\, to\; u = u_r$$

add

- √ -Adaptation variables
- √ -Possibility for firing patterns
- √ -Dynamic threshold ϑ
- -Noise

Neuronal Dynamics – Quiz 9.2. Nullclines for constant input

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) + w + RI(t)$$

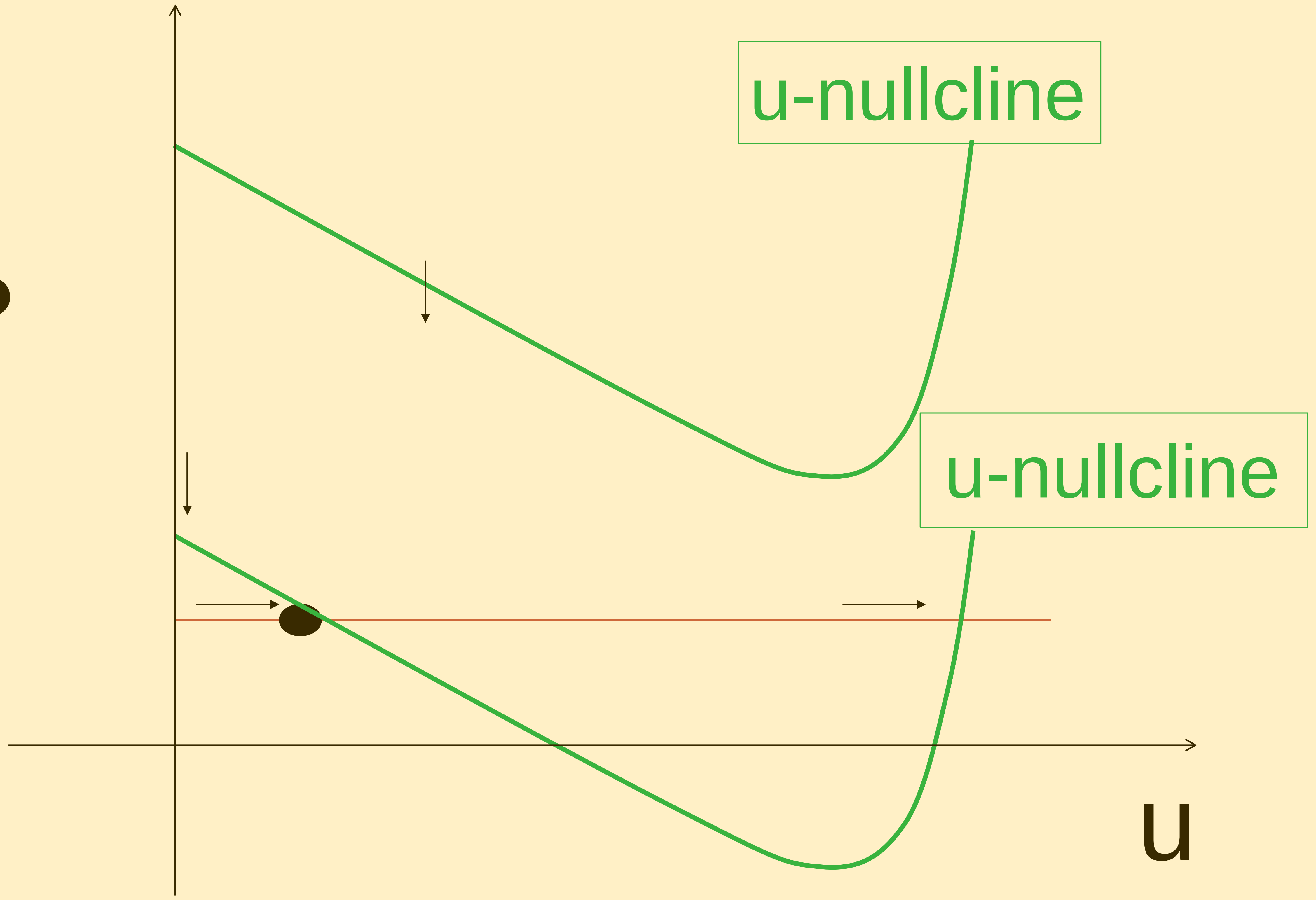
$$\tau_w \frac{dw}{dt} = a\,(u - u_{rest}) - w + b\,\tau_w \sum_f \delta(t - t^f)$$

Only during reset

a=0

What happens if input switches from I=0 to I>0?

[ ] u-nullcline moves horizontally
[ ] u-nullcline moves vertically
[ ] w-nullcine moves horizontally
[ ] w-nullcline moves vertically



Week 9 – part 3: Spike Response Model (SRM)

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

√ 9.1 What is a good neuron model?
- Models and data

√ 9.2 AdEx model
- Firing patterns and adaptation

9.3 Spike Response Model (SRM)
- Integral formulation

9.4 Generalized Linear Model
- Adding noise to the SRM

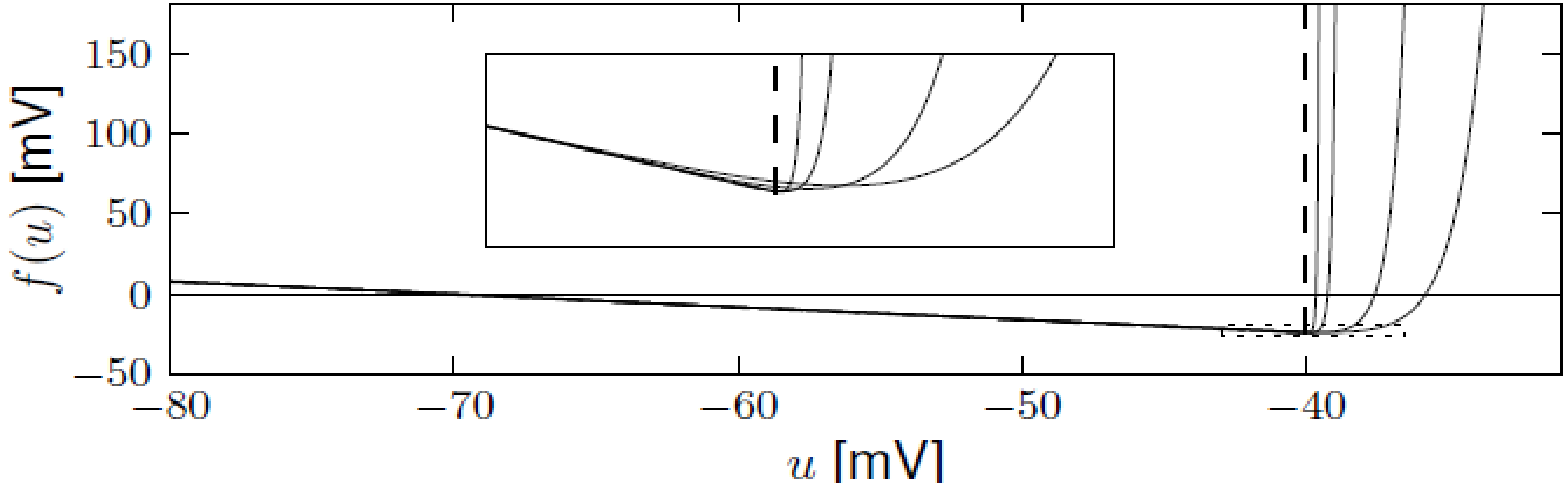
9.5 Parameter Estimation
- Quadratic and convex optimization

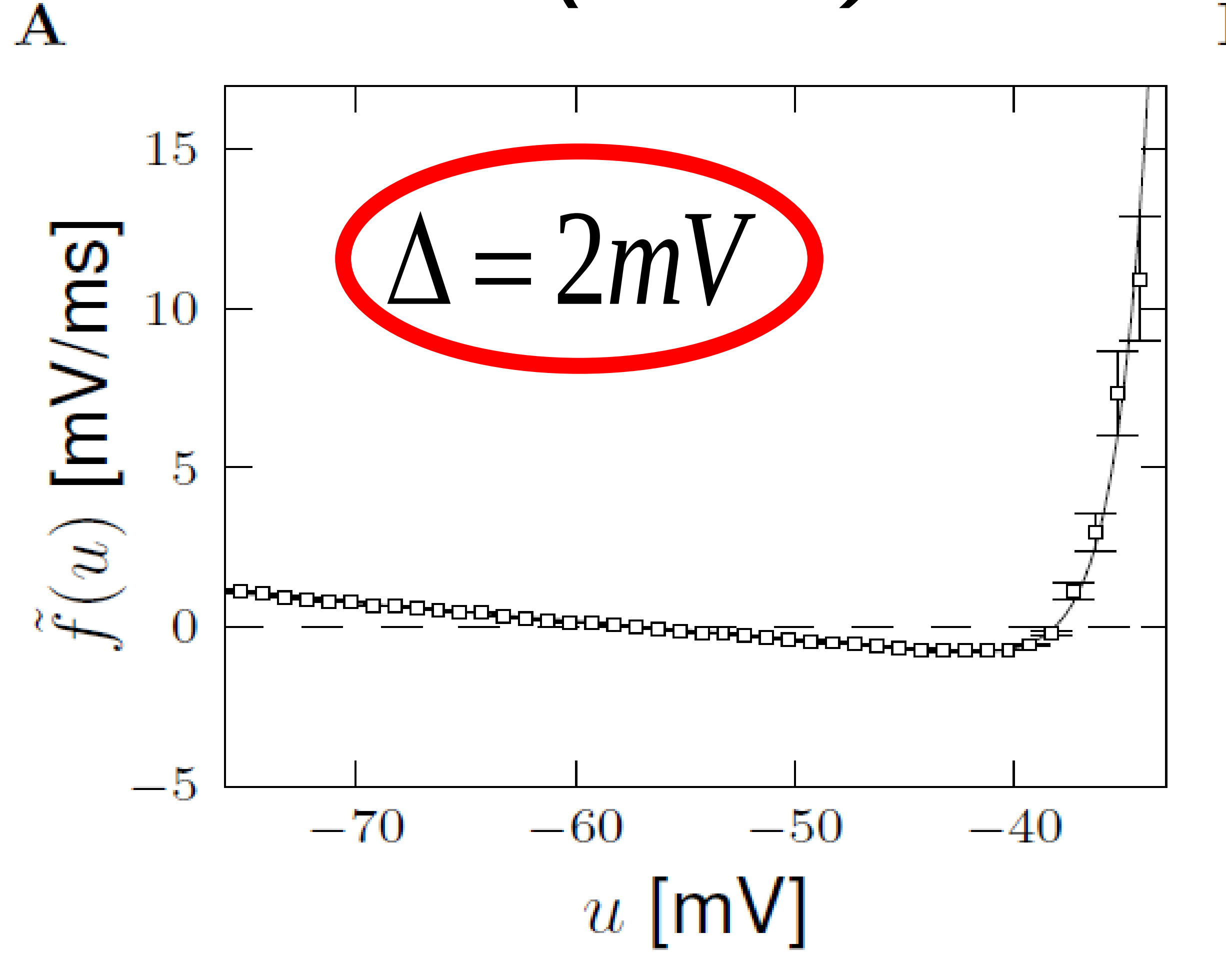
9.6. Modeling in vitro data
- how long lasts the effect of a spike?

Exponential versus Leaky Integrate-and-Fire

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) + RI(t)$$

Badel et al (2008)





$$\tau \frac{du}{dt} = -(u - u_{rest}) + RI(t)$$

Reset if u= ϑ

Leaky Integrate-and-Fire

$$\tau \frac{du}{dt} = -(u - u_{rest}) - R\sum_k w_k + RI(t)$$

$$\tau_k \frac{dw_k}{dt} = a_k(u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)$$

SPIKE AND RESET

after each spike

w_k jumps by an amount b_k

$If \quad u = \vartheta(t) \quad then \quad reset\, to\; u = u_r$

Dynamic threshold

$$\tau \frac{du}{dt} = -(u - u_{rest}) - R\sum_k w_k + RI(t)$$

$$\tau_k \frac{dw_k}{dt} = a_k(u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)$$

Adaptive leaky I&F

Linear equation → can be integrated!

$$u(t) = \sum_f \eta\,(t - t^f) + \int_0^\infty ds\, \kappa(s) I(t - s)$$

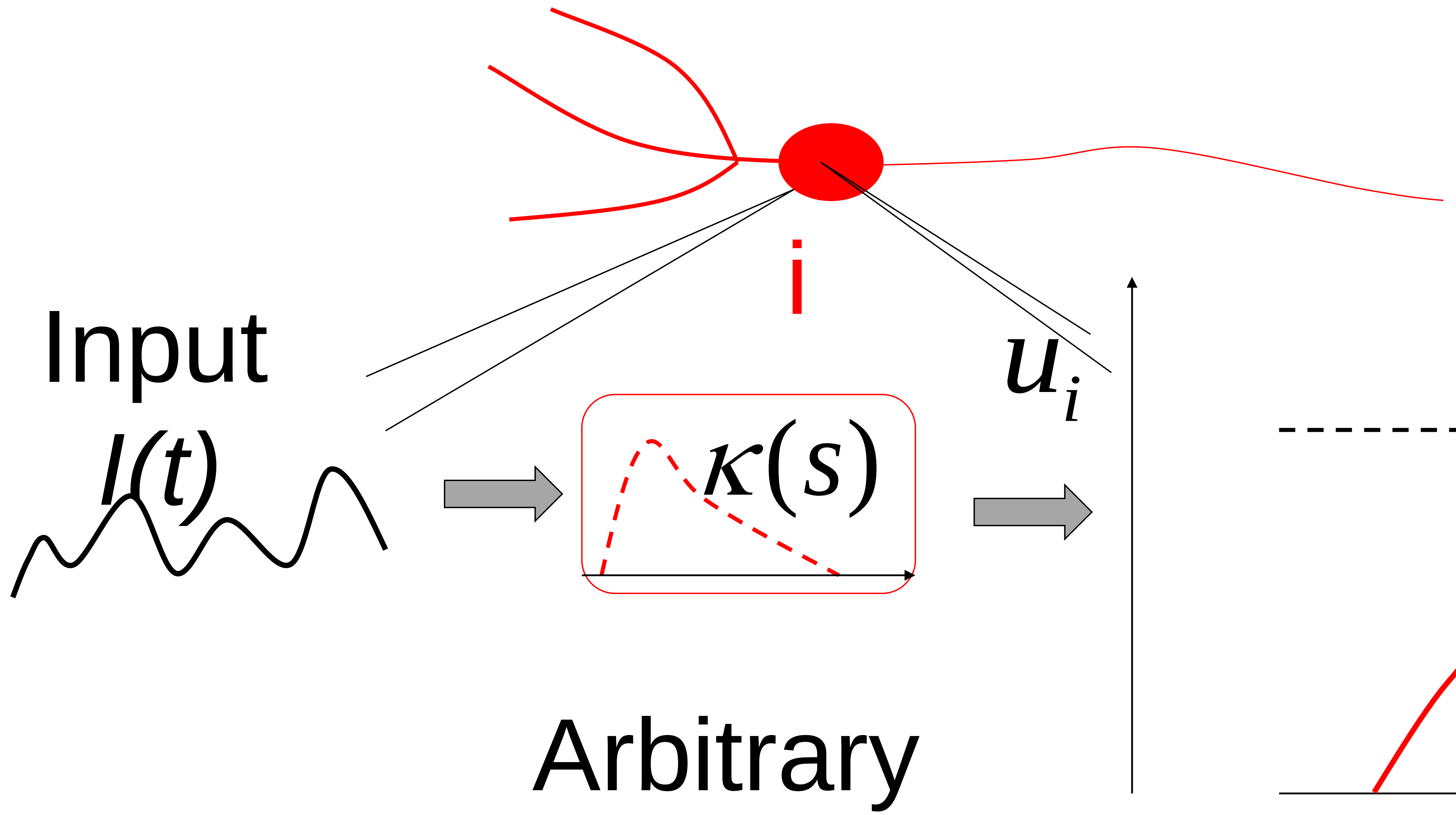
$$\vartheta(t) = \theta_0 + \sum_f \theta_1(t - t^f)$$

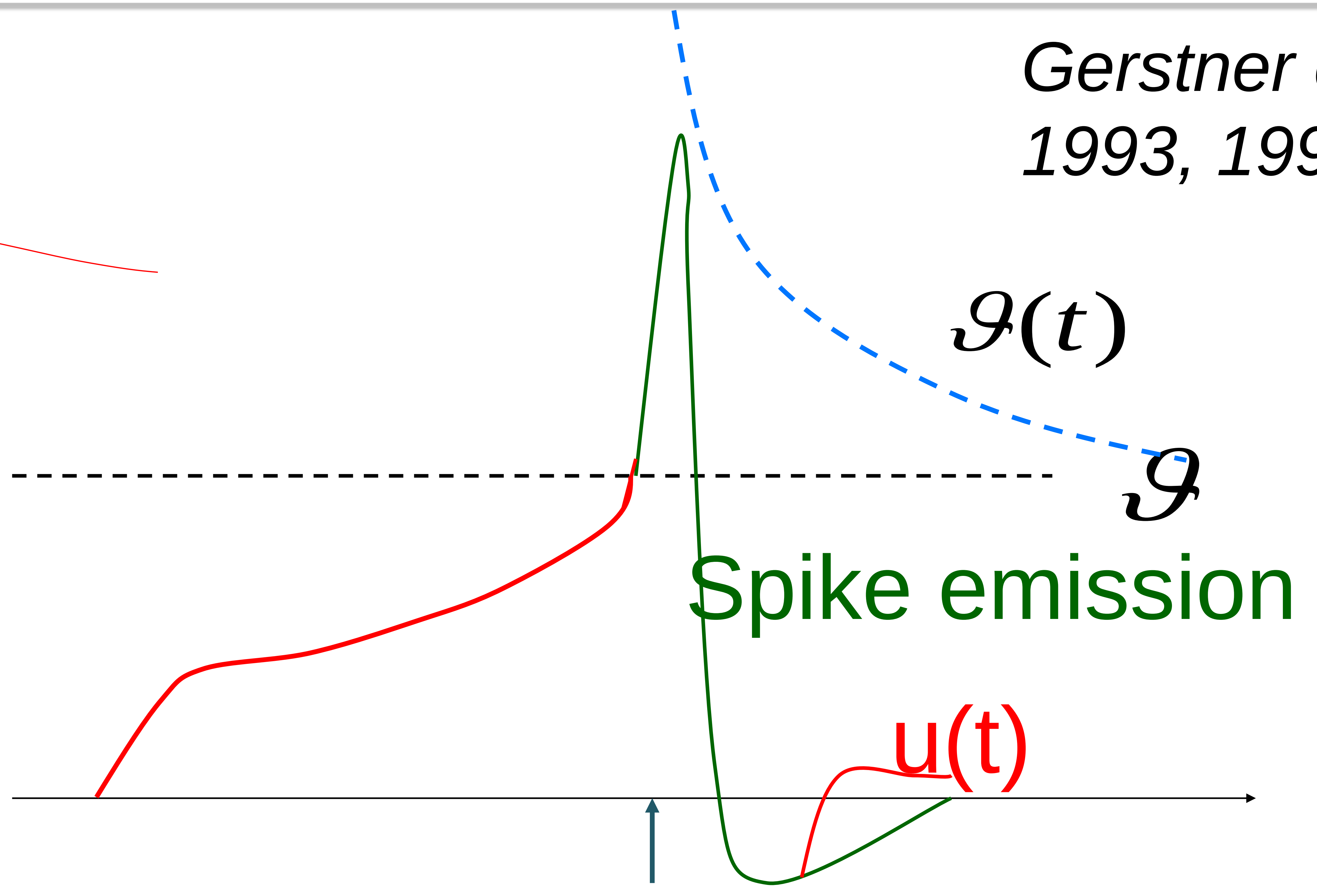
Spike Response Model (SRM)

Gerstner et al. (1996)

Neuronal Dynamics – 9.3 Spike Response Model (SRM)

Gerstner et al., 1993, 1996





Arbitrary
Linear filters

potential

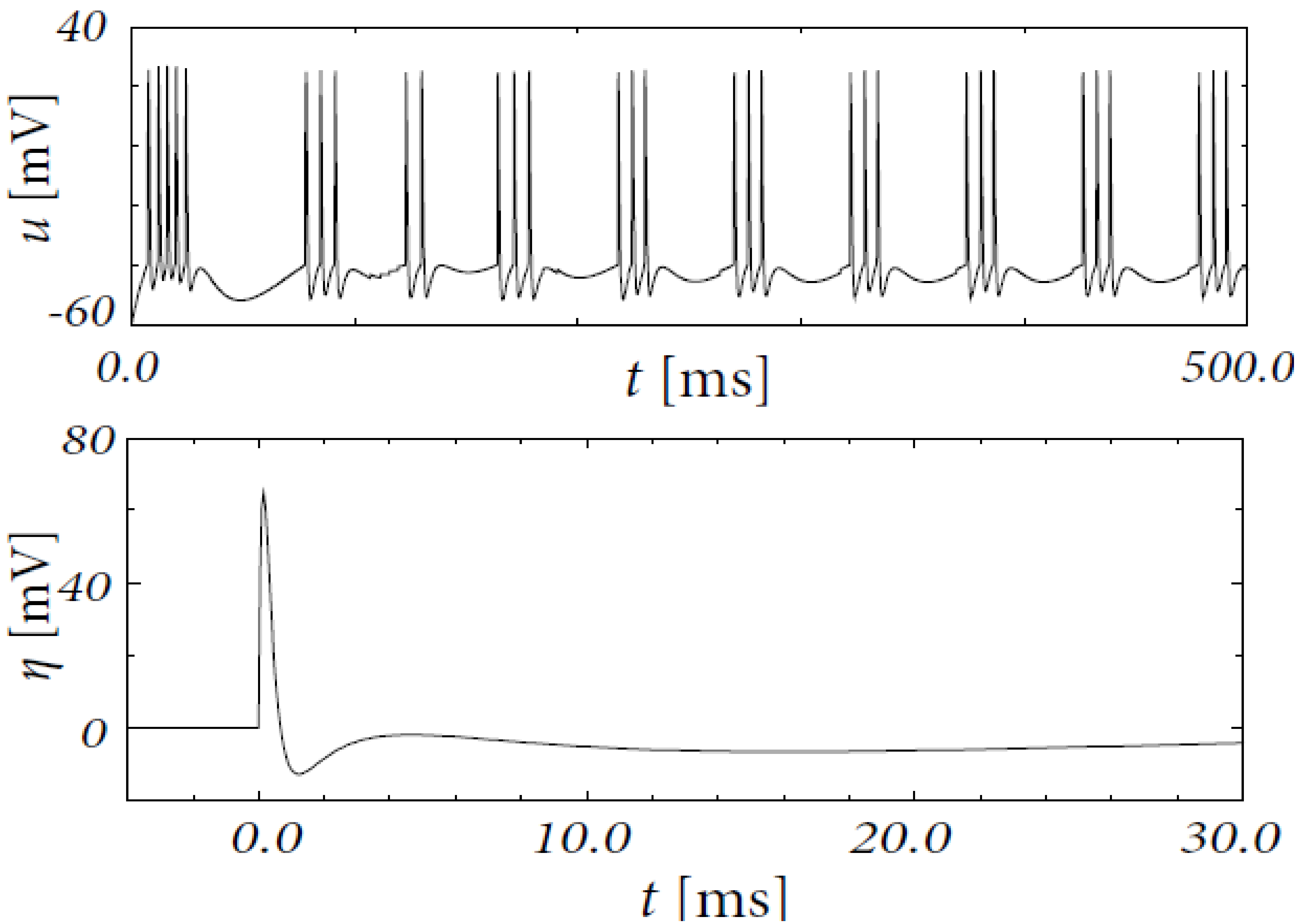
$$u(t) = \sum_{t'} \eta(t - t') + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$$

threshold

$$\vartheta(t) = \theta_0 + \sum_{t'} \theta_1(t - t')$$

Neuronal Dynamics – 9.3 Bursting in the SRM

SRM with appropriate η leads to bursting



$$u(t) = \sum_f \eta\,(t - t^f) + \int_0^\infty ds\, \kappa(s) I(t-s) + u_{rest}$$

$$u(t) = \int_0^\infty ds\, \eta(s)\, S(t-s) + \int_0^\infty ds\, \kappa(s) I(t-s) + u_{rest}$$

Quiz 9.3: from adaptive IF to SRM

$$\tau \frac{du}{dt} = -(u - u_{rest}) - w + RI(t)$$

$$\tau_w \frac{dw}{dt} = -w + b\,\tau_w \sum_f \delta(t - t^f)$$

$$If \quad u = \vartheta \; then \quad reset\, to \; u = u_r$$

Next lecture at 9:57

Integrate the above system of two differential equations so as to rewrite the equations as

potential $$u(t) = \int_0^\infty \eta(s) S(t-s)ds + \int_0^\infty \varepsilon(s) I(t-s)ds + u_{rest}$$

A – what is $\eta(s)$ **?**

B – what is $\varepsilon(s)$**?**

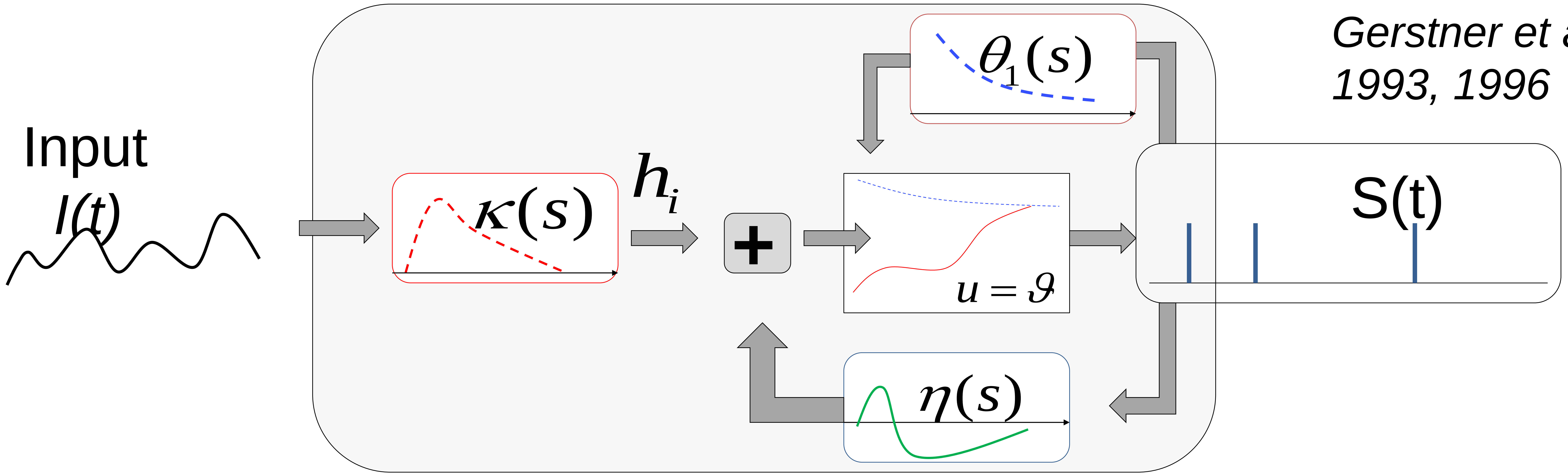
(i) $x(s) = \frac{R}{\tau}\exp(-\frac{s}{\tau})$

(ii) $x(s) = \frac{R}{\tau_w}\exp(-\frac{s}{\tau_w})$

(iii) $x(s) = C[\exp(-\frac{s}{\tau}) - \exp(-\frac{s}{\tau_w})]$

(iv) Combi of (i) + (iii)

Neuronal Dynamics – 9.3 Spike Response Model (SRM)



Gerstner et al., 1993, 1996

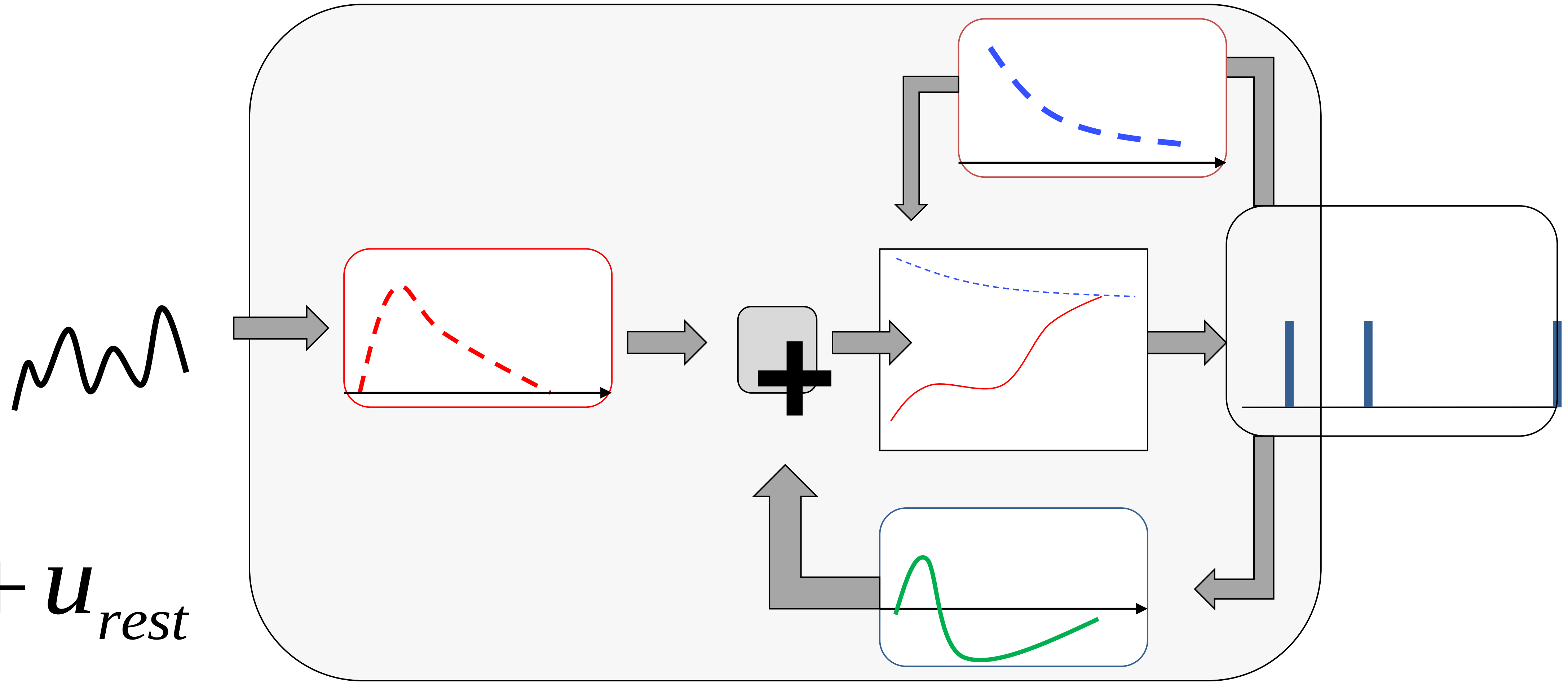
potential $$u(t)=\sum_{t'}\eta(t-t')+\int_0^\infty \kappa(s)I(t-s)ds+u_{rest}$$

threshold $$\vartheta(t)=\theta_0+\sum_{t'}\theta_1(t-t')$$

firing if $$u(t)=\vartheta(t)$$

potential

$$u(t)=\sum_{t'}\eta(t-t')+\int_0^\infty \kappa(s)I(t-s)ds+u_{rest}$$



threshold

$$\vartheta(t)=\theta_0+\sum_{t'}\theta_1(t-t')$$

Linear filters for

- input
- threshold
- refractoriness

Week 9 – part 4: Generalized Linear Model (GLM)

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

√ **9.1 What is a good neuron model?**
- Models and data

√ **9.2 AdEx model**
- Firing patterns and adaptation

√ **9.3 Spike Response Model (SRM)**
- Integral formulation

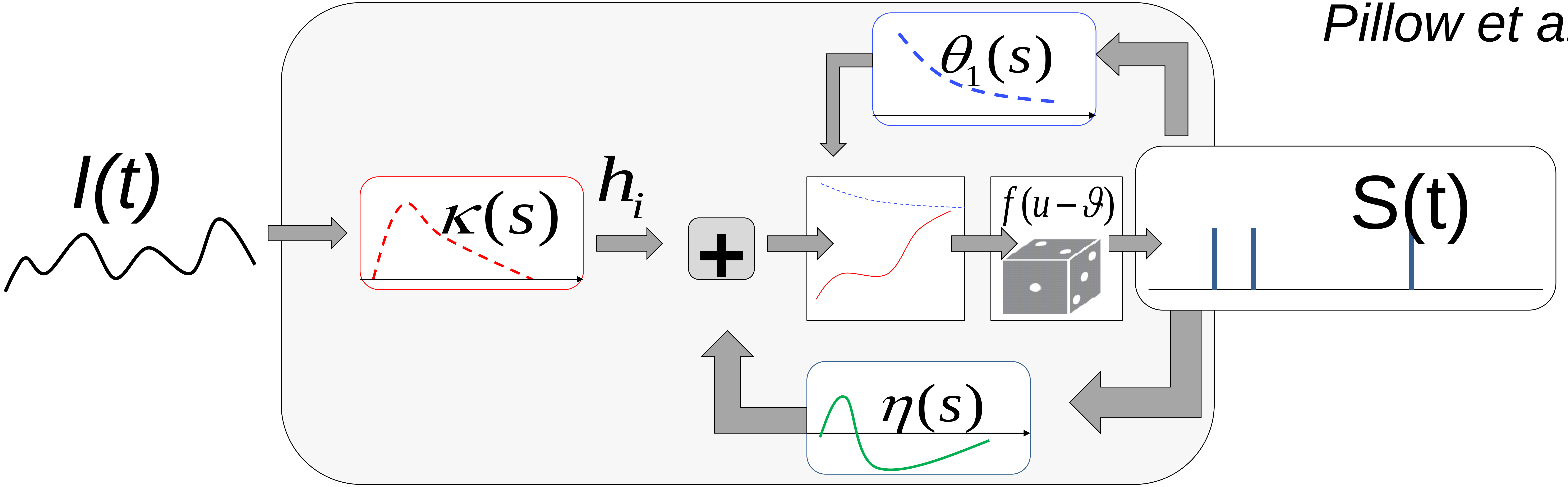
9.4 Generalized Linear Model
- Adding noise to the SRM

9.5 Parameter Estimation
- Quadratic and convex optimization

9.6. Modeling in vitro data
- how long lasts the effect of a spike?

Spike Response Model (SRM)
Generalized Linear Model GLM

Gerstner et al., 1992,2000
Truccolo et al., 2005
Pillow et al. 2008



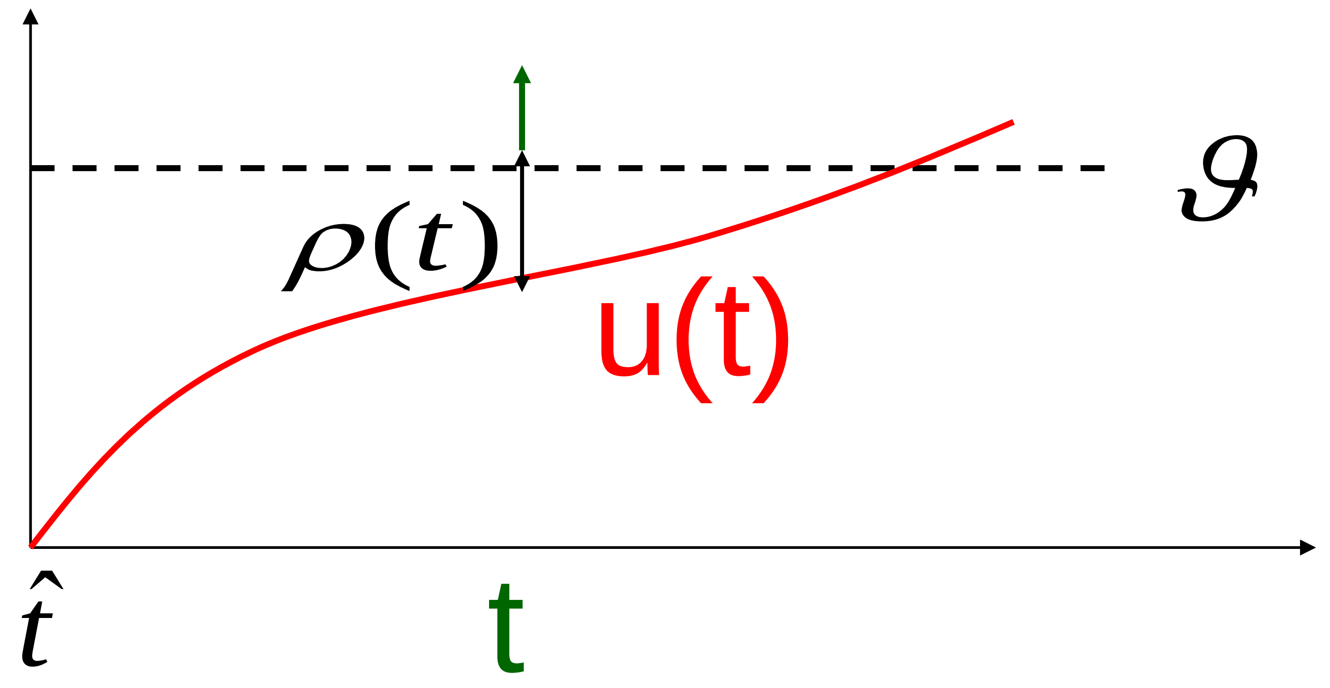
potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

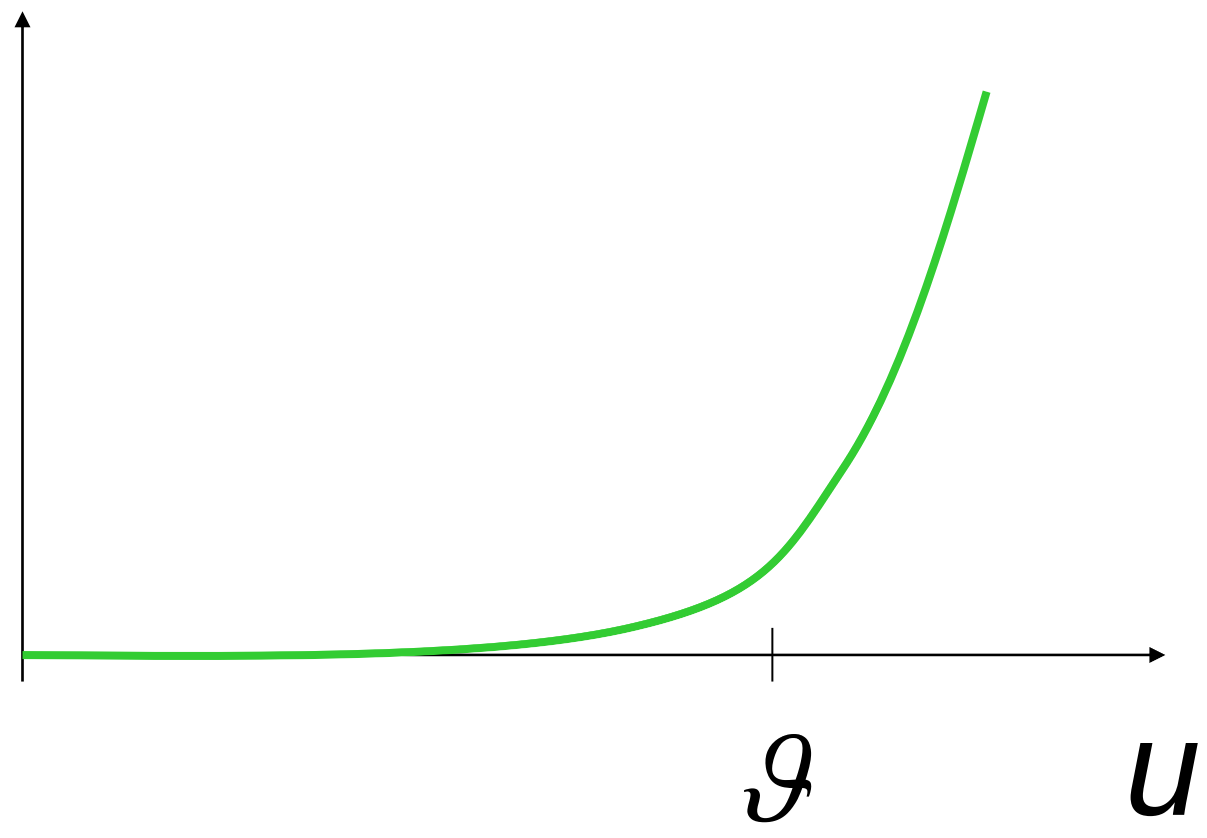
Neuronal Dynamics – review from week 8: Escape noise

escape process



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$



escape rate

$$\rho(t) = \frac{1}{\Delta} \exp\left(\frac{u(t) - \vartheta}{\Delta}\right)$$

Example: leaky integrate-and-fire model

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

$$\text{if spike at } t^f \Rightarrow u\left(t^f + \delta\right) = u_r$$

Exerc. 1: leaky and non-leaky IF with escape rates

$$\tau \frac{du}{dt} = -(u - u_{rest}) + RI(t)$$

$$reset\ to\ u_{rest} = u_r = 0$$

$$\frac{du}{dt} = \frac{R}{\tau} I(t) = \frac{1}{C} I(t)$$

$$reset\ to\ u_r = 0$$

Integrate for constant input (repetitive firing)

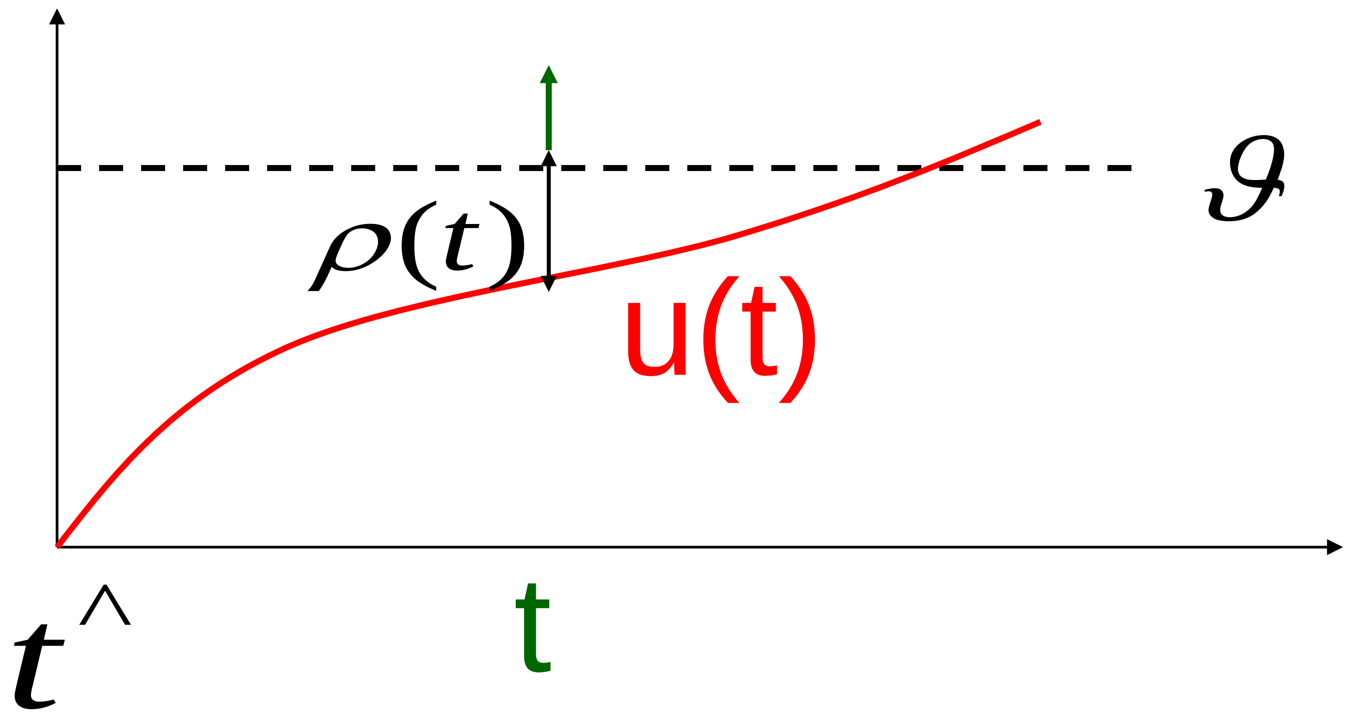
Next lecture at 10:40

Calculate

- potential $u(t-\hat{t})$
- hazard $\rho(t-\hat{t}) = \beta \cdot [u(t-\hat{t}) - \vartheta]_+$
- survivor function $S(t-\hat{t})$
- interval distrib. $P_0(t-\hat{t})$

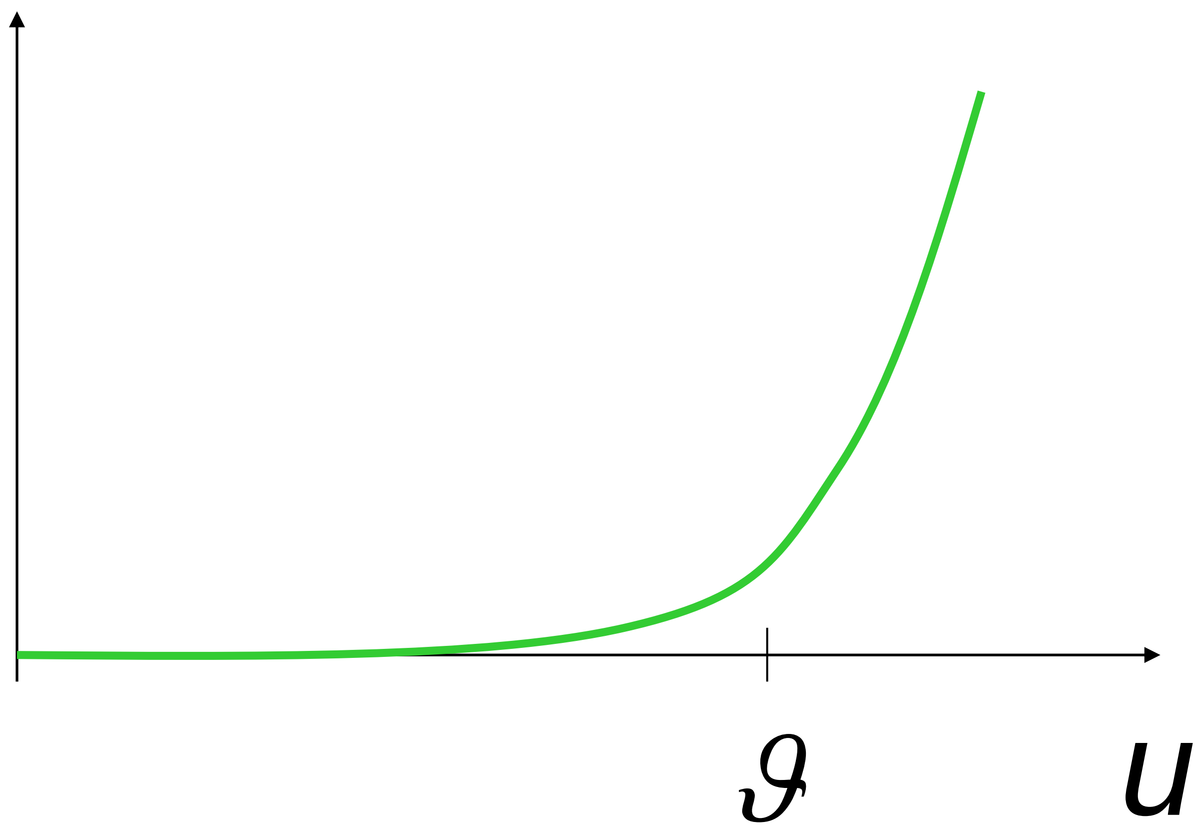
Neuronal Dynamics – review from week 8: Escape noise

escape process



escape rate

$$\rho(t) = f(u(t) - \vartheta(t))$$



Survivor function

$$\frac{d}{dt} S_I(t|\hat{t}) = -\rho(t)\, S_I(t|\hat{t})$$

$$S_I(t|\hat{t}) = \exp\left(-\int_{t^\wedge}^{t} \rho(t')dt'\right)$$

Interval distribution

$$P_I(t|\hat{t}) = \rho(t) \cdot \exp\left(-\int_{t^\wedge}^{t} \rho(t')dt'\right)$$

escape rate

Survivor function

Good choice

$$\rho(t) = f(u(t) - \vartheta(t)) = \rho_0 \exp\left[\frac{u(t) - \vartheta(t)}{\Delta u}\right]$$

Neuronal Dynamics – 9.4 Likelihood of a spike train in GLMs

$$S(t) = \sum_f \delta(t - t^f)$$

→Blackboard

$t^1, t^2, \ldots t^N$

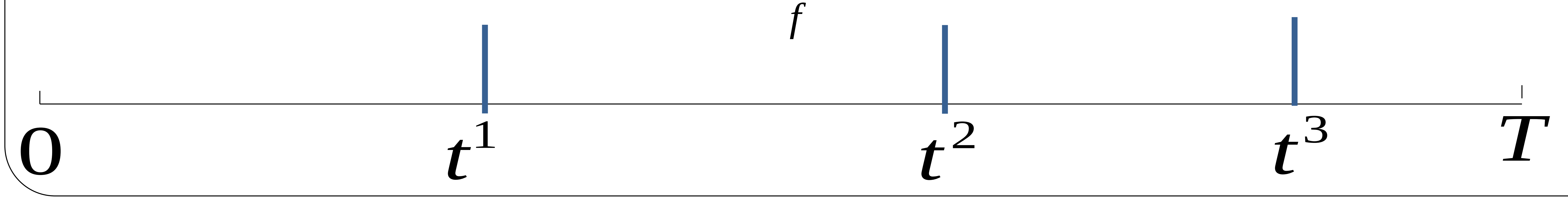
Measured spike train with spike times

Likelihood L that this spike train
could have been generated by model?

$$L(t^1, \ldots, t^N) = \exp\left(-\int_0^{t^1} \rho(t')dt'\right)\rho(t^1) \cdot \exp\left(-\int_{t^1}^{t^2} \rho(t')dt'\right) \ldots$$

Neuronal Dynamics – 9.4 Likelihood of a spike train

$$S(t) = \sum_f \delta(t - t^f)$$

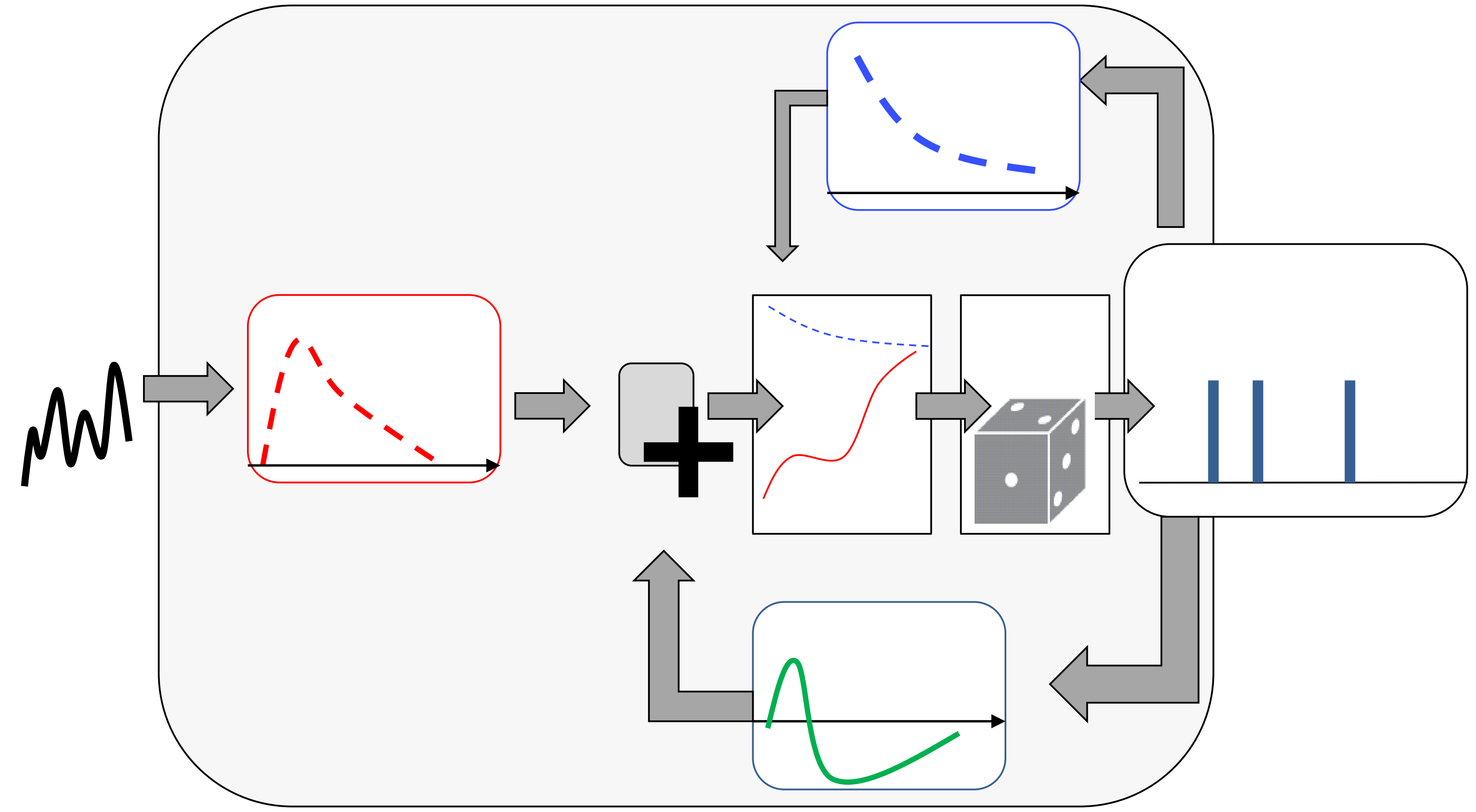


$$L(t^1, \ldots, t^N) = \exp\left(-\int_0^{t^1} \rho(t')dt'\right)\rho(t^1) \cdot \exp\left(-\int_{t^1}^{t^2} \rho(t')dt'\right)\rho(t^2) \ldots \cdot \exp\left(-\int_{t^N}^{T} \rho(t')dt'\right)$$

$$L(t^1, \ldots, t^N) = \exp\left(-\int_0^{T} \rho(t')dt'\right)\prod_f \rho(t^f)$$

$$\log L(t^1, \ldots, t^N) = -\int_0^{T} \rho(t')dt' + \sum_f \log \rho(t^f)$$

Neuronal Dynamics – 9.4 SRM with escape noise = GLM



-linear filters
-escape rate
→likelihood of observed spike train

→parameter optimization of neuron model

Week 9 – part 5: Parameter Estimation

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

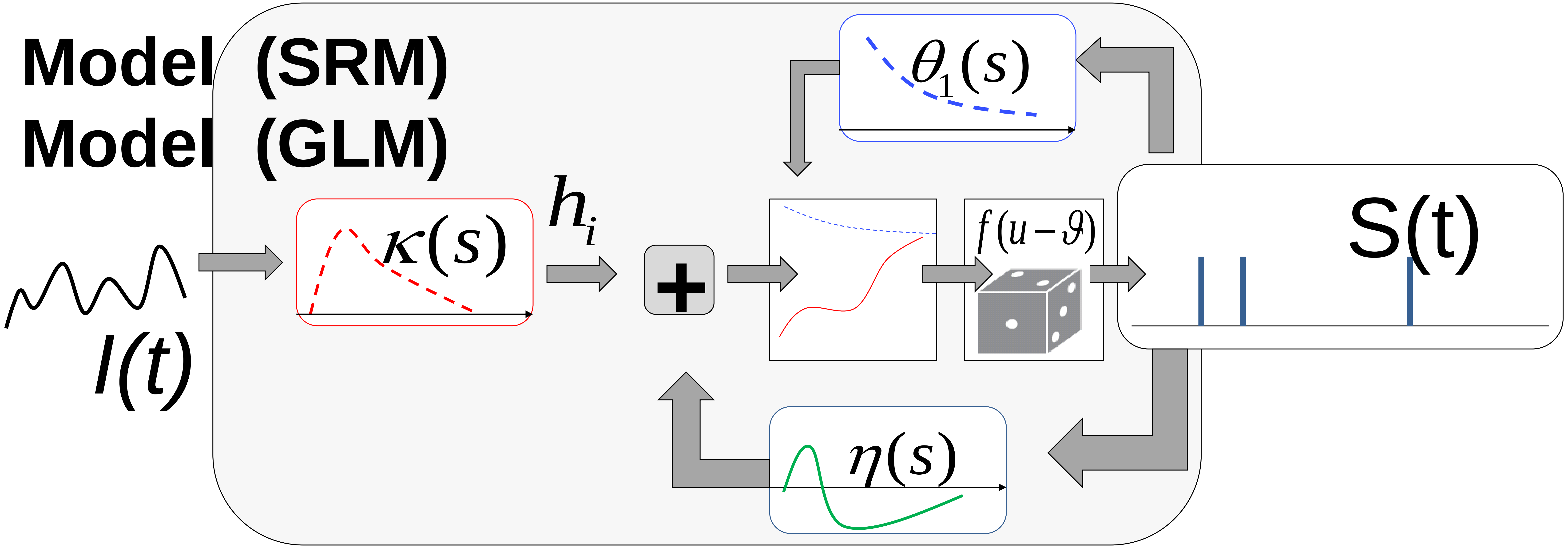
Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner
EPFL, Lausanne, Switzerland

- √ **9.1 What is a good neuron model?**
  - Models and data
- √ **9.2 AdEx model**
  - Firing patterns and adaptation
- √ **9.3 Spike Response Model (SRM)**
  - Integral formulation
- √ **9.4 Generalized Linear Model**
  - Adding noise to the SRM
- **9.5 Parameter Estimation**
  - Quadratic and convex optimization
- **9.6. Modeling in vitro data**
  - how long lasts the effect of a spike?

Neuronal Dynamics – 9.5 Parameter estimation: voltage

Spike Response Model (SRM)
Generalized Lin. Model (GLM)



Subthreshold potential

$$u(t) = \int \eta(s) S(t-s) ds + \int_0^{\infty} \kappa(s) I(t-s) ds + u_{rest}$$

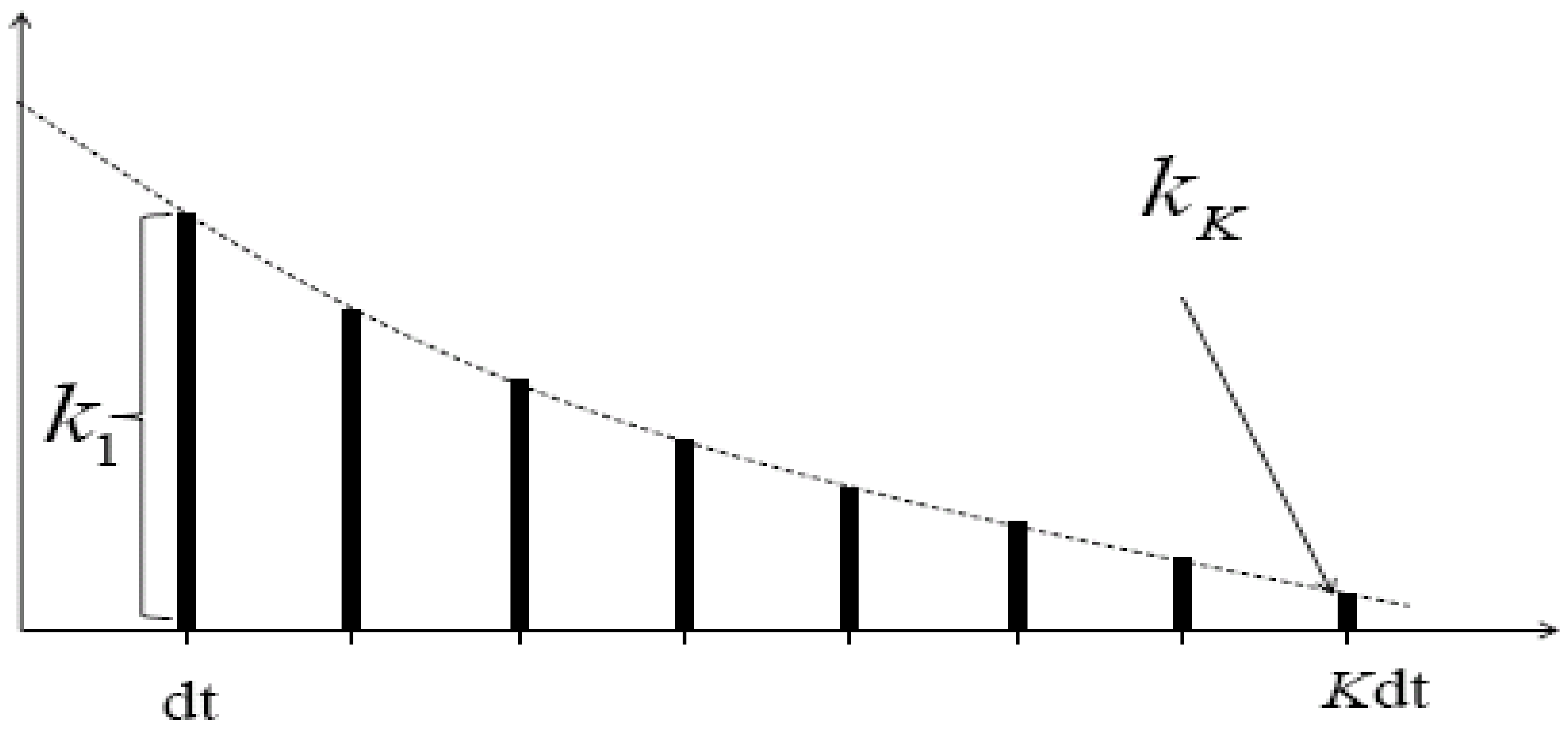
known spike train known input

Linear filters/linear in parameters

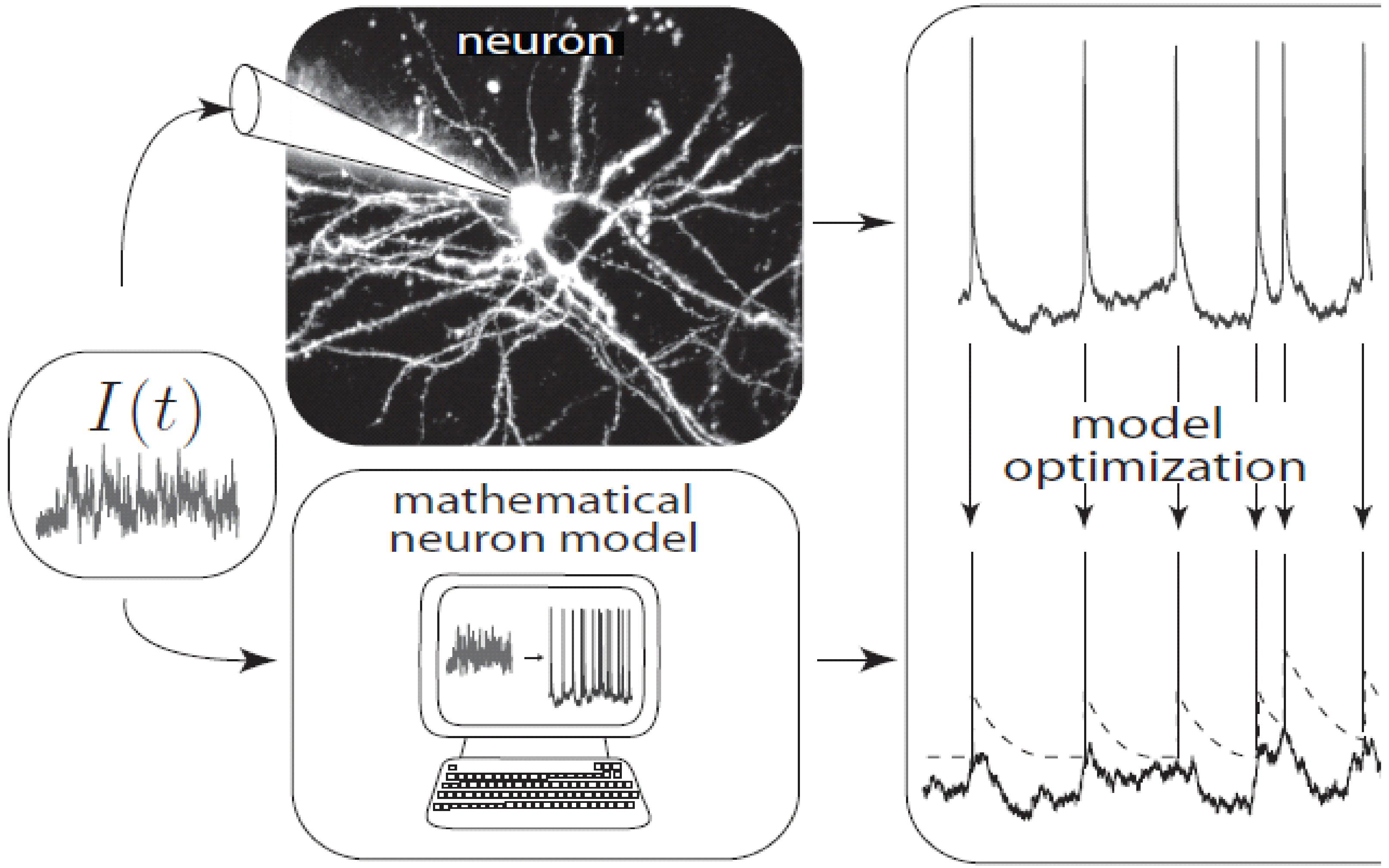
Linear in parameters = linear fit = quadratic problem

$$u(t) = \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$$

$$u(t_n) = \sum k_k I_{n-k} + u_{rest}$$



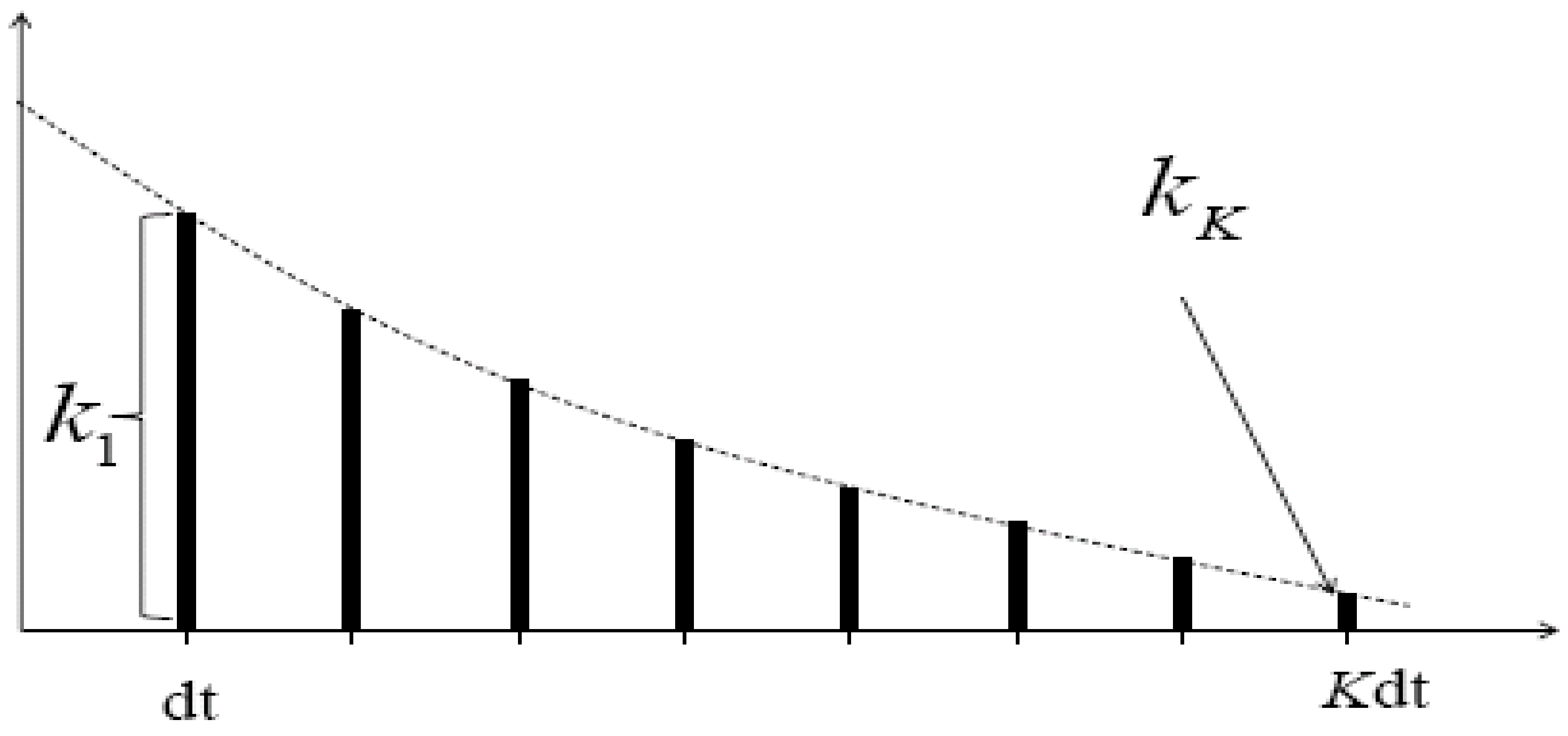
comparison model-data

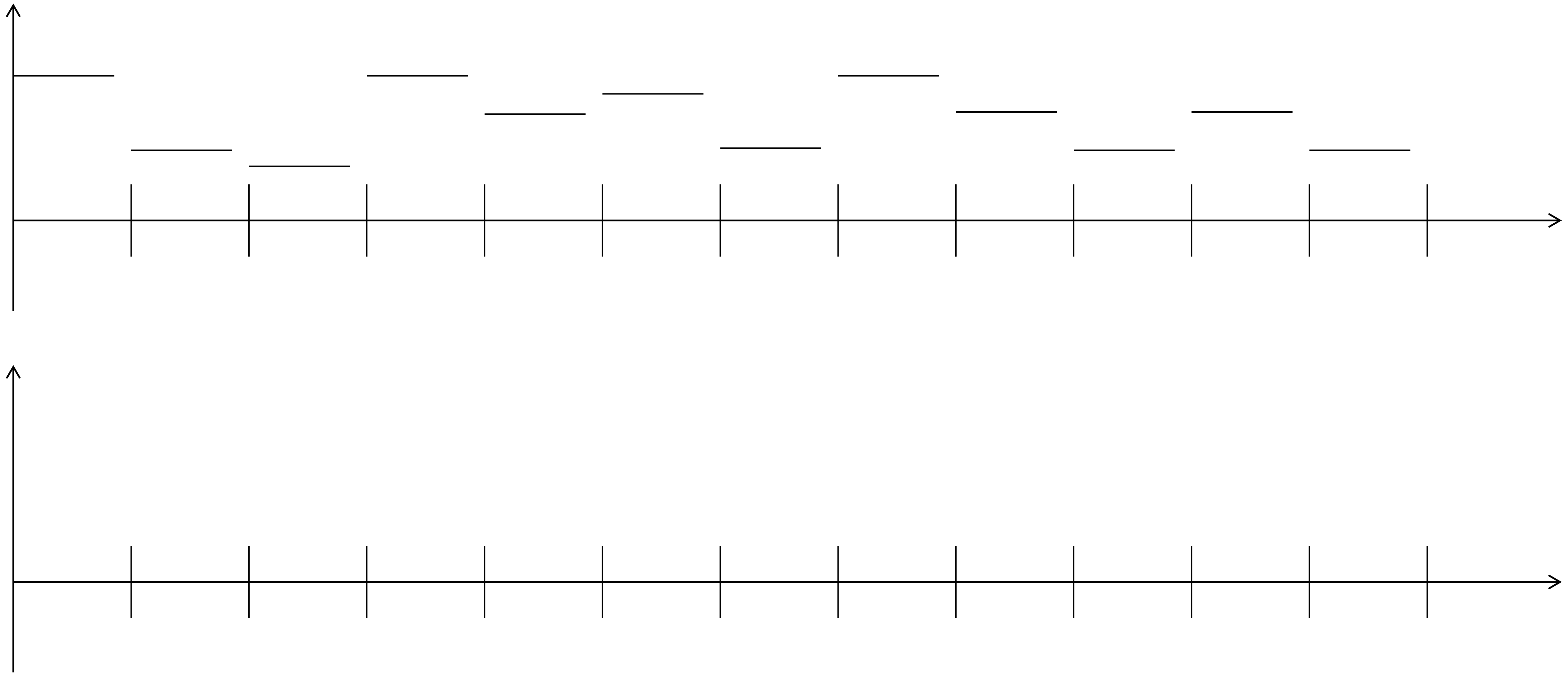


Linear in parameters = linear fit

$$u(t) = \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$$

$$u(t_n) = \sum k_k I_{n-k} + u_{rest}$$





$$E = \sum_n [u^{data}(t_n) - \sum_{k=1}^{K} k_k I_{n-k} - u_{rest}]^2$$

Neuronal Dynamics – 9.5 Parameter estimation: voltage

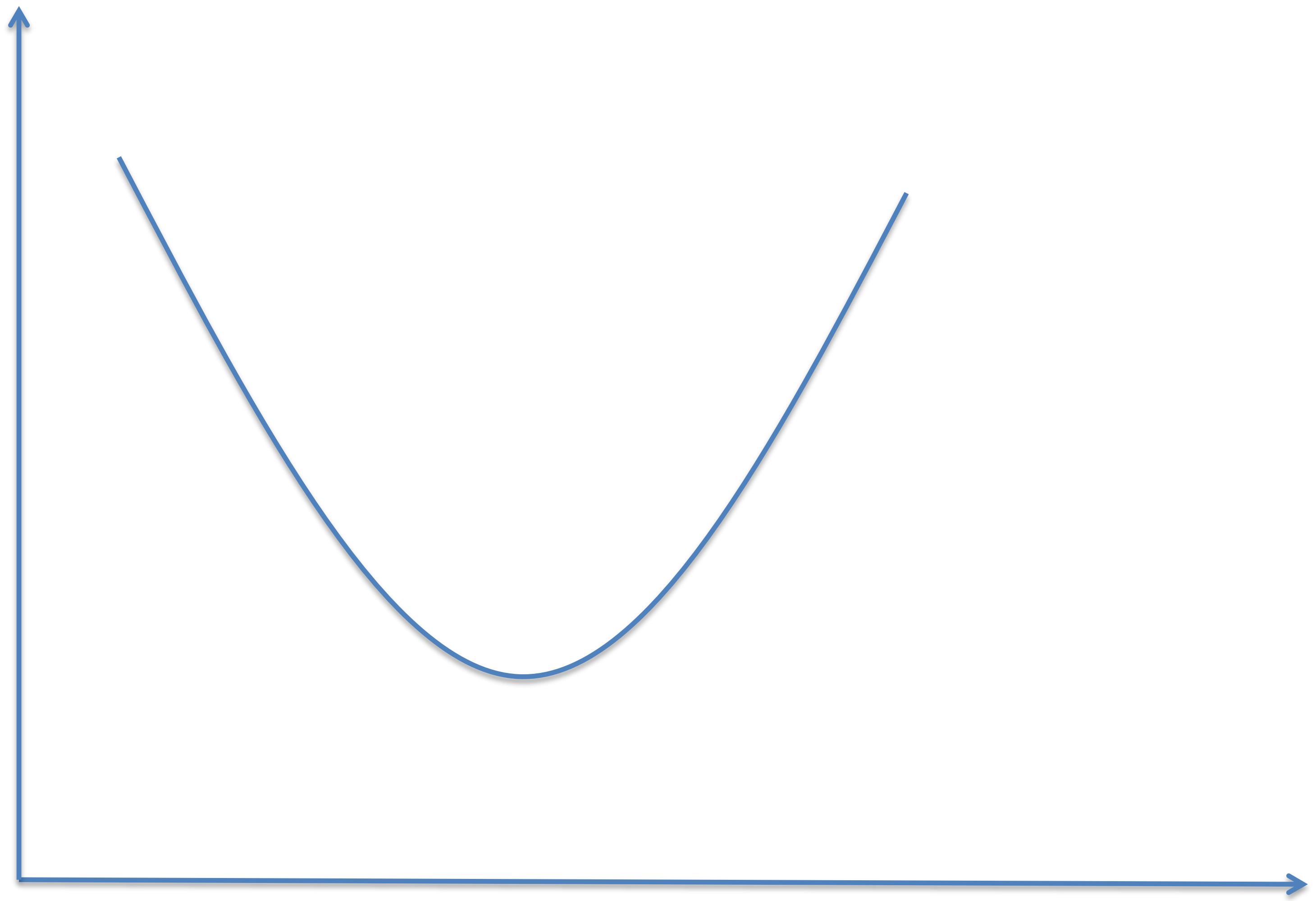
Linear in parameters = linear fit = quadratic optimization

Model

$$u(t) = \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$$

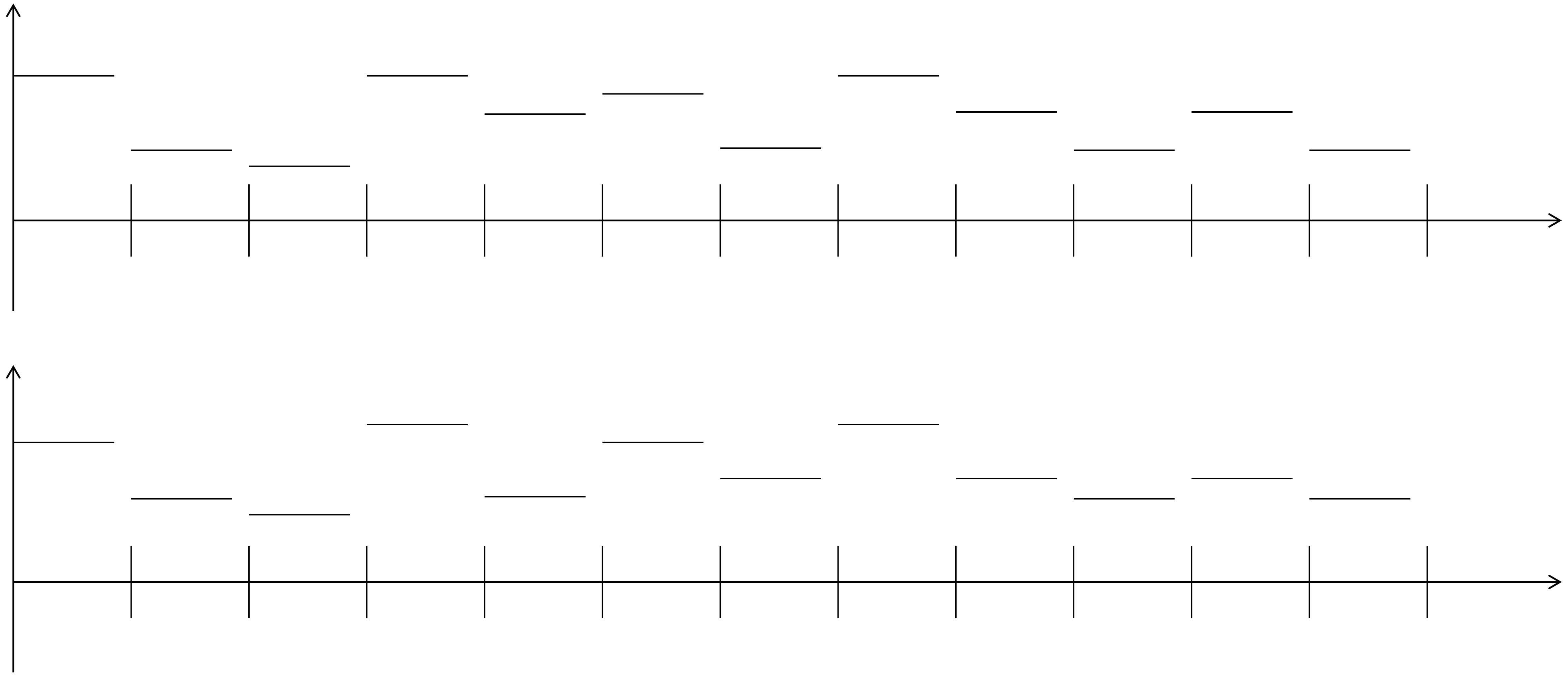
$$u(t_n) = \sum_k k_k \, I_{n-k} + u_{rest}$$

Data $\quad u^{data}(t)$



I

$u^{data}(t)$



$$E = \sum_n [u^{data}(t_n) - \sum_{k=1}^{K} k_k \, I_{n-k} - u_{rest}]^2$$

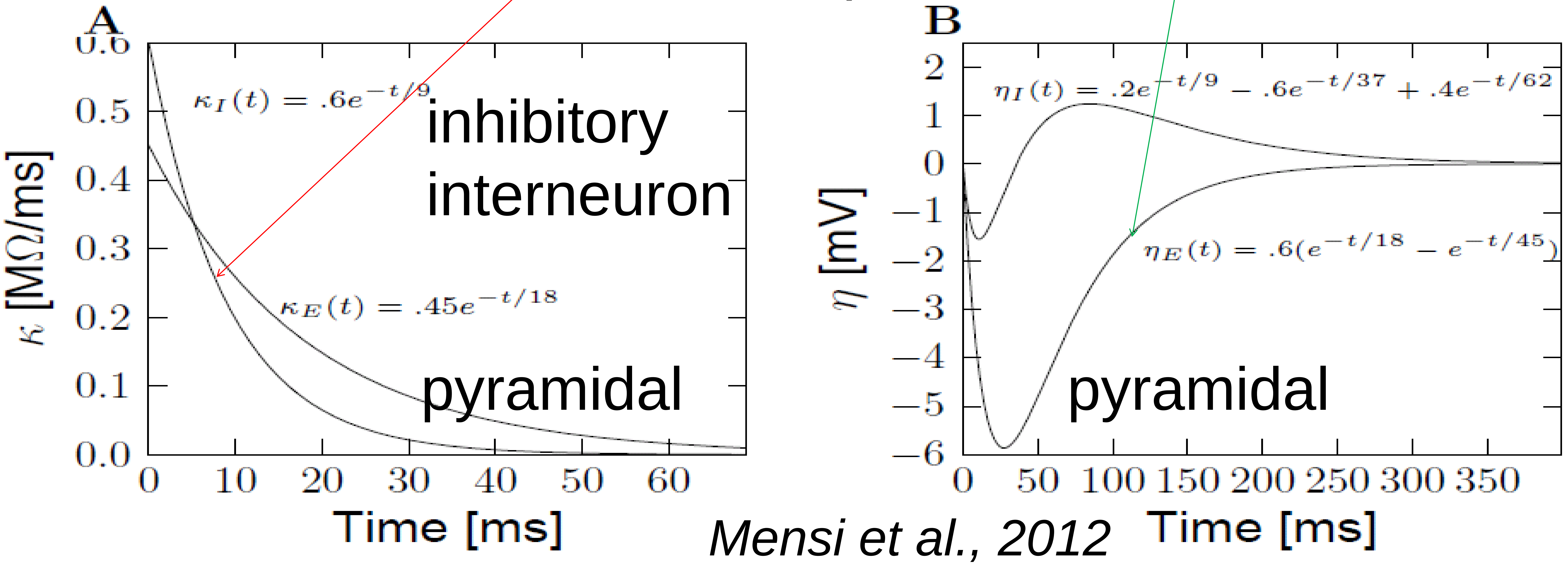
Neuronal Dynamics – 9.5 Extracted parameters: voltage

Subthreshold potential

$$u(t) = \int_0^\infty \kappa(s) I(t-s)ds + u_{rest} + \int \eta(s) S(t-s)ds$$

known input

known spike train



Mensi et al., 2012

Exercise 3 NOW: optimize 1 free parameter

Model

$$u_n = RI_n$$

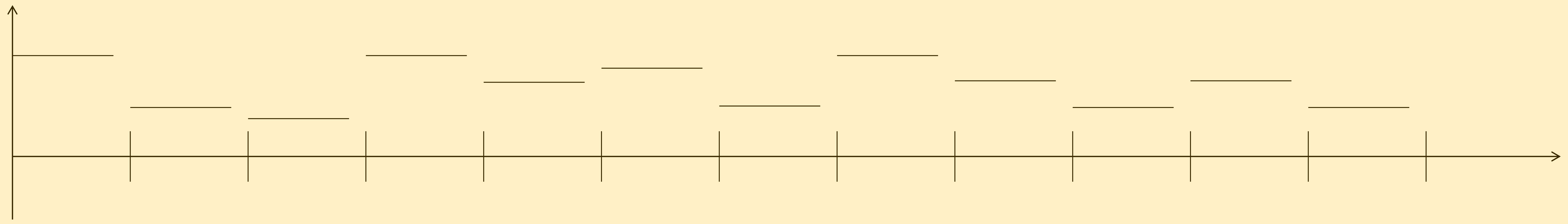
Data

$$u^{data}(t_n)$$

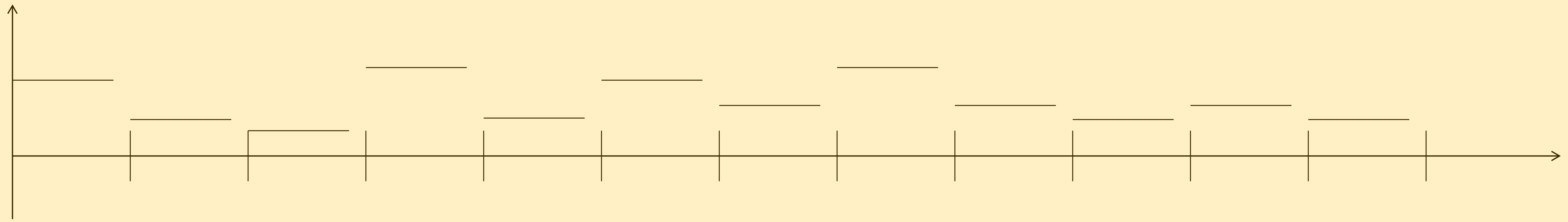
Optimize parameter R, so as to have a minimal error

$$E = \sum_n [u^{data}(t_n) - RI_n]^2$$

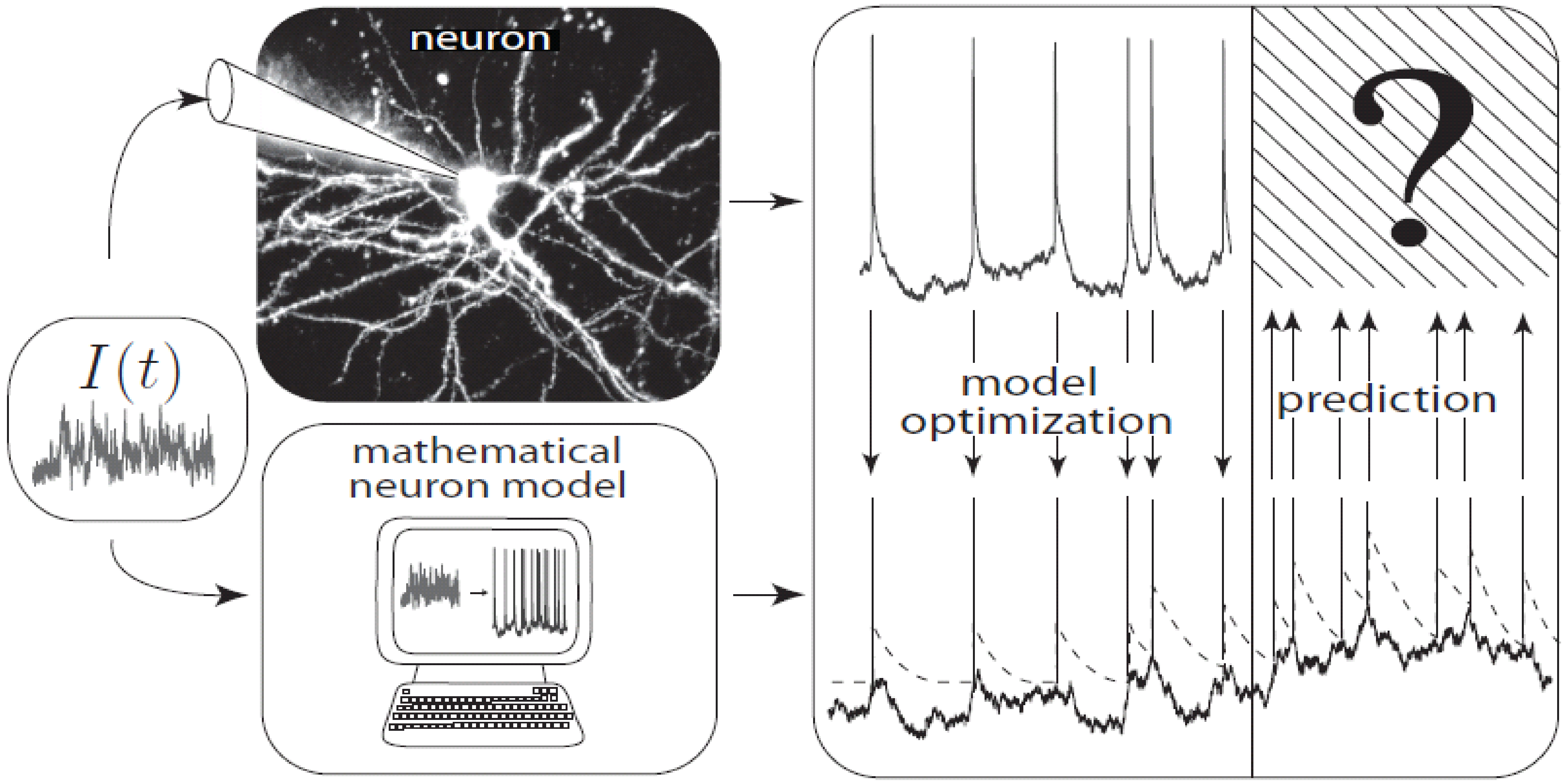
I



$u^{data}(t)$



Neuronal Dynamics – What is a good neuron model?



A) Predict spike times
B) Predict subthreshold voltage
C) Easy to interpret (not a 'black box')
D) Flexible
E) Systematic: 'optimize' parameters

Week 9 – part 5b: Quadratic and Convex Optimization

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

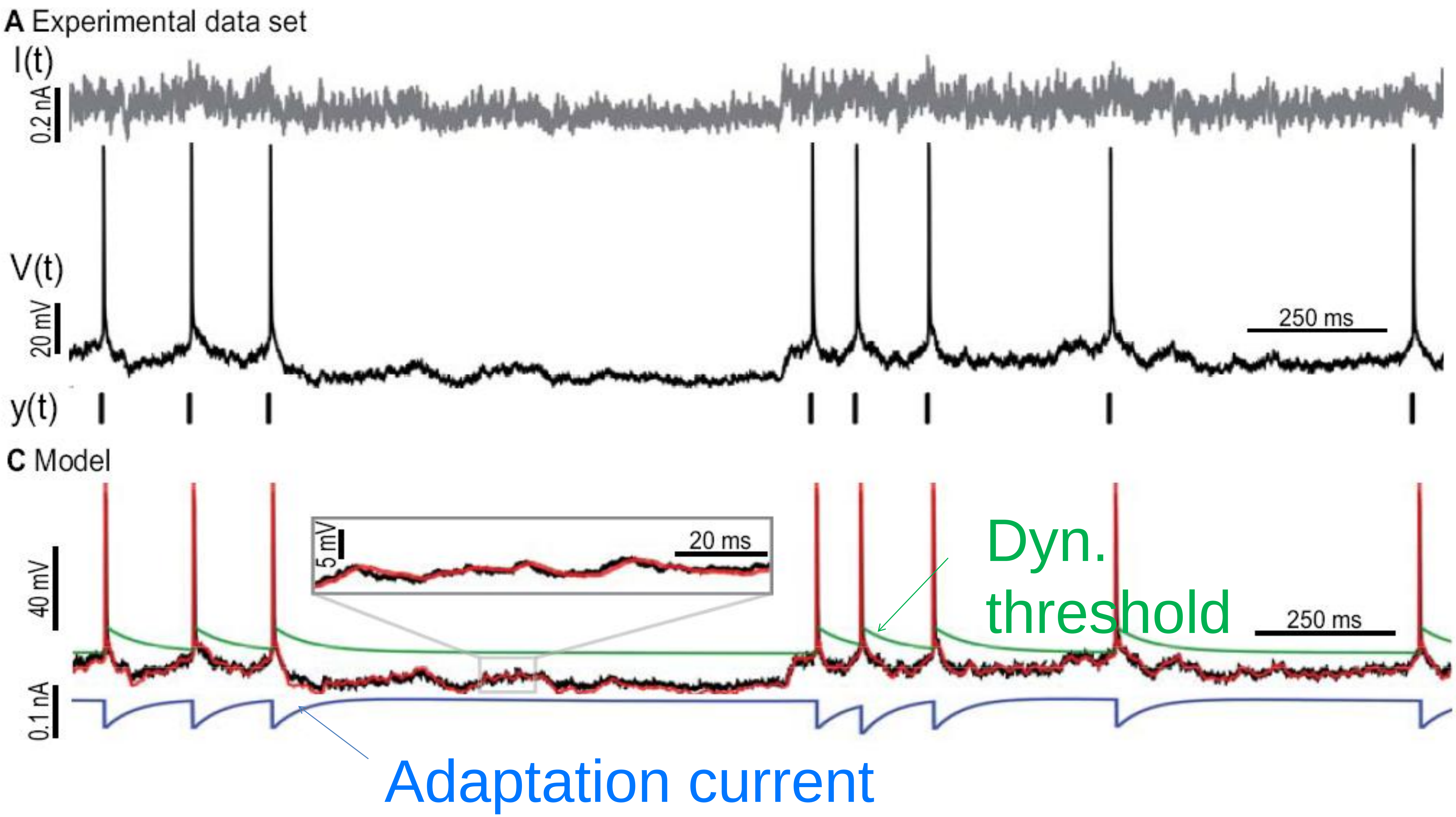
Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

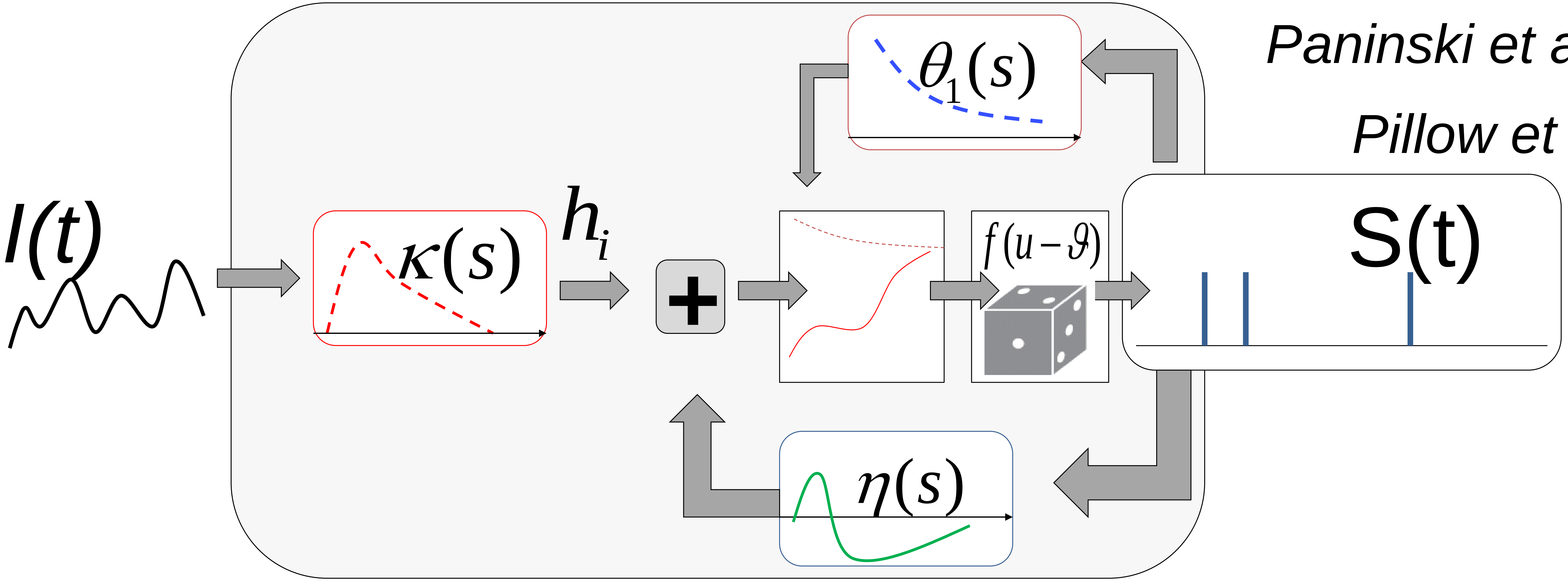
EPFL, Lausanne, Switzerland

Fitting models to data: so far ‘subthreshold’



Neuronal Dynamics – 9.5 Threshold: Predicting spike times

Jolivet&Gerstner, 2005
Paninski et al., 2004
Pillow et al. 2008



potential $u(t) = \int \eta(s) S(t-s)\,ds + \int_0^{\infty} \kappa(s) I(t-s)\,ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s)\,ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

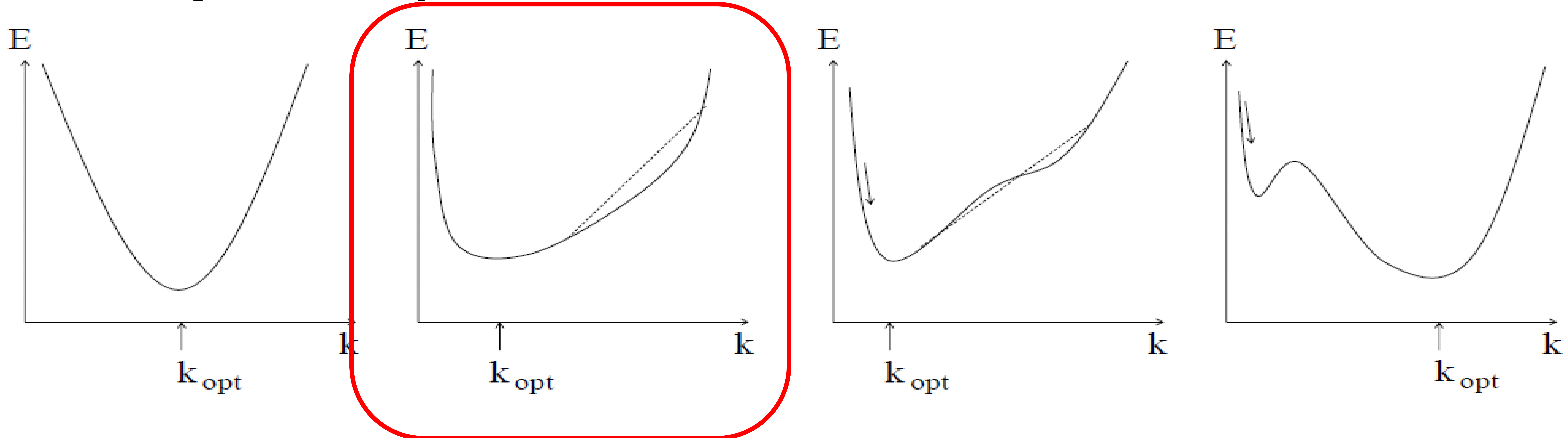
Neuronal Dynamics – 9.5 Generalized Linear Model (GLM)

$$\log L(t^1,\ldots,t^N) = -\int_0^T \rho(t')dt' + \sum_f \log\rho(t^f) = -E$$

potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$



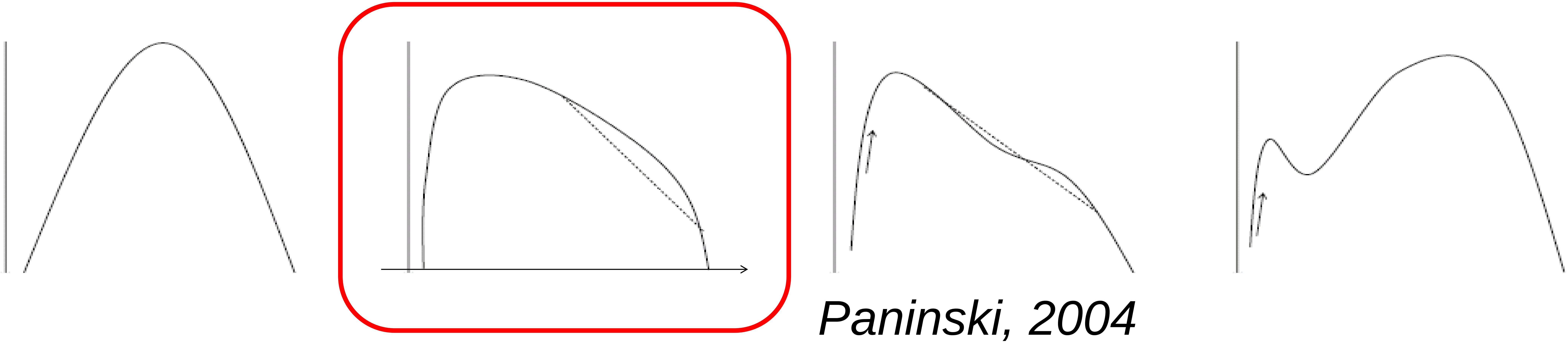
Neuronal Dynamics – 9.5 GLM: concave error function

potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

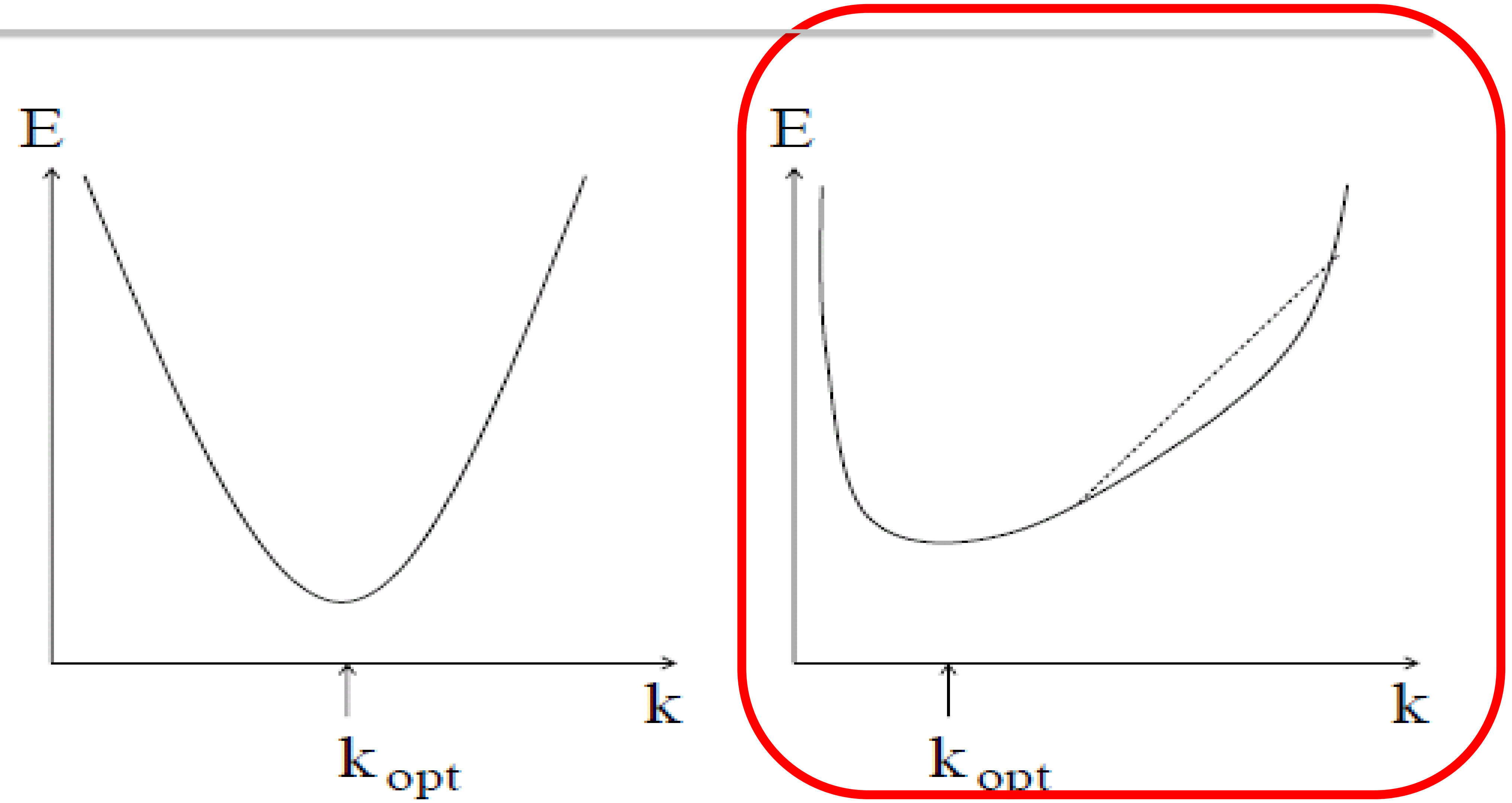
firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

$$\log L(t^1, \ldots, t^N) = -\int_0^T \rho(t') dt' + \sum_f \log \rho(t^f)$$



Paninski, 2004

Neuronal Dynamics – 9.5 quadratic and convex/concave optimization



Voltage/subthreshold

- linear in parameters

  → quadratic error function

Spike times

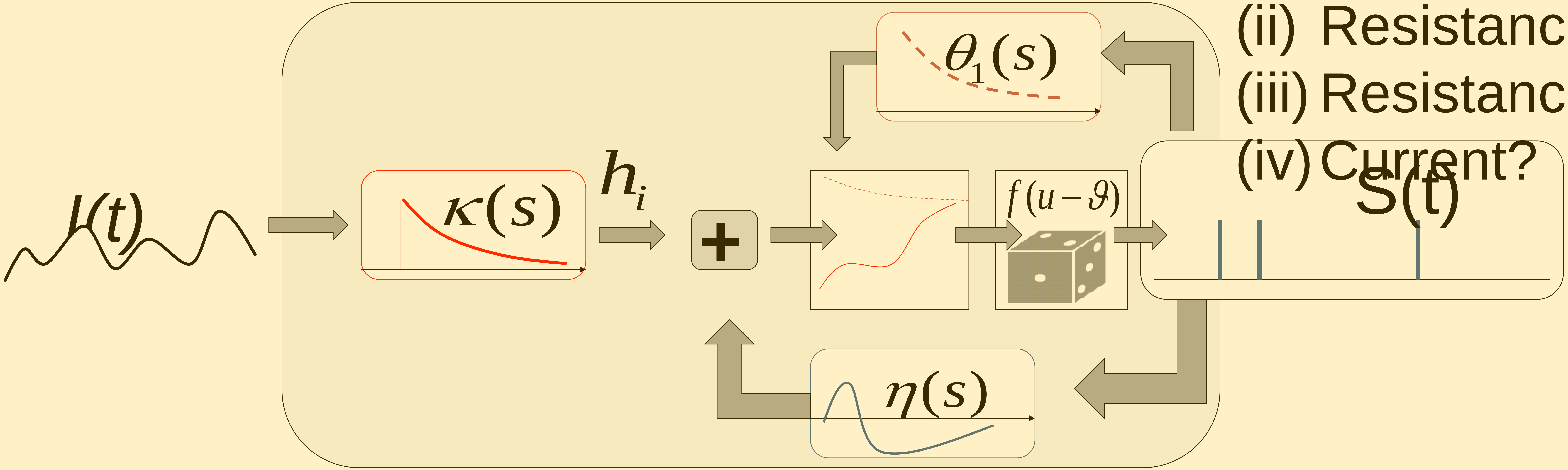
- nonlinear, but GLM

  → convex error function

Quiz 3 NOW :

What are the units of $\eta(s)$?

(i) Voltage?
(ii) Resistance?
(iii) Resistance/s
(iv) Current?



potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

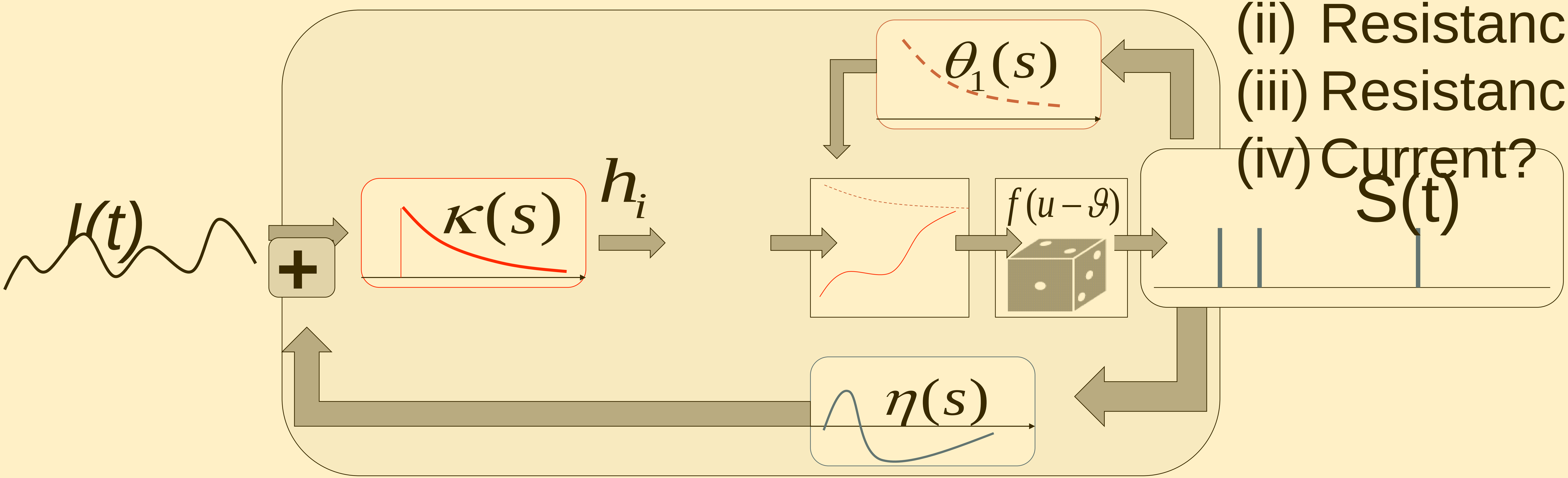
threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

Quiz NOW:

What are the units of $\kappa(s)$ **?**

(i) Voltage?
(ii) Resistance?
(iii) Resistance/s?
(iv) Current?



potential $u(t) = \int \eta(s) S(t-s)ds + \int_0^\infty \kappa(s) I(t-s)ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s)ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

Week 9 – part 6: Modeling in vitro data

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

√ **9.1 What is a good neuron model?**
- Models and data

√ **9.2 AdEx model**
- Firing patterns and adaptation

√ **9.3 Spike Response Model (SRM)**
- Integral formulation

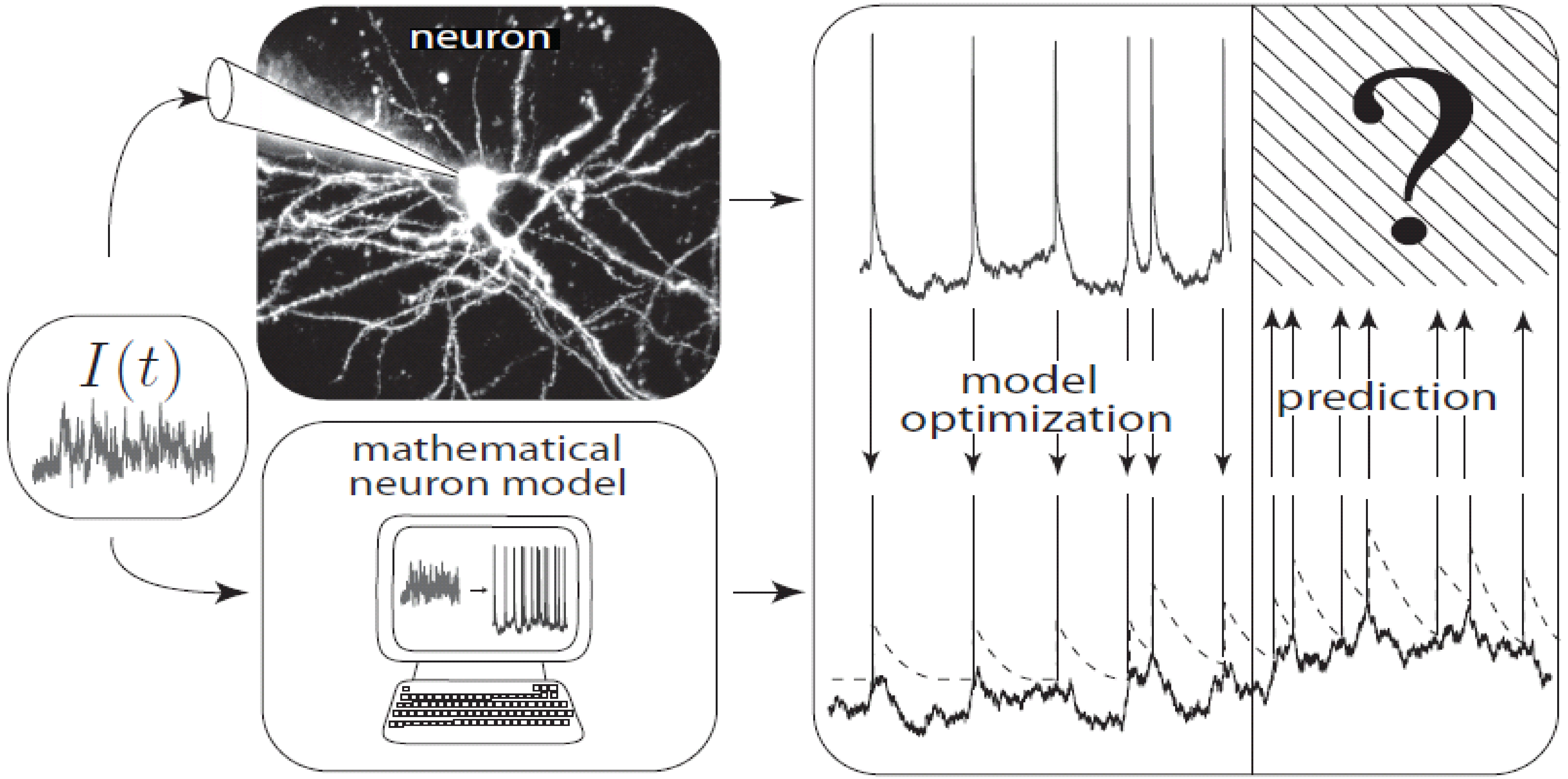
√ **9.4 Generalized Linear Model**
- Adding noise to the SRM

√ **9.5 Parameter Estimation**
- Quadratic and convex optimization

9.6. Modeling in vitro data
- how long lasts the effect of a spike?

9.7. Helping Humans

comparison model-data



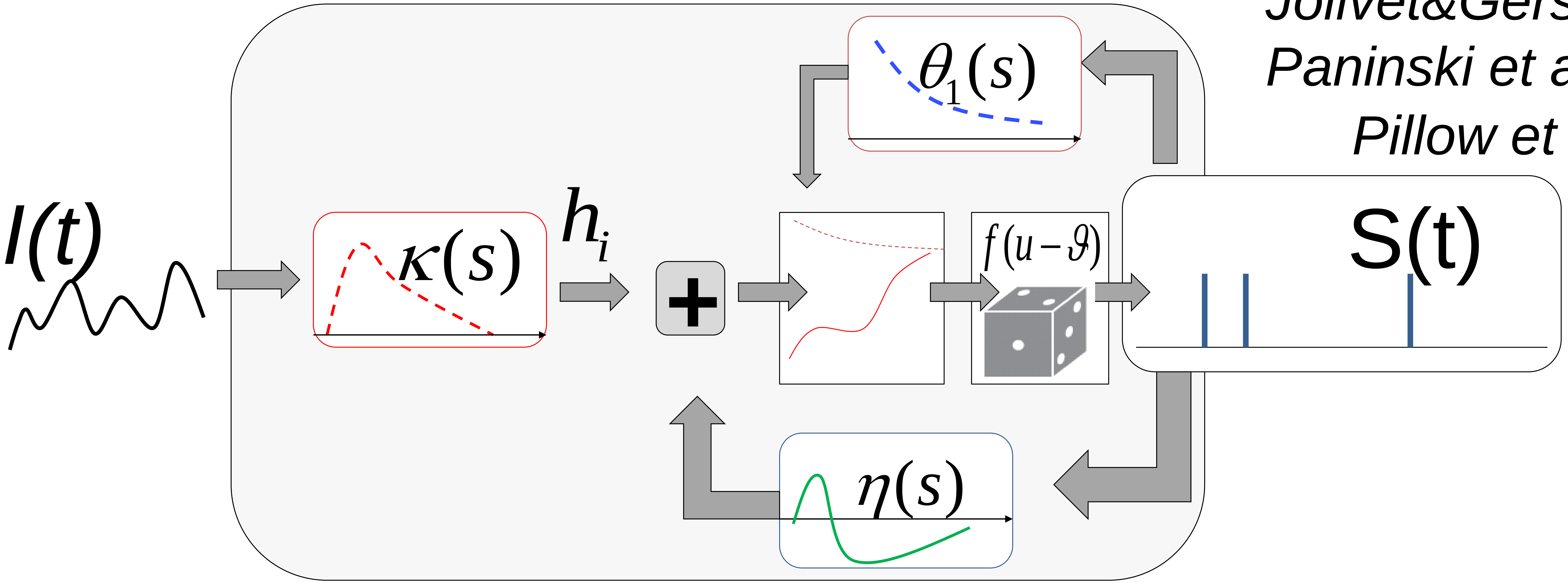
Predict

-Subthreshold voltage

-Spike times

Neuronal Dynamics – 9.6 GLM/SRM with escape noise

Jolivet&Gerstner, 2005
Paninski et al., 2004
Pillow et al. 2008

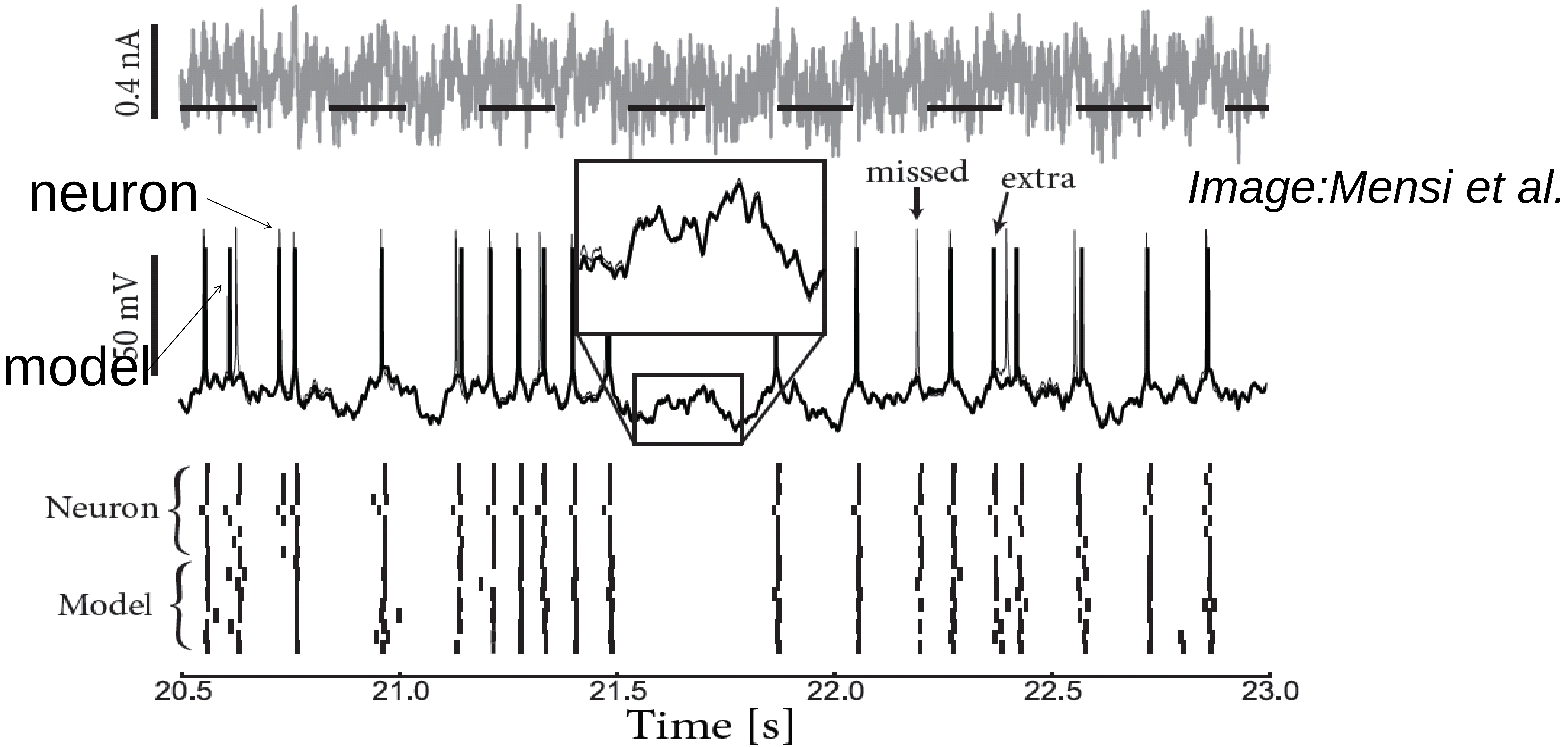


potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

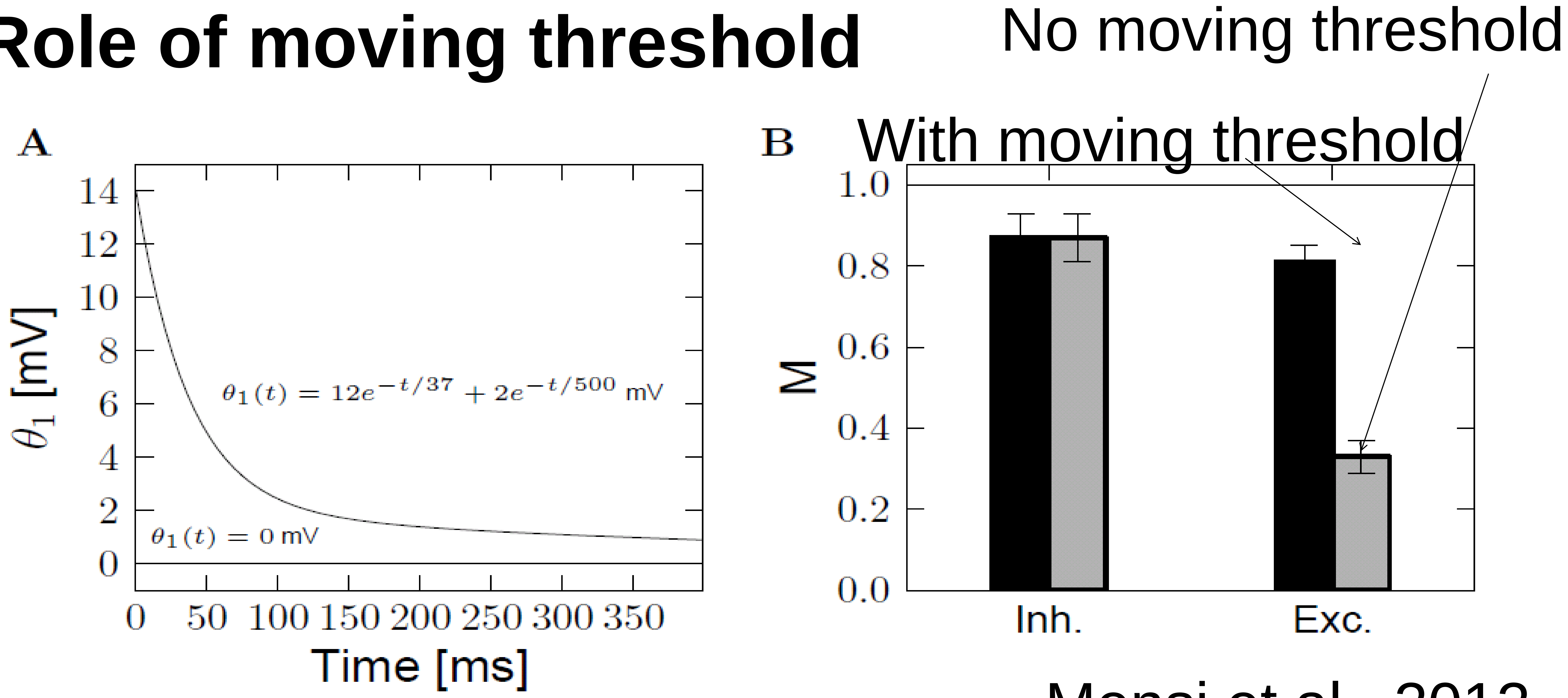
threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

Neuronal Dynamics – 9.6 GLM/SRM predict subthreshold voltage



Role of moving threshold

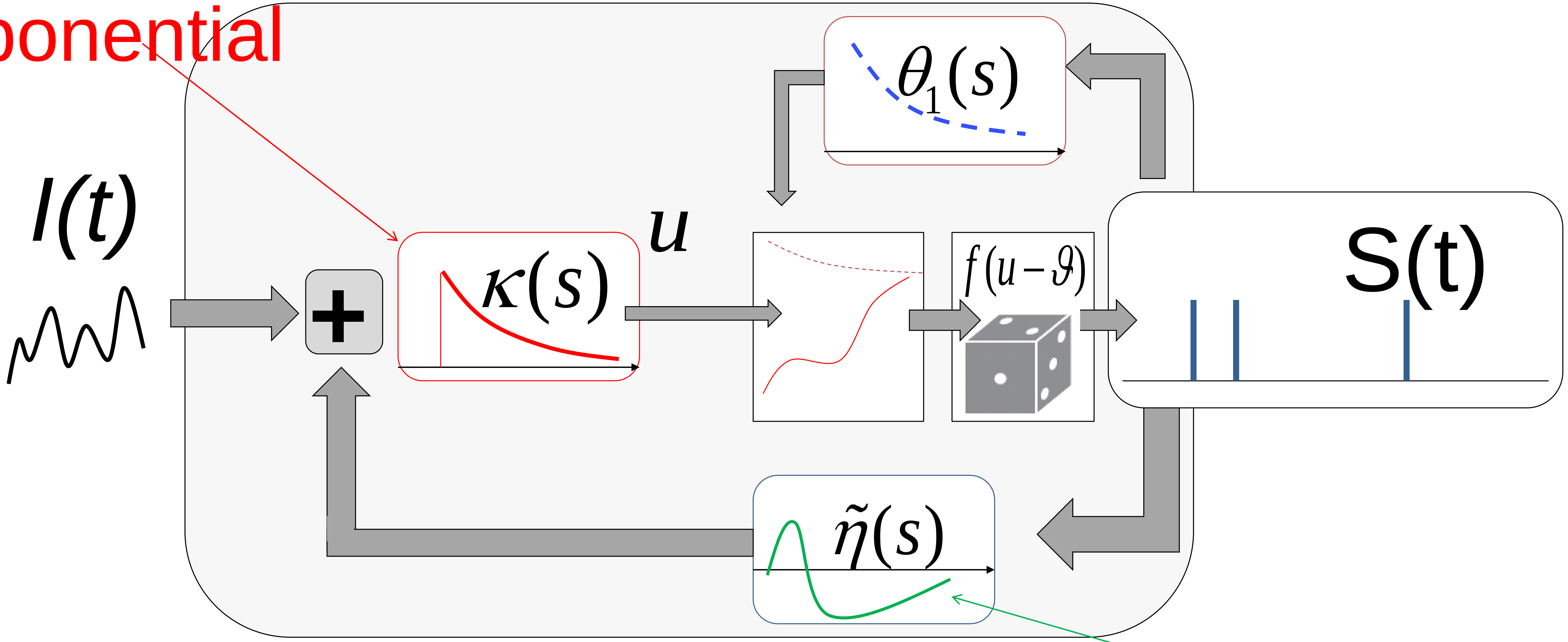


Mensi et al., 2012

Change in model formulation:

What are the units of …. ?

'soft-threshold adaptive IF model'



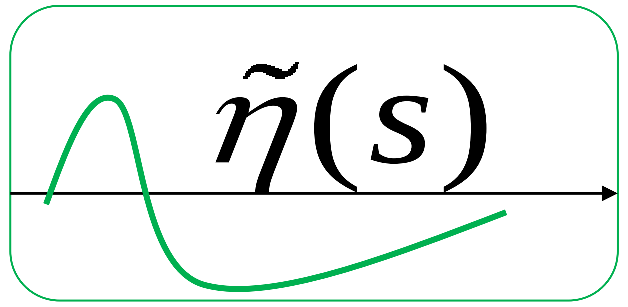
adaptation current

potential $$C\frac{d}{dt}u(t) = \int \tilde{\eta}(s) S(t-s)\,ds \quad + I(t)$$

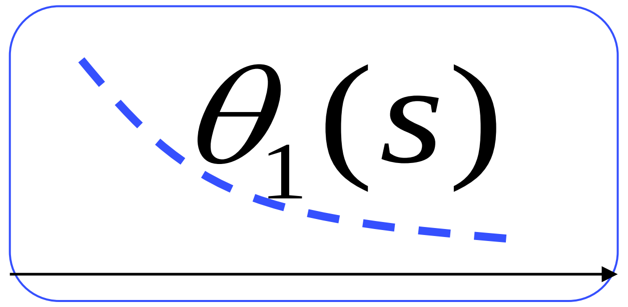
threshold $$\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s)\,ds$$

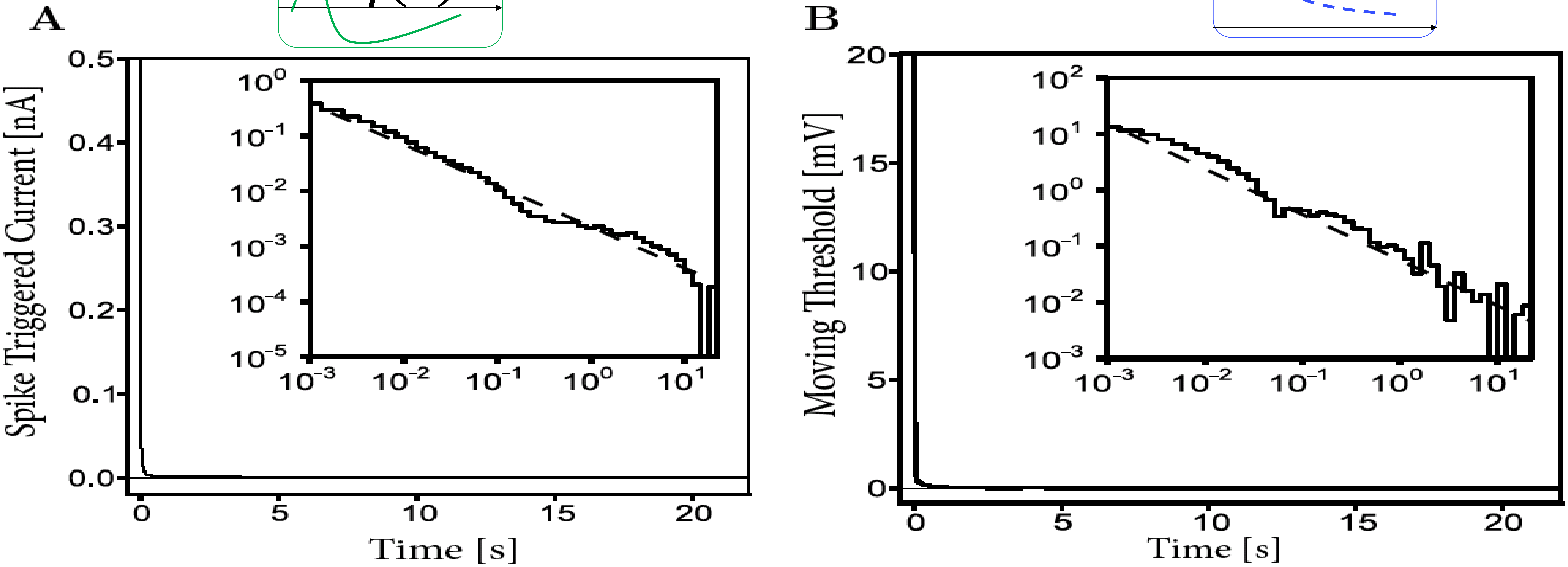
firing intensity $$\rho(t) = f(u(t) - \vartheta(t))$$

Time scale of filters?



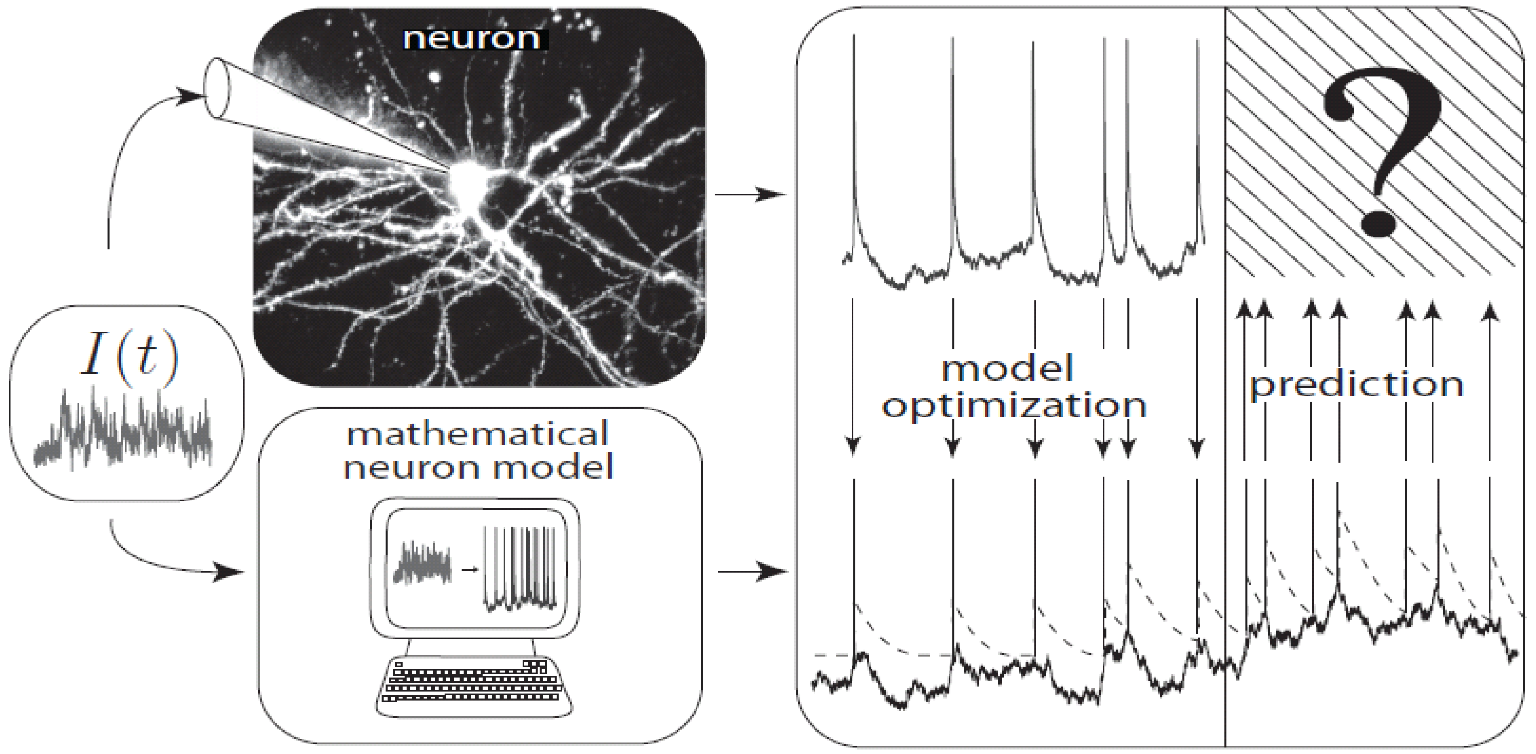
→ **Power law**





Pozzorini et al. 2013

A single spike has a measurable effect more than 10 seconds later!



-Predict spike times
-Predict subthreshold voltage
-Easy to interpret (not a ‘black box’)
-Variety of phenomena
-Systematic: ‘optimize’ parameters

BUT so far limited to in vitro

Neuronal Dynamics week 7– Suggested Reading/selected references

Reading: W. Gerstner, W.M. Kistler, R. Naud and L. Paninski,
Neuronal Dynamics: from single neurons to networks and models of cognition. Ch. 6,10,11: Cambridge, 2014

Nonlinear and adaptive IF

Fourcaud-Trocme, N., Hansel, D., van Vreeswijk, C., and Brunel, N. (2003). How spike …. *J. Neuroscience*, 23:11628-11640.
Badel, L., et al. (2008a). Extracting nonlinear integrate-and-fire, *Biol. Cybernetics*, 99:361-370.
Brette, R. and Gerstner, W. (2005). Adaptive exponential integrate-and-fire *J. Neurophysiol*., 94:3637- 3642.
Izhikevich, E. M. (2003). Simple model of spiking neurons. IEEE Trans Neural Netw, 14:1569-1572.
Gerstner, W. (2008). Spike-response model. *Scholarpedia*, 3(12):1343.

Optimization methods for neuron models, max likelihood, and GLM

-Brillinger, D. R. (1988). Maximum likelihood analysis of spike trains of interacting nerve cells. *Biol. Cybern*., 59:189-200.
-Truccolo, et al. (2005). A point process framework for relating neural spiking activity to spiking history, neural ensemble, and extrinsic covariate effects. Journal of Neurophysiology, 93:1074-1089.
- Paninski, L. (2004). Maximum likelihood estimation of … *Network: Computation in Neural Systems*, 15:243-262.
- Paninski, L., Pillow, J., and Lewi, J. (2007). Statistical models for neural encoding, decoding, and optimal stimulus design. In Cisek, P., et al. , *Comput. Neuroscience: Theoretical Insights into Brain Function*. Elsevier Science.
Pillow, J., ET AL.(2008). Spatio-temporal correlations and visual signalling… . Nature, 454:995-999.

Encoding and Decoding

Rieke, F., Warland, D., de Ruyter van Steveninck, R., and Bialek, W. (1997). Spikes - Exploring the neural code. MIT Press,
Keat, J., Reinagel, P., Reid, R., and Meister, M. (2001). Predicting every spike … *Neuron*, 30:803-817.
Mensi, S., et al. (2012). Parameter extraction and classication …. *J. Neurophys.,*107:1756-1775.
Pozzorini, C., Naud, R., Mensi, S., and Gerstner, W. (2013). Temporal whitening by . *Nat. Neuroscience,*
Georgopoulos, A. P., Schwartz, A.,Kettner, R. E. (1986). Neuronal population coding of movement direction. *Science*, 233:1416-1419.
Donoghue, J. (2002). Connecting cortex to machines: recent advances in brain interfaces. Nat. Neurosci., 5:1085-1088.

The END