Week 9 – adaptation and firing patterns

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

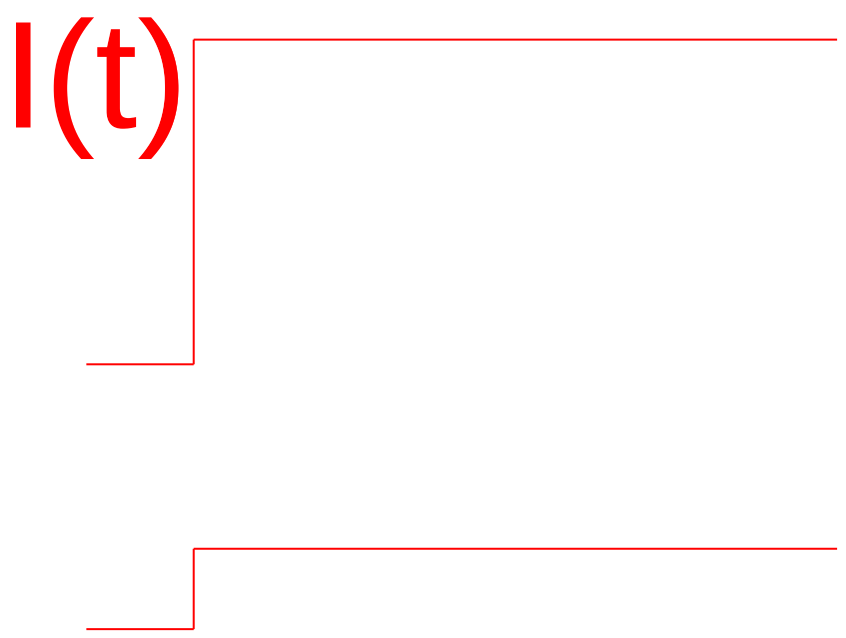
Biological Modeling of Neural Networks:

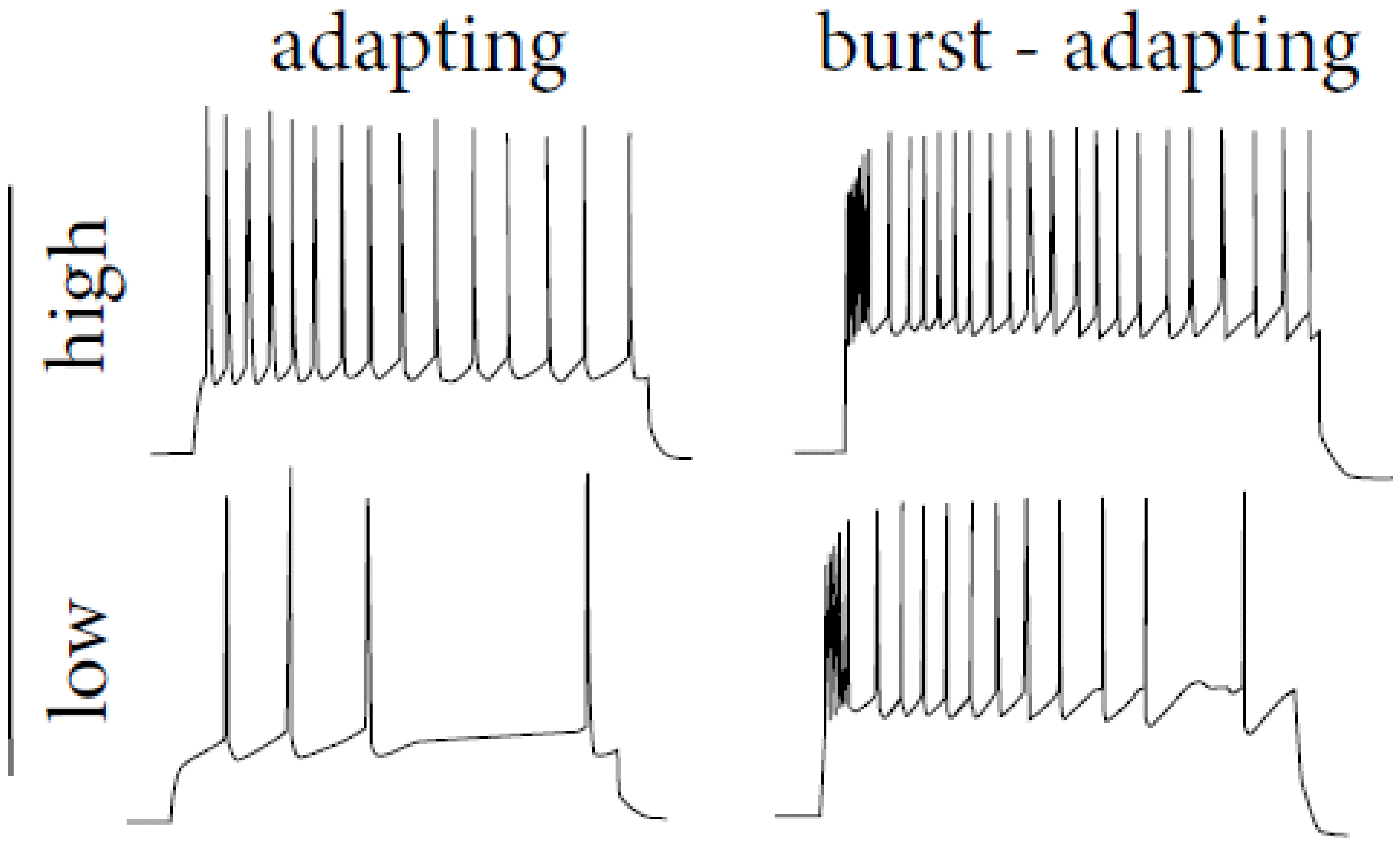
Week 9 – Adaptation and firing patterns

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Step current input – neurons show adaptation



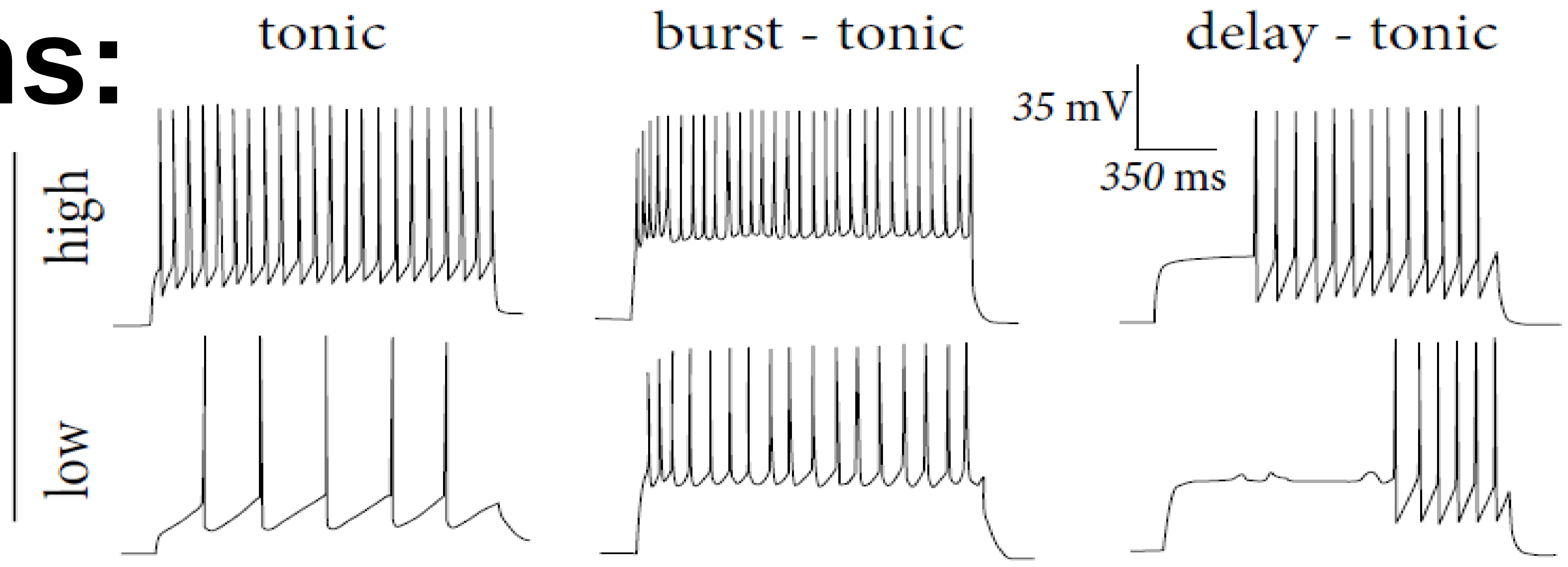


Data:
Markram et al.
(2004)

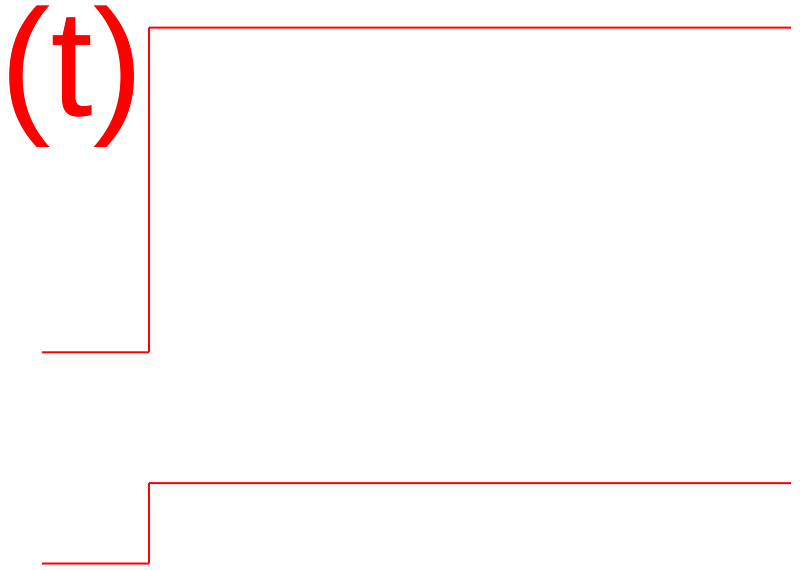
1-dimensional (nonlinear) integrate-and-fire model cannot do this!

Firing patterns:

Response to
Step currents,
Exper. Data,
Markram et al.
(2004)



I(t)



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Add adaptation variables:

Blackboard !

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u - \vartheta}{\Delta}) - R \sum_k w_k$$

$$\tau_k \frac{dw_k}{dt} = a_k (u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)$$

Exponential I&F
+ **1 adaptation** var.
= AdEx

SPIKE AND RESET

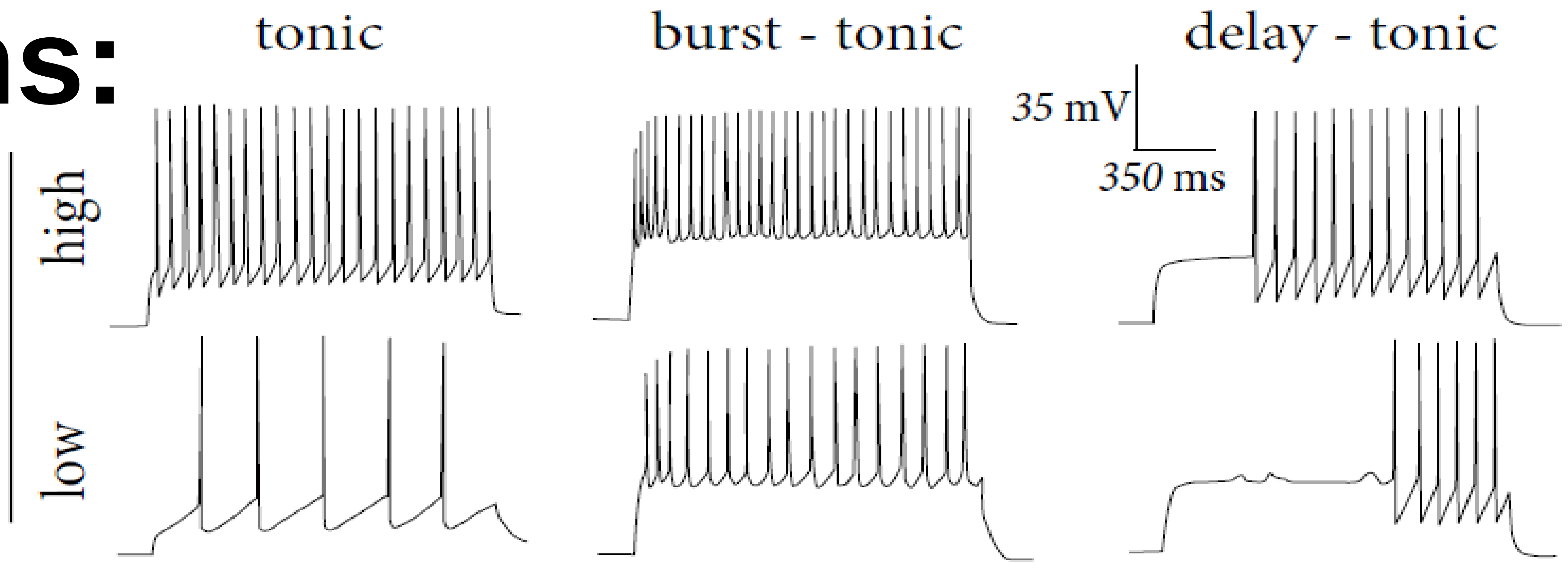
after each spike w_k jumps by an amount b_k

$If \;\; u = \theta_{reset} \;\; then \;\; reset\, to\; u = u_r$

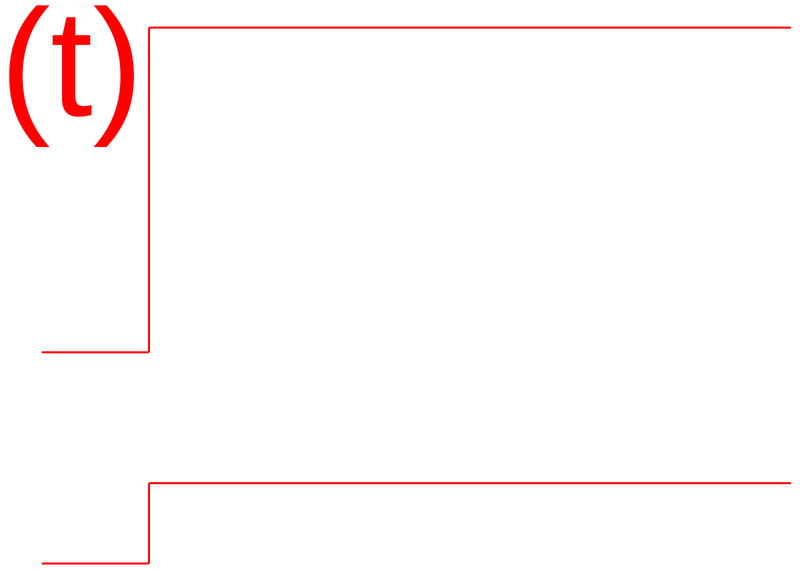
AdEx model, Brette&Gerstner (2005):

Firing patterns:

Response to
Step currents,
Exper. Data,
Markram et al.
(2004)

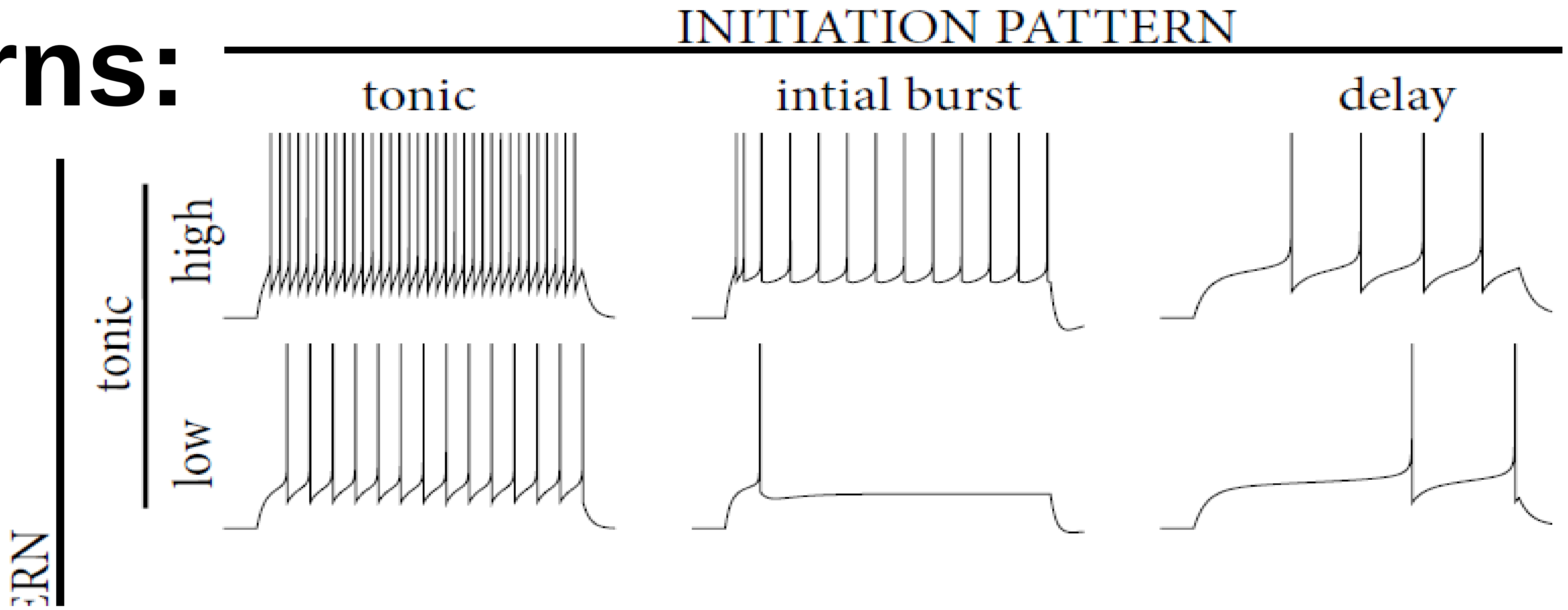


I(t)



Firing patterns:

Response to
Step currents,
AdEx Model,
Naud&Gerstner



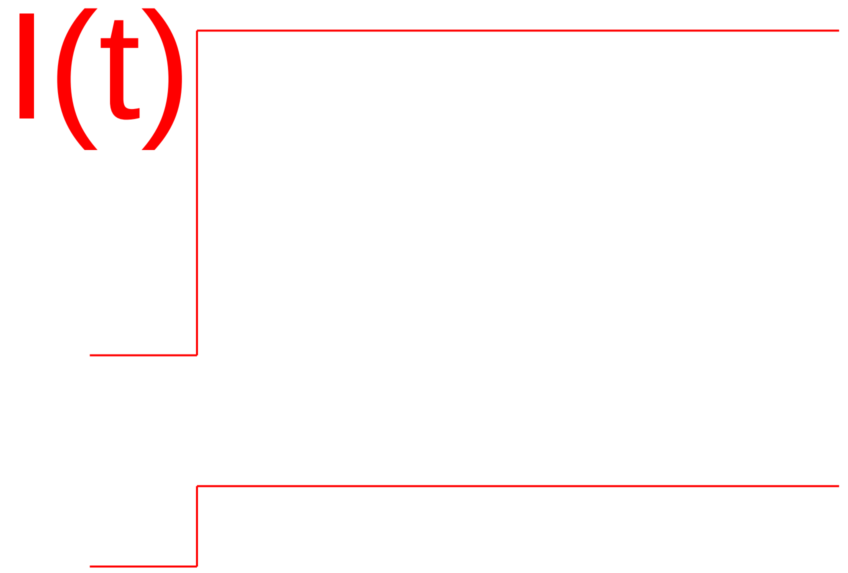


Image:
Neuronal Dynamics,
Gerstner et al.
Cambridge (2002)

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\ \tau_w \sum_f \delta(t - t^f)$$

AdEx model

Phase plane analysis!

Can we understand the different firing patterns?

Neuronal Dynamics – Quiz 9.1. Nullclines of AdEx

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u - \vartheta}{\Delta}) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w$$

A - What is the qualitative shape of the w-nullcline?

[ ] constant
[ ] linear, slope a
[ ] linear, slope 1
[ ] linear + quadratic
[ ] linear + exponential

B - What is the qualitative shape of the u-nullcline?

[ ] linear, slope 1
[ ] linear, slope 1/R
[ ] linear + quadratic
[ ] linear w. slope 1/R+ exponential

1 minute
Restart at 9:38

Week 9 – part 2b : Firing Patterns

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

9.1 What is a good neuron model?
- Models and data

9.2 AdEx model
- Firing patterns and adaptation

9.3 Spike Response Model (SRM)
- Integral formulation

9.4 Generalized Linear Model
- Adding noise to the SRM

9.5 Parameter Estimation
- Quadratic and convex optimization

9.6. Modeling in vitro data
- how long lasts the effect of a spike?

AdEx model

after each spike
u is reset to **u_r**

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u-\vartheta}{\Delta}) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\ \tau_w \sum_f \delta(t - t^f)$$

after each spike
w jumps by an amount ***b***

parameter ***a*** **– slope of *w*-nullcline**

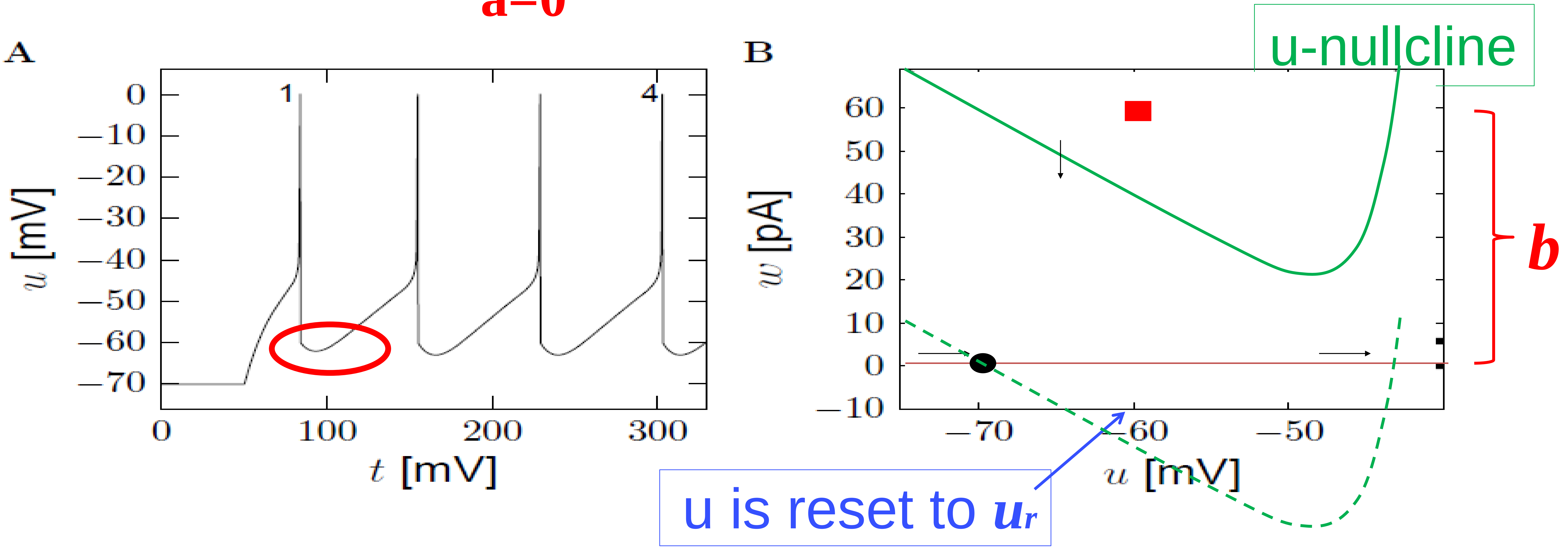
Can we understand the different firing patterns?

AdEx model – phase plane analysis: large *b*

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\,(u - u_{rest}) - w + b\,\tau_w \sum_f \delta(t - t^f)$$

a=0

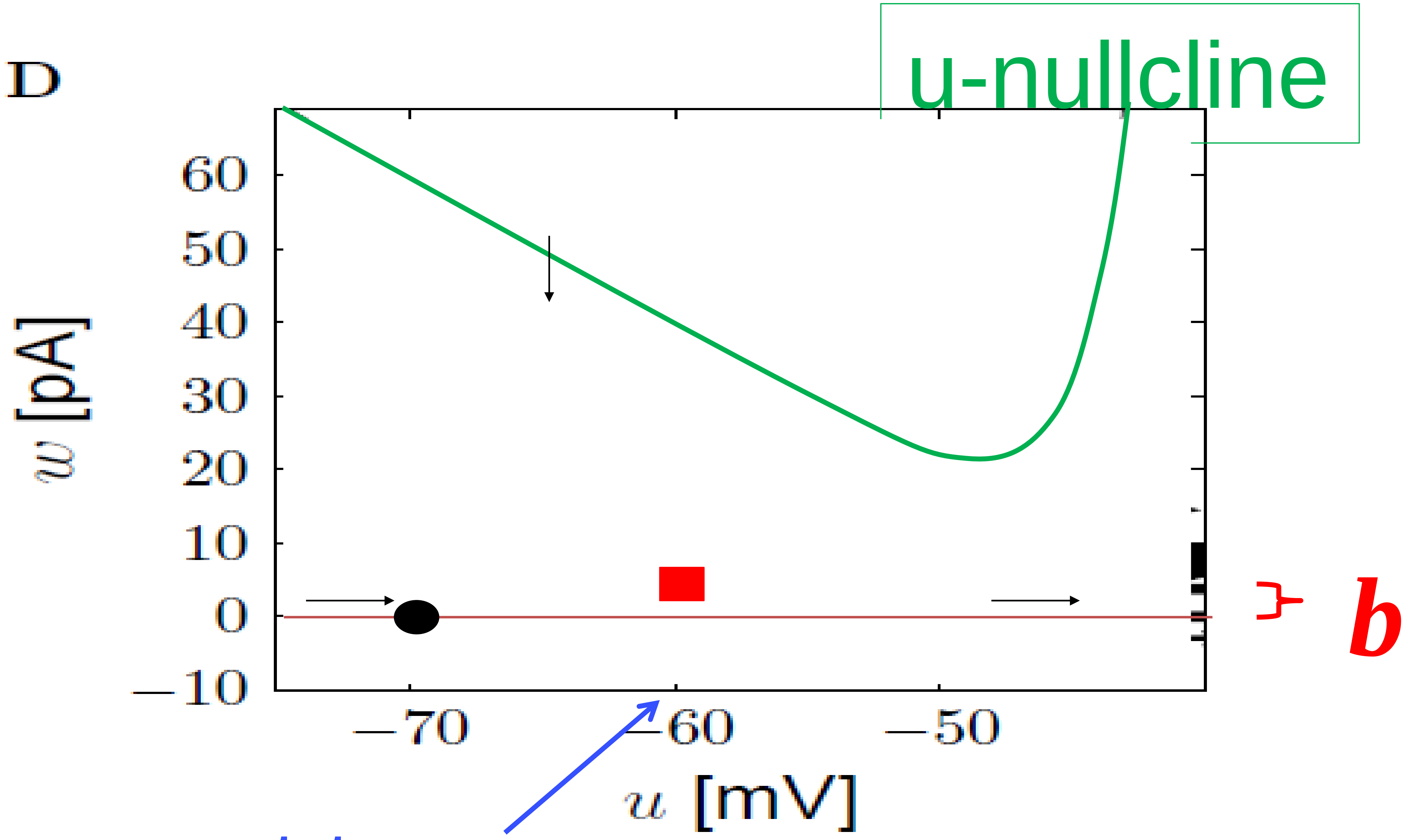


AdEx model – phase plane analysis: small b

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\,(u - u_{rest}) - w + b\,\tau_w \sum_f \delta(t - t^f)$$

adaptation



u is reset to u_r

Quiz 9.2: AdEx model – phase plane analysis

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) + b\ \tau_w \sum_f\ \delta(t - t^f)$$

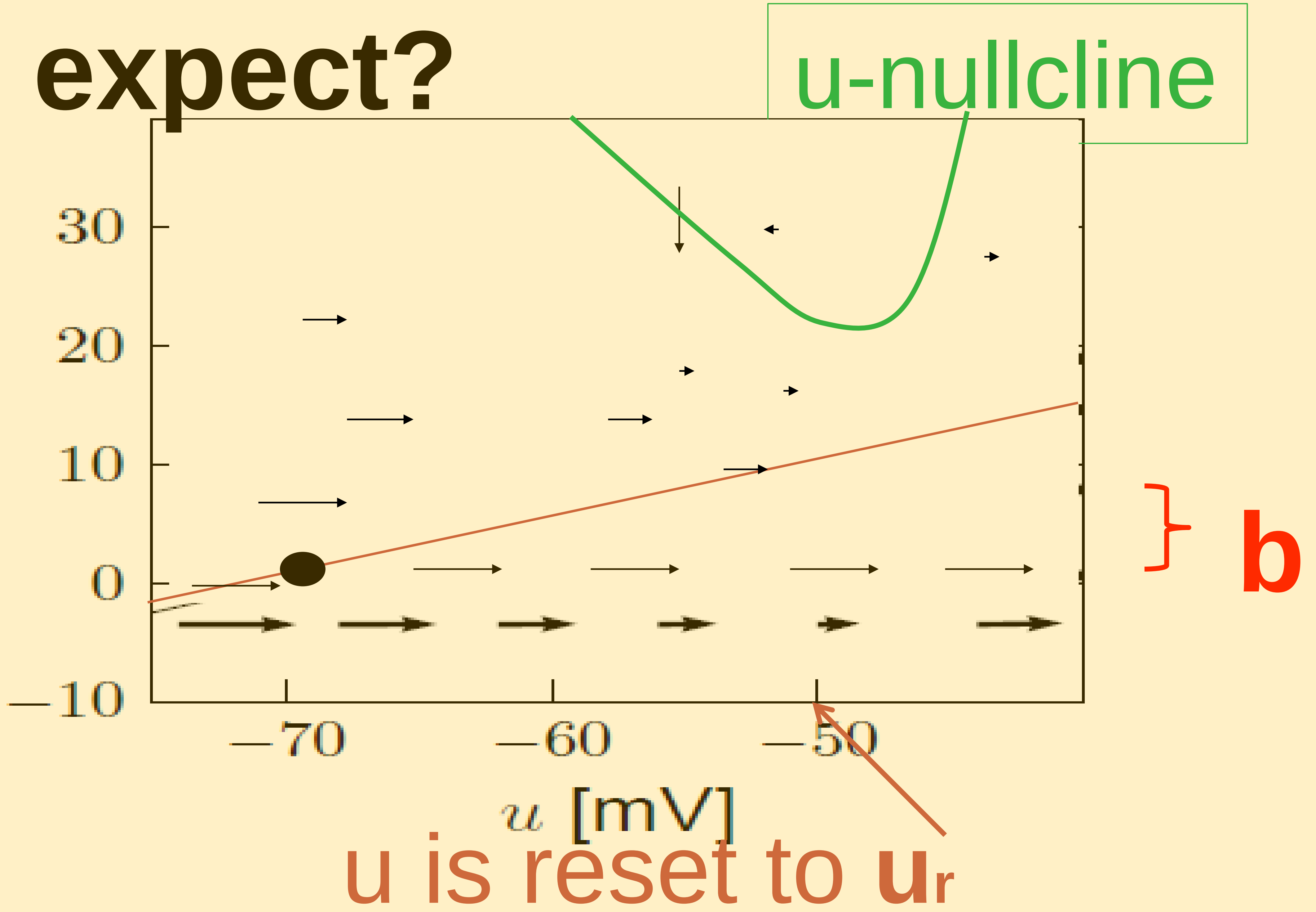
What firing pattern do you expect?

(i) Adapting

(ii) Bursting

(iii) Initial burst

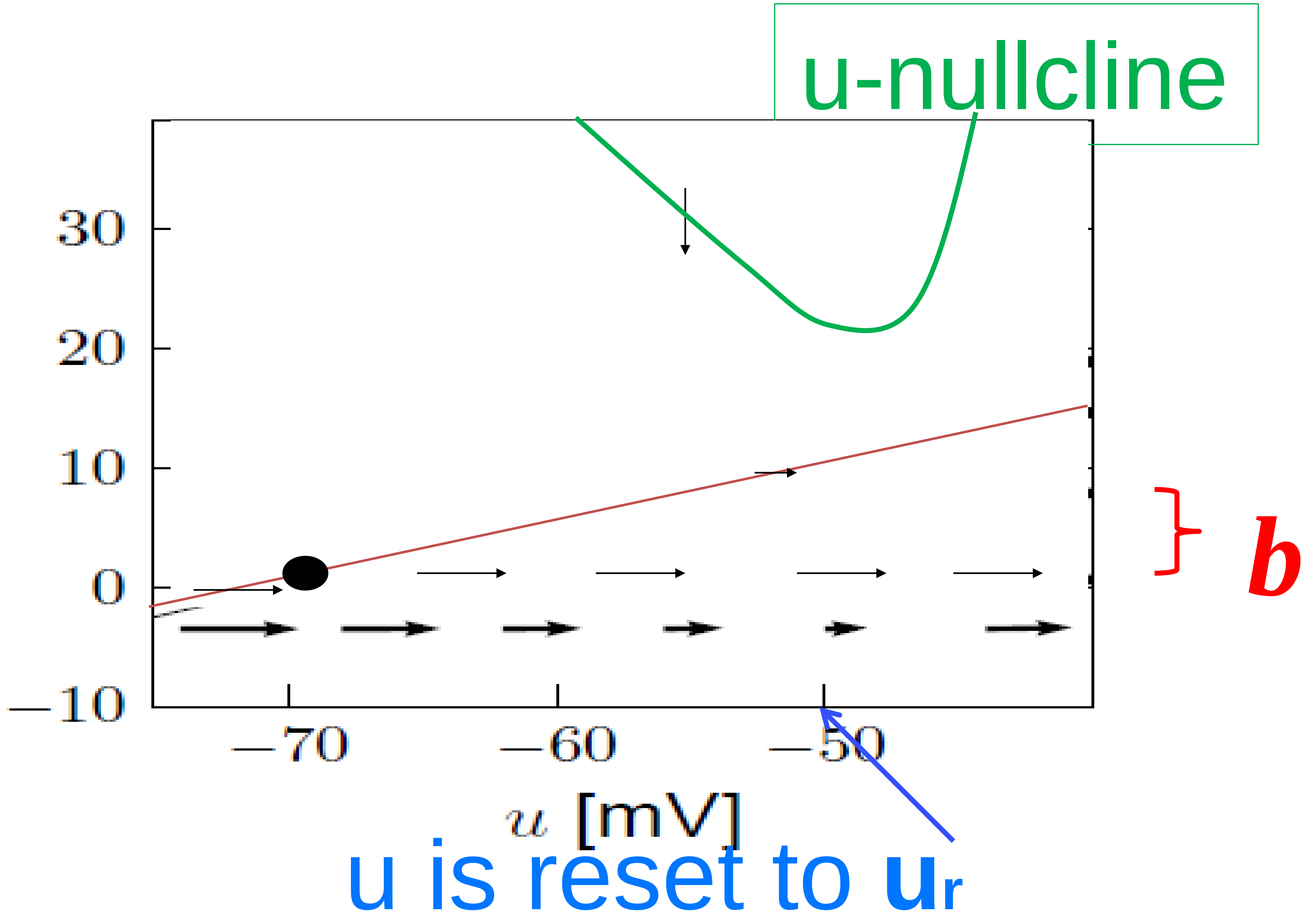
(iv)Non-adapting



AdEx model – phase plane analysis: a>0

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u-\vartheta}{\Delta}) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\ \tau_w \sum_f \delta(t - t^f)$$



Neuronal Dynamics – 9.2 AdEx model and firing patterns

after each spike u is reset to **u**$_r$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\,(u - u_{rest}) - w + b\,\tau_w \sum_f \delta(t - t^f)$$

after each spike
w jumps by an amount **b**

parameter **a – slope of w nullcline**

Firing patterns arise from different parameters!

See Naud et al. (2008), see also Izikhevich (2003)

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(1) $$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

(2) $$If \quad u = \theta_{reset} \quad then \quad reset\,to\; u = u_r$$

Best choice of ***f*** : linear + exponential

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right)$$

BUT: **Limitations – need to add**

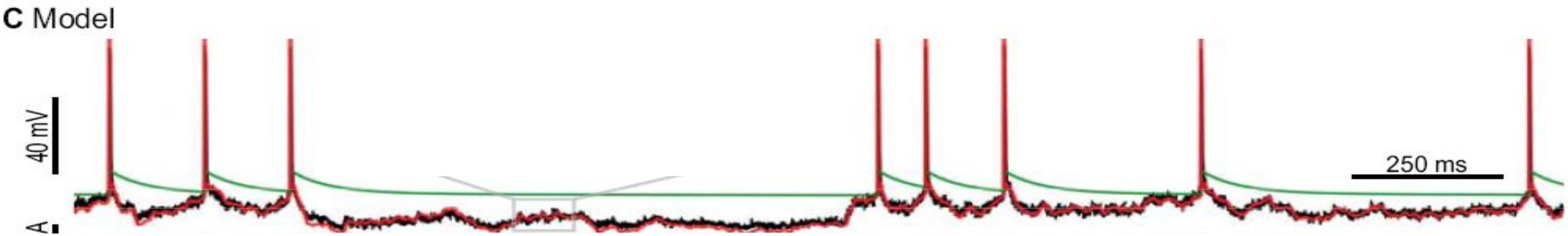
- ✓ -Adaptation on slower time scales
- ✓ -Possibility for a diversity of firing patterns
- -Increased threshold ϑ after each spike
- -Noise

Add dynamic threshold:

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u - \vartheta}{\Delta}) - R\sum_k w_k + RI(t)$$

Threshold increases after each spike

$$\vartheta = \theta_0 + \sum_f \theta_1(t - t^f)$$



$$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

$$If \quad u = \theta_{reset} \quad then \quad reset\,to\; u = u_r$$

add

√ -Adaptation variables
√ -Possibility for firing patterns
√ -Dynamic threshold ϑ
-Noise

Week 9 – part 3: Spike Response Model (SRM)

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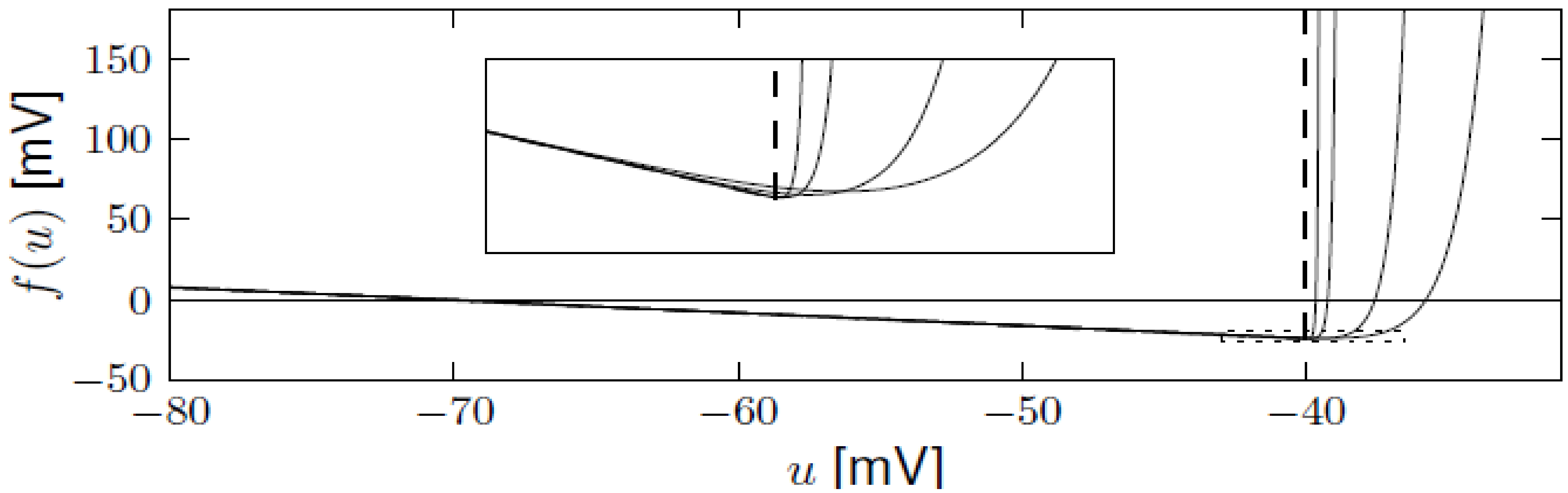
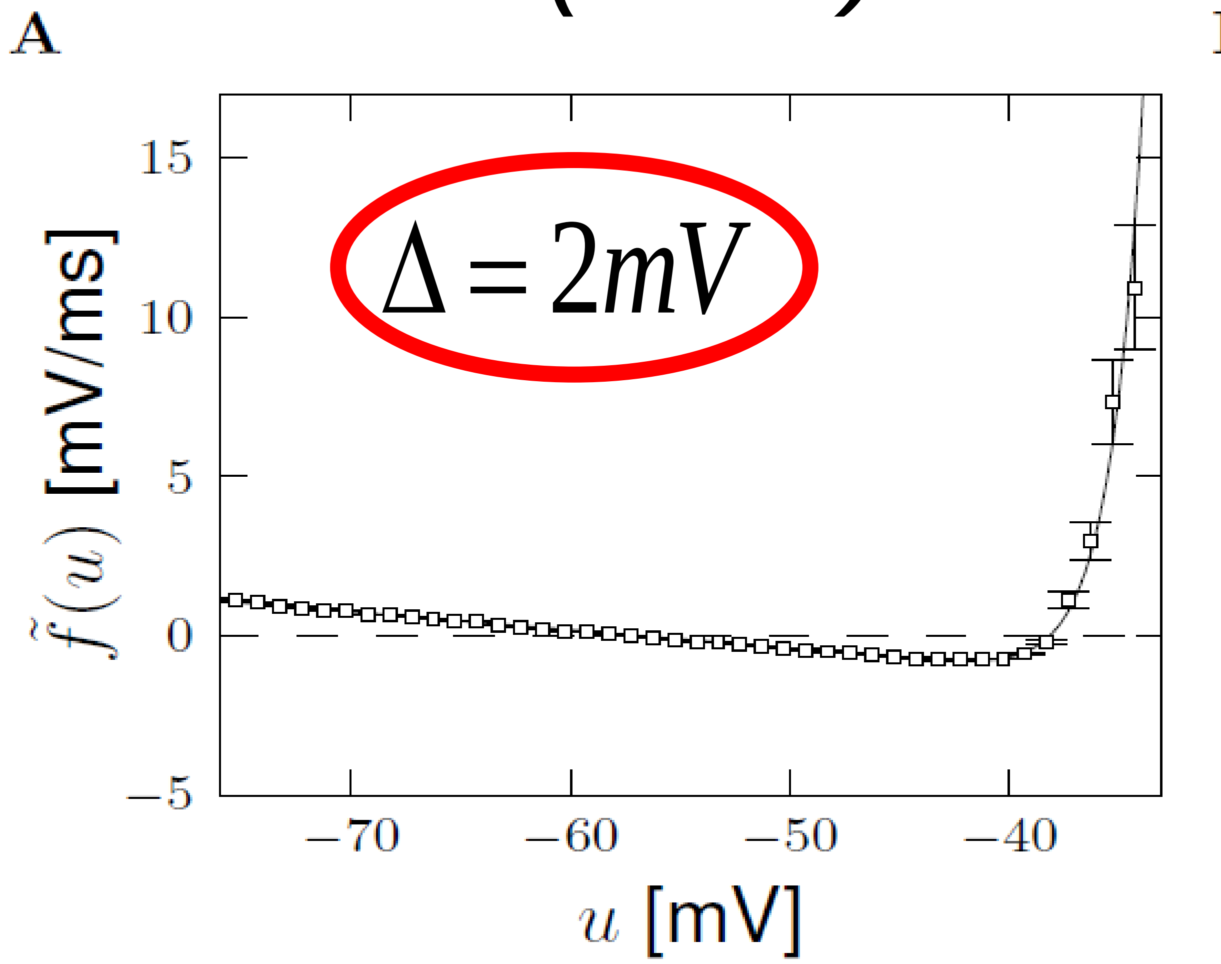
Exponential versus Leaky Integrate-and-Fire

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) + RI(t)$$

Badel et al (2008)





$$\tau \frac{du}{dt} = -(u - u_{rest}) + RI(t)$$

Reset if u= ϑ

Leaky Integrate-and-Fire:
Replace nonlinear kink by threshold

Neuronal Dynamics – 9.3 Adaptive leaky integrate-and-fire

$$\tau \frac{du}{dt} = -(u - u_{rest}) - R\sum_k w_k + RI(t)$$

$$\tau_k \frac{dw_k}{dt} = a_k (u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)$$

SPIKE AND RESET

after each spike

w_k jumps by an amount b_k

If $u = \vartheta(t)$ *then reset to* $u = u_r$

Dynamic threshold

$$\tau \frac{du}{dt} = -(u - u_{rest}) - R\sum_k w_k + RI(t)$$

$$\tau_k \frac{dw_k}{dt} = a_k (u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)$$

Adaptive leaky I&F

Linear equation → can be integrated!

$$u(t) = \sum_f \eta\,(t - t^f) + \int_0^\infty ds\, \kappa(s) I(t - s)$$

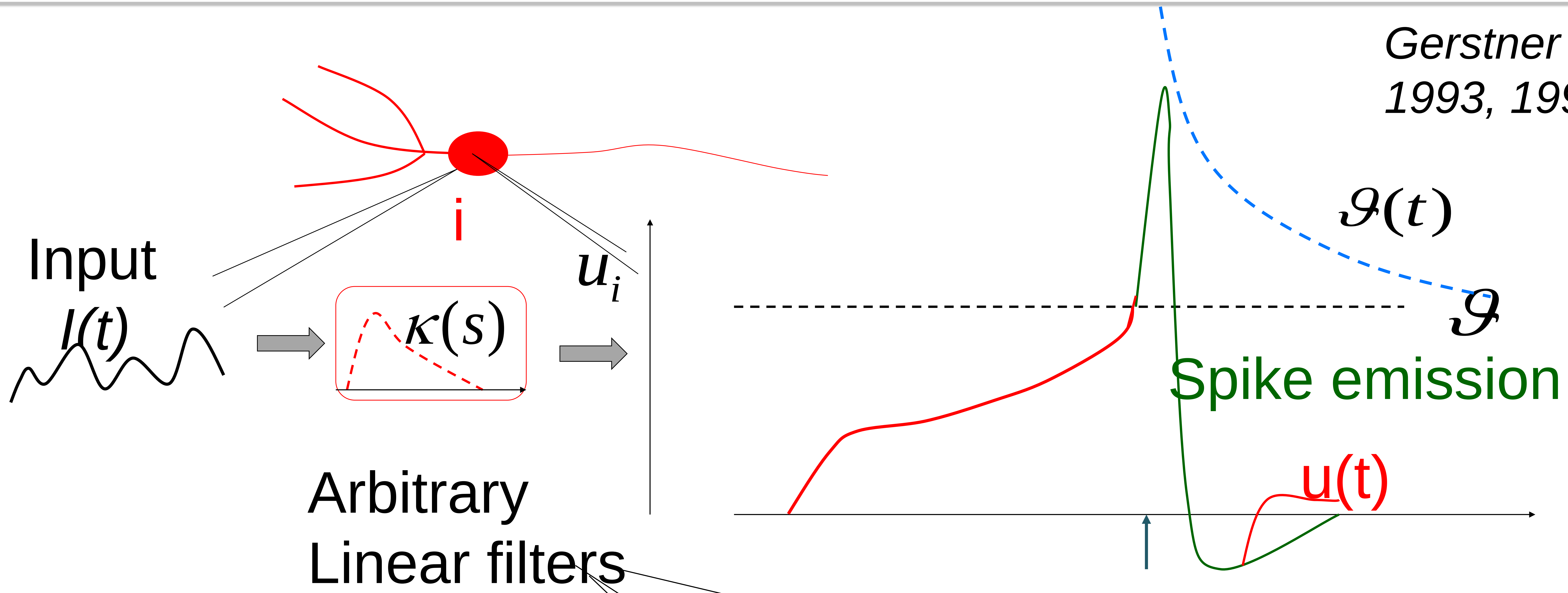
$$\vartheta(t) = \theta_0 + \sum_f \theta_1 (t - t^f)$$

Spike Response Model (SRM)

Gerstner et al. (1996)

Neuronal Dynamics – 9.3 Spike Response Model (SRM)

Gerstner et al., 1993, 1996



potential

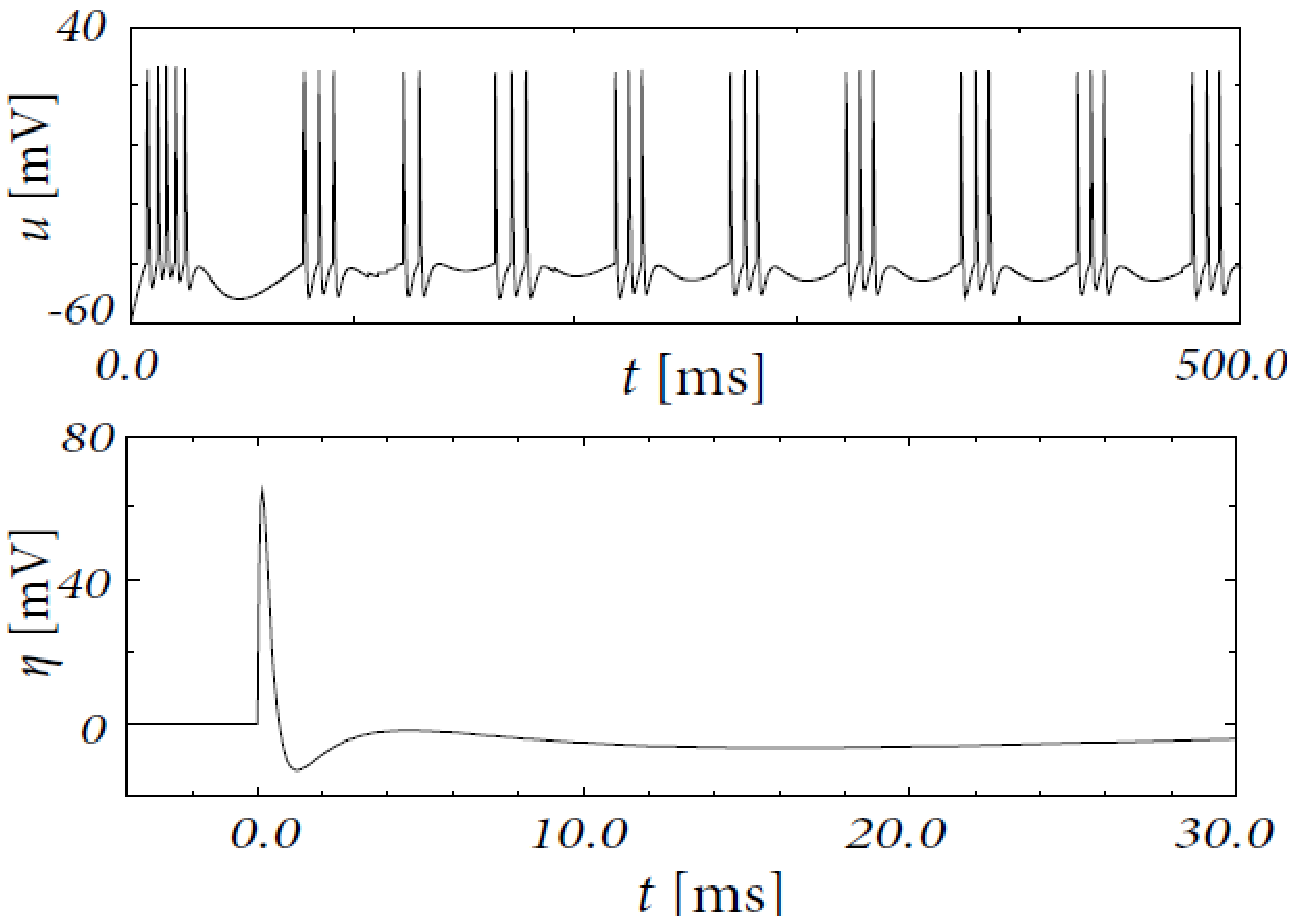
$$u(t) = \sum_{t'} \eta(t-t') + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$$

threshold

$$\vartheta(t) = \theta_0 + \sum_{t'} \theta_1(t-t')$$

Neuronal Dynamics – 9.3 Bursting in the SRM

SRM with appropriate η leads to bursting



$$u(t) = \sum_f \eta\,(t - t^f) + \int_0^\infty ds\, \kappa(s) I(t-s) + u_{rest}$$

$$u(t) = \int_0^\infty ds\, \eta(s)\, S(t-s) + \int_0^\infty ds\, \kappa(s) I(t-s) + u_{rest}$$

Exercise 1: from adaptive IF to SRM

$$\tau \frac{du}{dt} = -(u - u_{rest}) - w + RI(t)$$

$$If \quad u = \vartheta \; then \;\; reset\, to \; u = u_r$$

$$\tau_w \frac{dw}{dt} = -w + b\,\tau_w \sum_f \delta(t - t^f)$$

Next lecture at 9:57/10:15

Integrate the above system of two differential equations so as to rewrite the equations as

potential $$u(t) = \int_0^\infty \eta(s) S(t-s) ds + \int_0^\infty \varepsilon(s) I(t-s) ds + u_{rest}$$

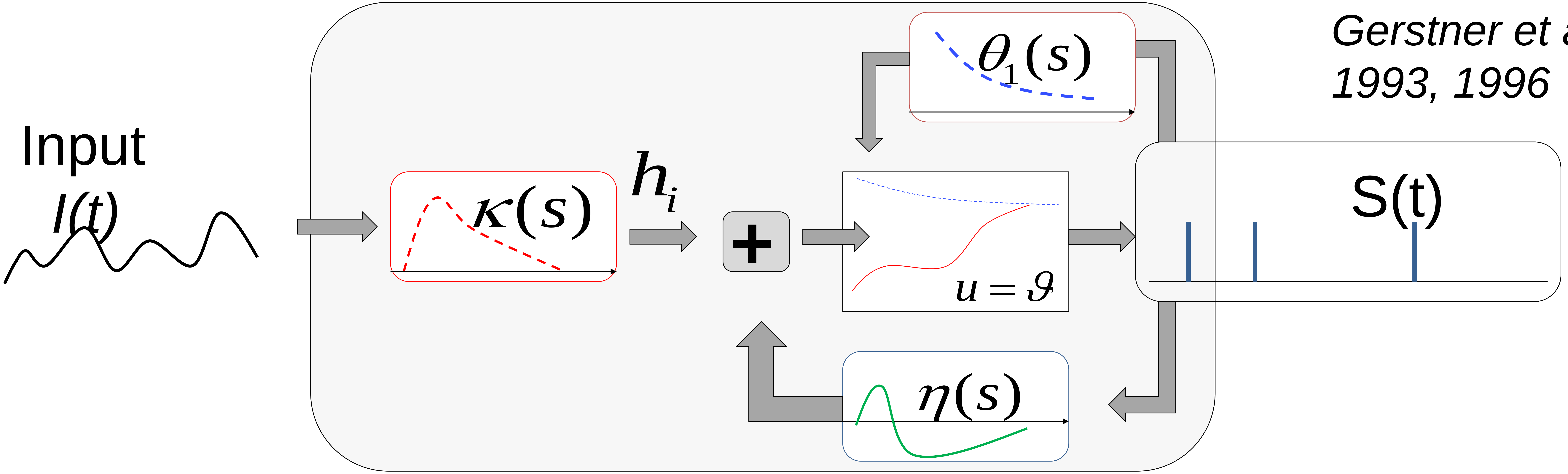
A – what is $\eta(s)$ **?** **(i)** $x(s) = \frac{R}{\tau} \exp(-\frac{s}{\tau})$ **(ii)** $x(s) = \frac{R}{\tau_w} \exp(-\frac{s}{\tau_w})$

B – what is $\varepsilon(s)$ **?**

(iii) $x(s) = C[\exp(-\frac{s}{\tau}) - \exp(-\frac{s}{\tau_w})]$ **(iv) Combi of (i) + (iii)**

Neuronal Dynamics – 9.3 Spike Response Model (SRM)

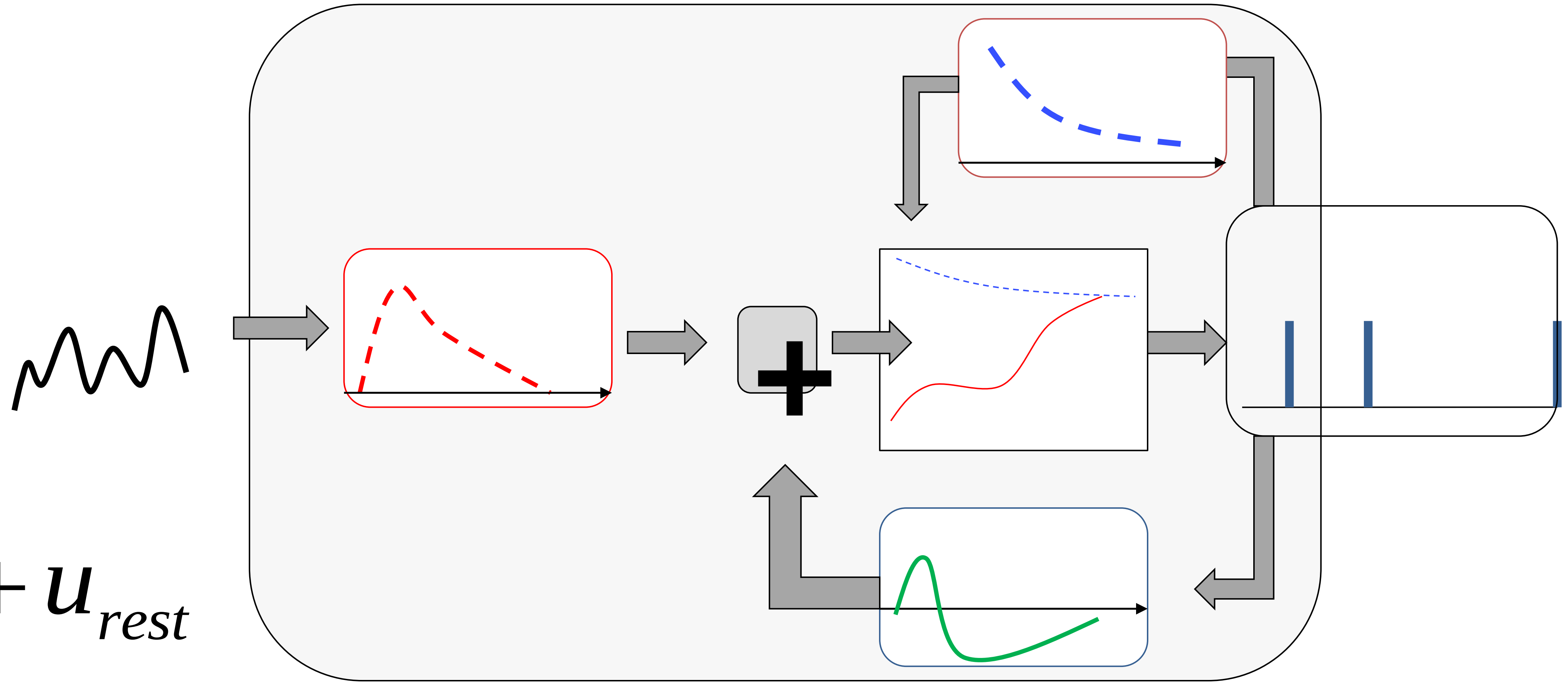
Gerstner et al., 1993, 1996



potential $$u(t)=\sum_{t'}\eta(t-t')+\int_0^\infty \kappa(s)I(t-s)ds+u_{rest}$$

threshold $$\vartheta(t)=\theta_0+\sum_{t'}\theta_1(t-t')$$

firing if $$u(t)=\vartheta(t)$$



potential

$$u(t)=\sum_{t'}\eta(t-t')+\int_0^\infty \kappa(s)I(t-s)ds+u_{rest}$$

threshold

$$\vartheta(t)=\theta_0+\sum_{t'}\theta_1(t-t')$$

Linear filters for

- input
- threshold
- refractoriness