Artificial Neural Networks

Convolutional Neural Networks

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Part 1: Inductive Bias in Machine Learning

Objectives for today:

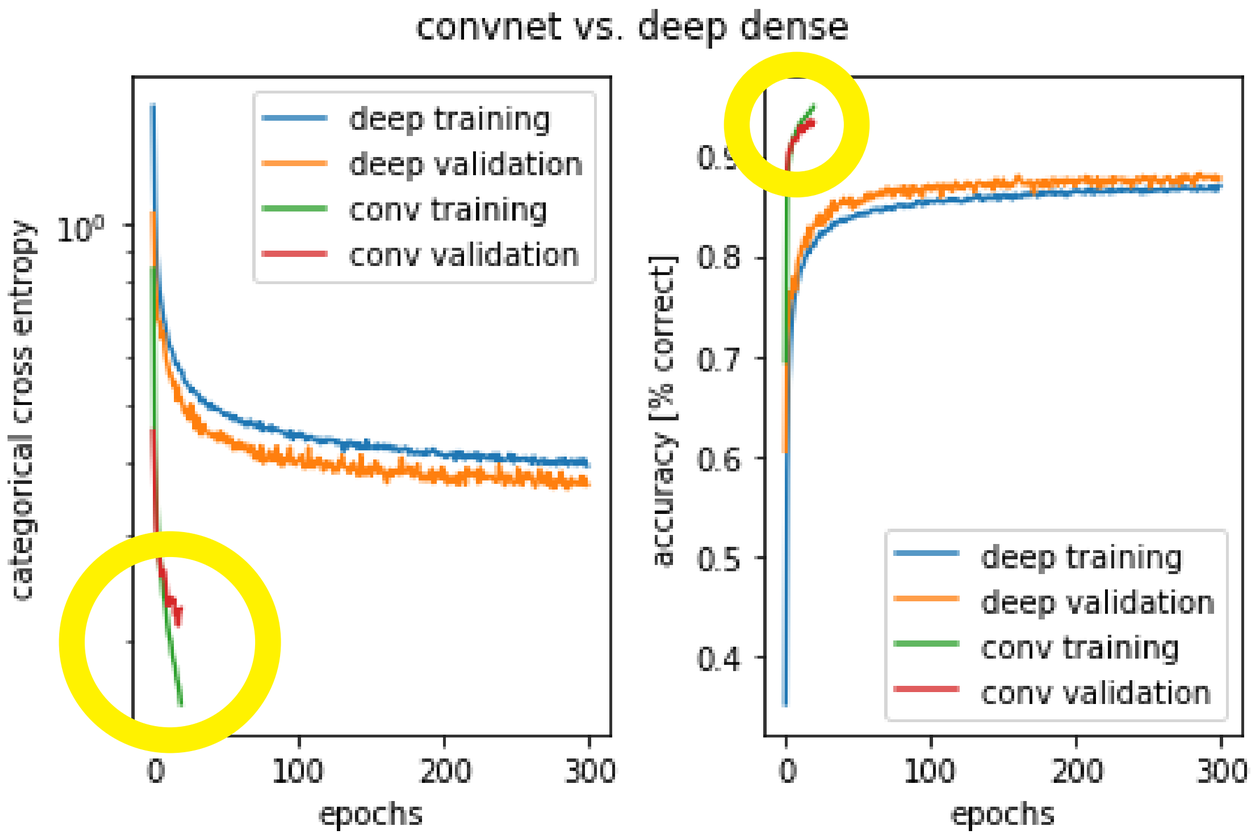
- Inductive bias in machine learning
- Convolution filters as inductive bias for images
- Max Pooling for local translation invariance
- ImageNet competition and modern ConvNets
- Training ConvNets with AutoDiff
- Applications beyond object recognition

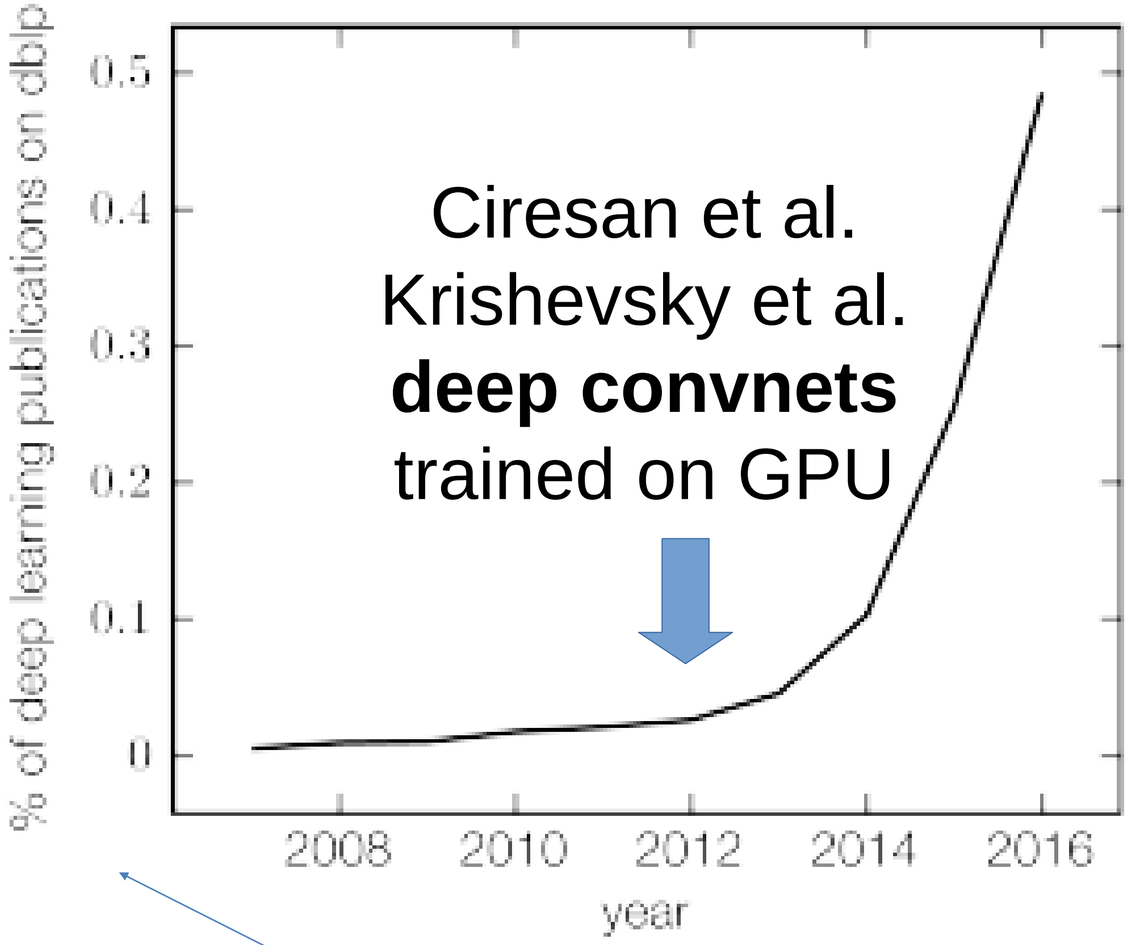
Previous slide.

This lecture has been prepared and taught several times by Dr. Johanni Brea (EPFL) for the class ‘Artificial Neural Networks’ at EPFL.

Motivation: Convolutional networks (convnet) work well!

An image recognition task





Fukushima (1982): Neocognitron
McClelland et al. (1996): Parallel Distributed Processing

Previous slide.

Left: Networks with convolutional layers reach significantly better training and test perfomances than those without, and converge more rapidly.

Right: Convolutional networks have been around since 1982 (Fukushima: Neocognitron) and multi-layer networks since the early 1990ies. But the “deep learning revolution” started only around 2012, after the publication of several really deep convolutional networks that out-performed other approaches on e.g. difficult object recognition tasks. Afterwards the number of deep learning publications increased dramatically.

Convolutional networks and No free lunch theorem

The big success of Deep Neural Networks came when they worked on large image data bases.

Why are convolutional networks better than other networks on image tasks?

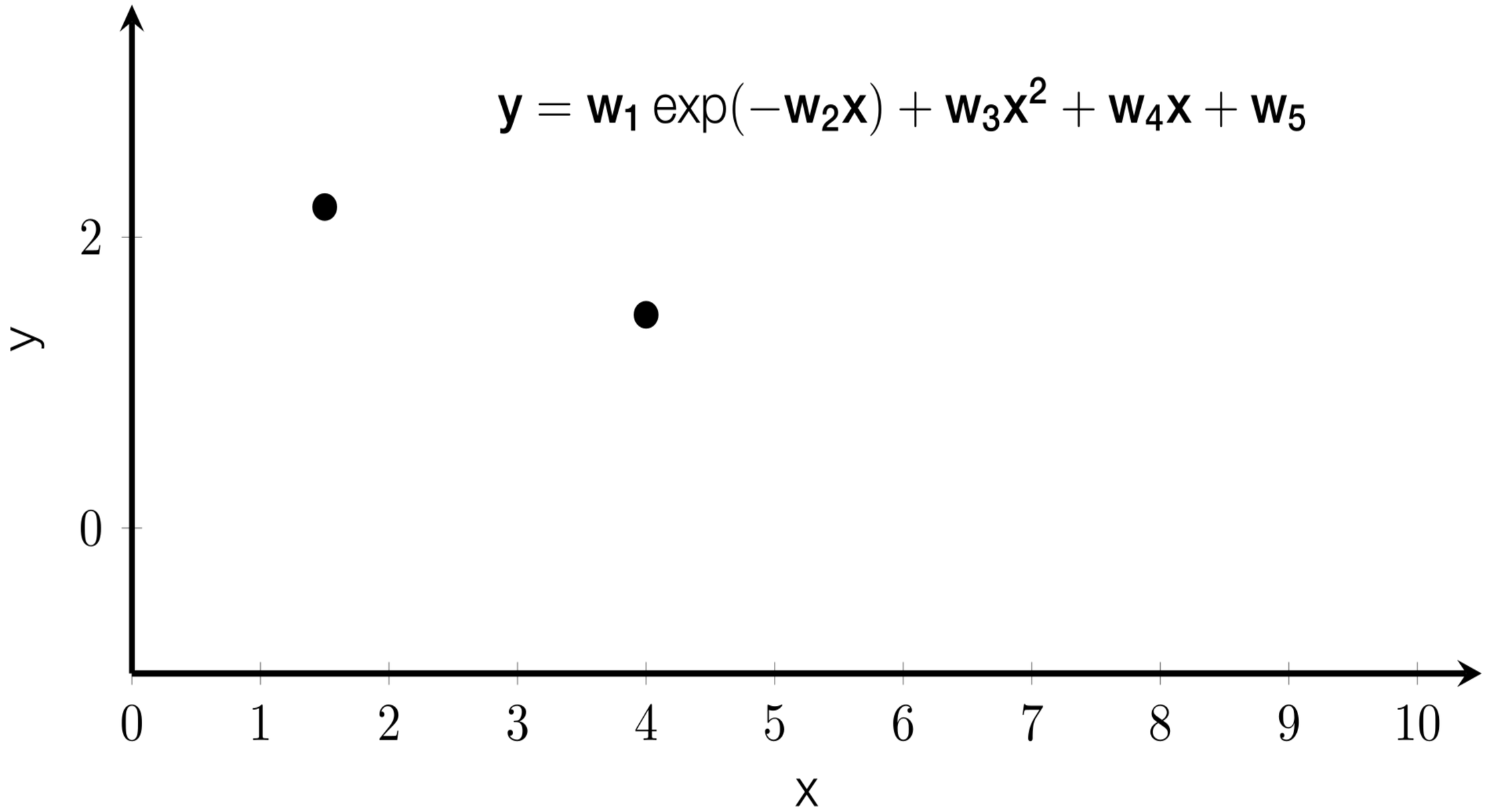
→ explicit inductive bias

Previous slide.

Why are convolutional networks much better than other approaches on these tasks? Because they have the right “inductive bias”.

Before we actually start looking at convolutional networks, we will illustrate the term “inductive bias” with a toy example and look at the different ways to find good inductive biases.

review: No Free-lunch Theorem (weak inductive bias)



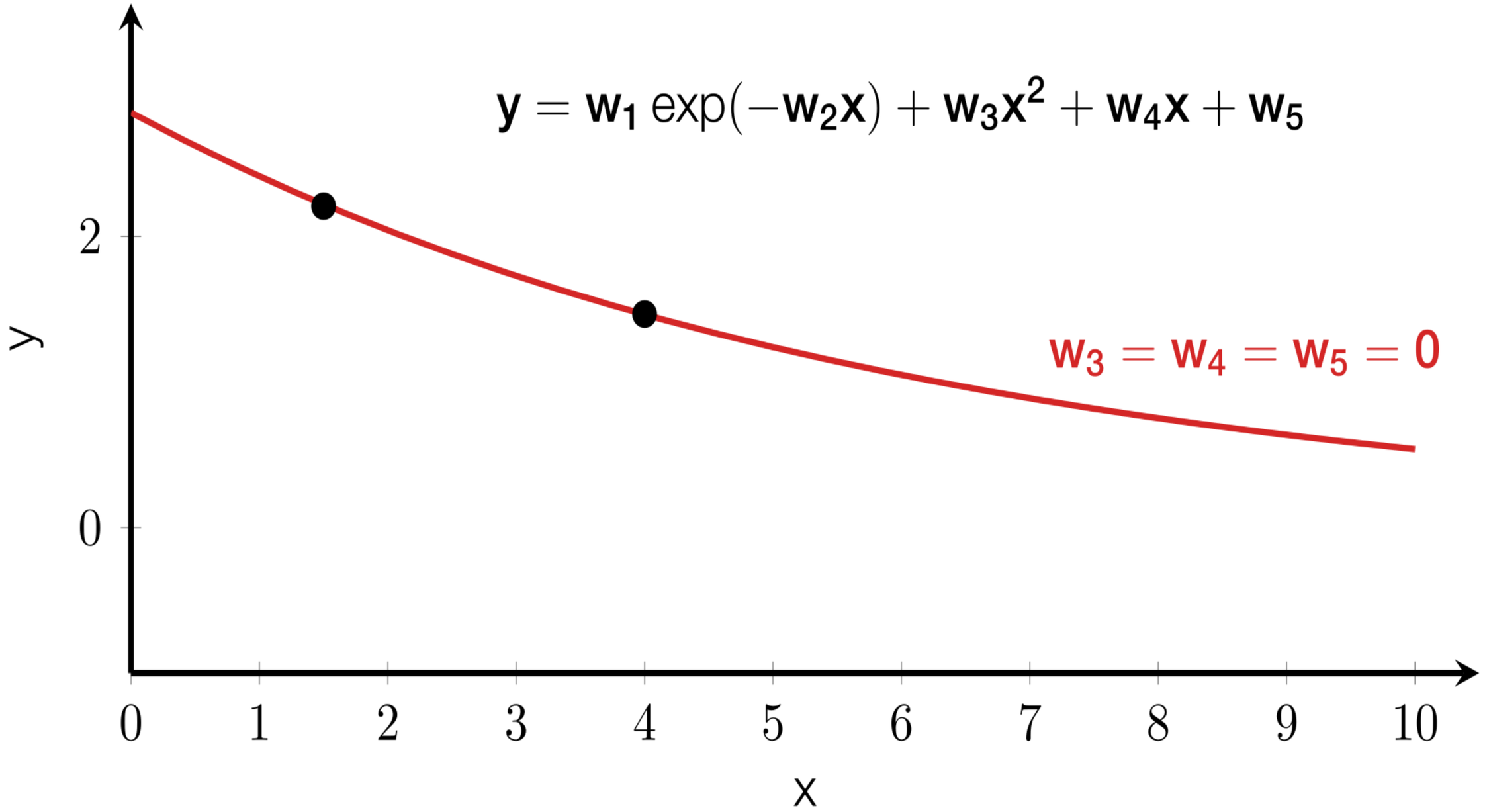
Previous slide.

Let us suppose we have two data points in our training set. It is a regression task with input x and real-valued target y; if it were a classification task, y would be discrete.

There are infinitely many possibilities to fit these two data points, but let us assume that our initial inductive bias is that x and y can be linked through the displayed formula.

The fitting problem still is under-constraint, since we have more parameters than data points; we still have have infinitely many possibilities to fit these two data points with the displayed formula.

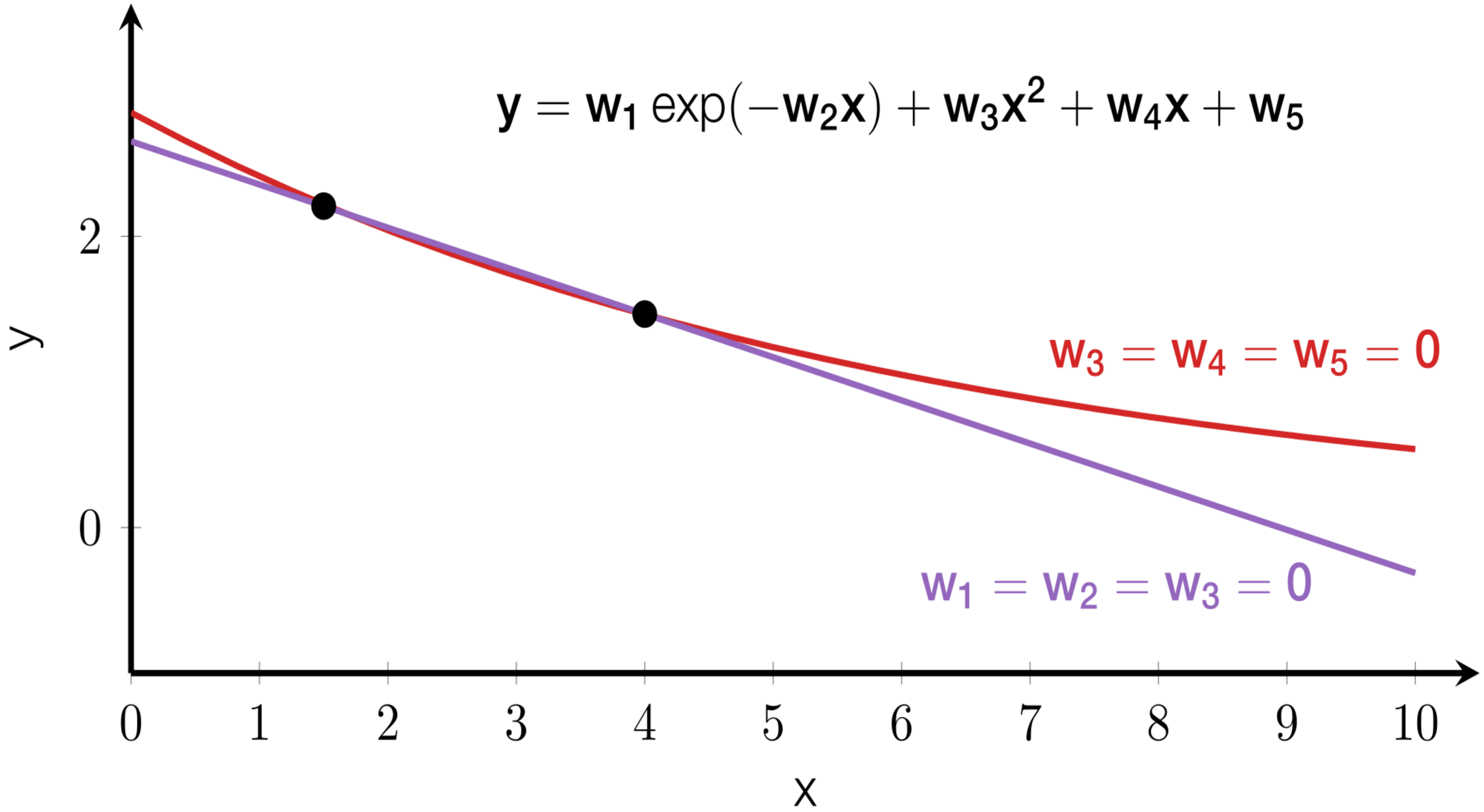
review: No Free-lunch Theorem (strong inductive bias 1)



Previous slide.

But let us assume we had reasons to consider even stronger inductive biases.
As a first example, exponential decay with $w_3 = w_4 = w_5 = 0$.

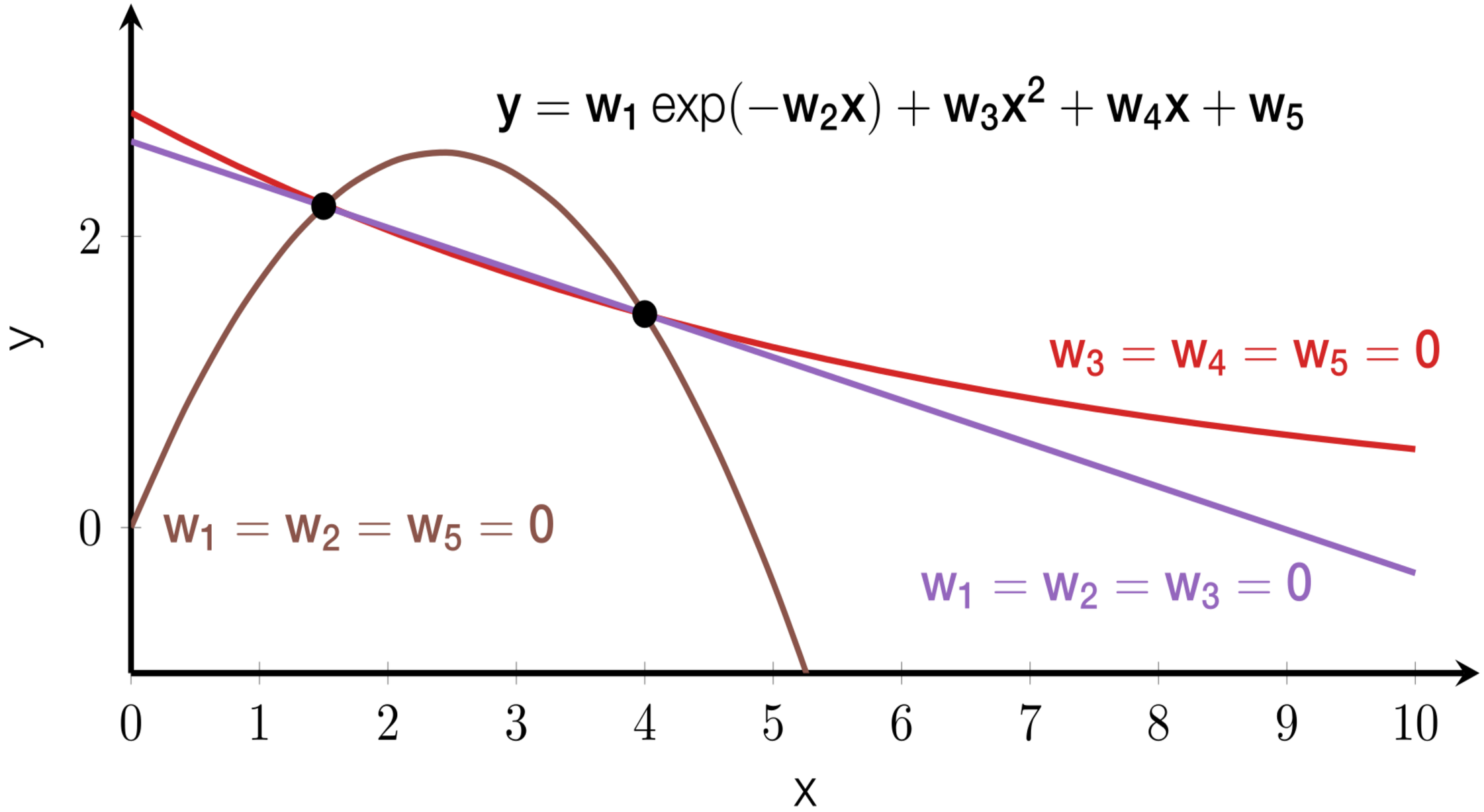
review: No Free-lunch Theorem (strong inductive bias 2)



Previous slide.

Second a straight line with $w_1 = w_2 = w_3 = 0$.

review: No Free-lunch Theorem (strong inductive bias 3)

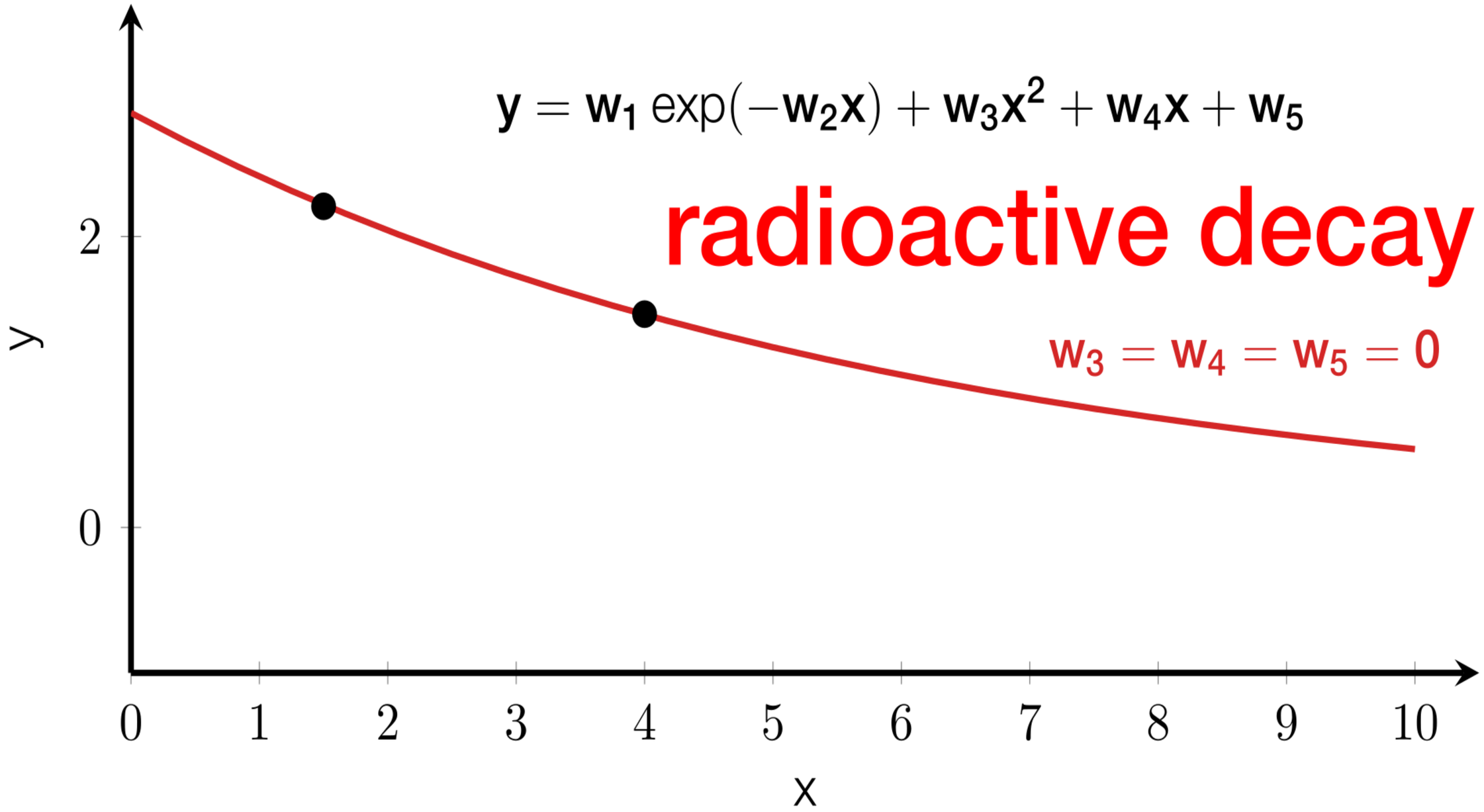


Previous slide.

And third, a parabola through the origin with $w_1 = w_2 = w_5 = 0$.
Any other way of fixing at least three parameters would be an valid alternative.

How should we choose between these inductive biases?

review: No Free-lunch Theorem (transfered inductive bias)



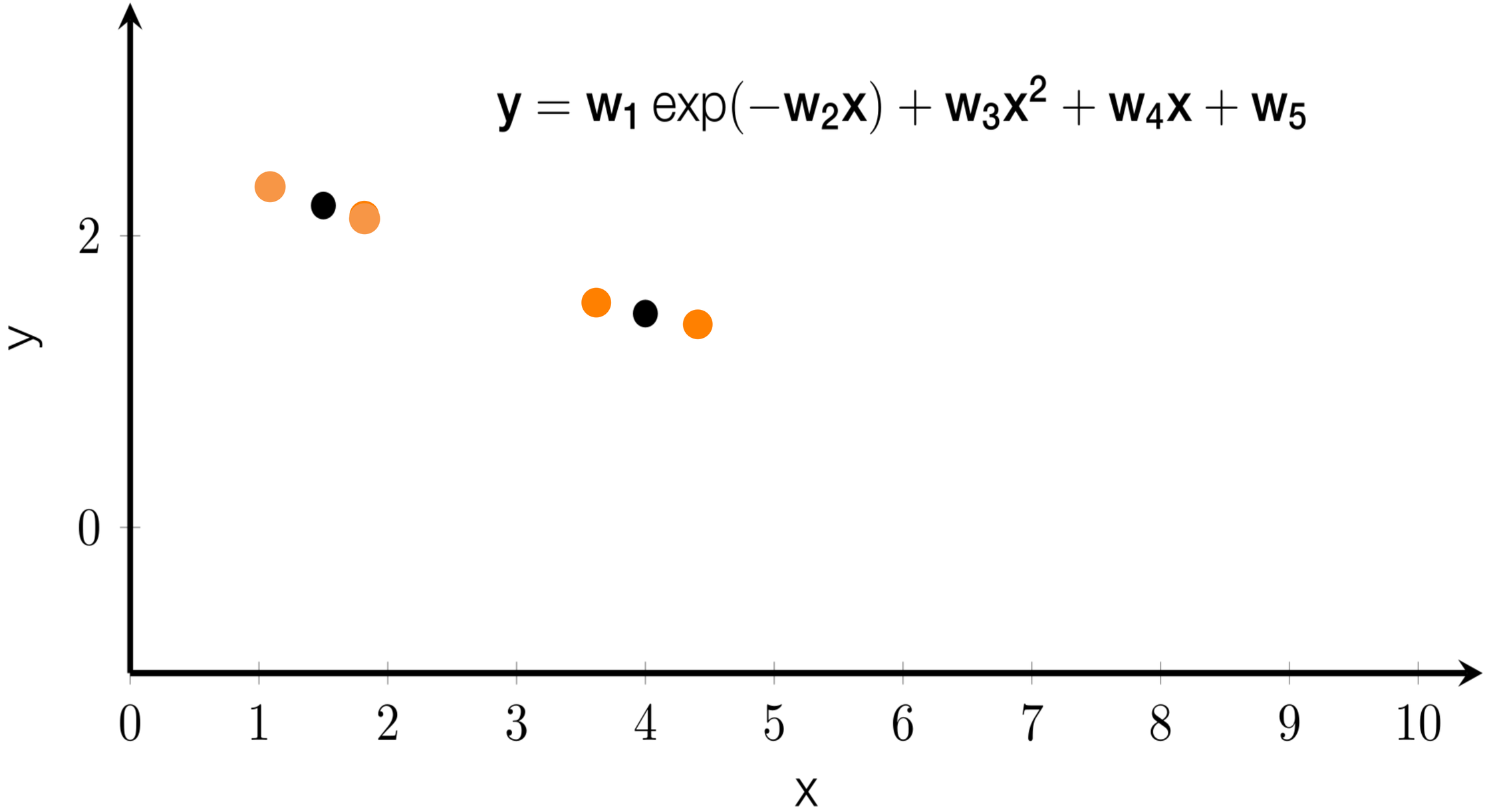
Previous slide.

If we have more information about the data, e.g. it comes from measurements of radioactive decay, we can transfer our knowledge about this type of problem, select a strong inductive bias and fit the other parameters.

This transfer can happen in different ways.

1. (The data scientist approach) We have already several times fitted all parameters w_1 to w_5 to similar data and we have always observed that w_3 to w_5 were close to 0.
2. (The physicist approach) There is some law that dictates a certain form of the function.

review: No Free-lunch Theorem (data augmentation)



Previous slide.

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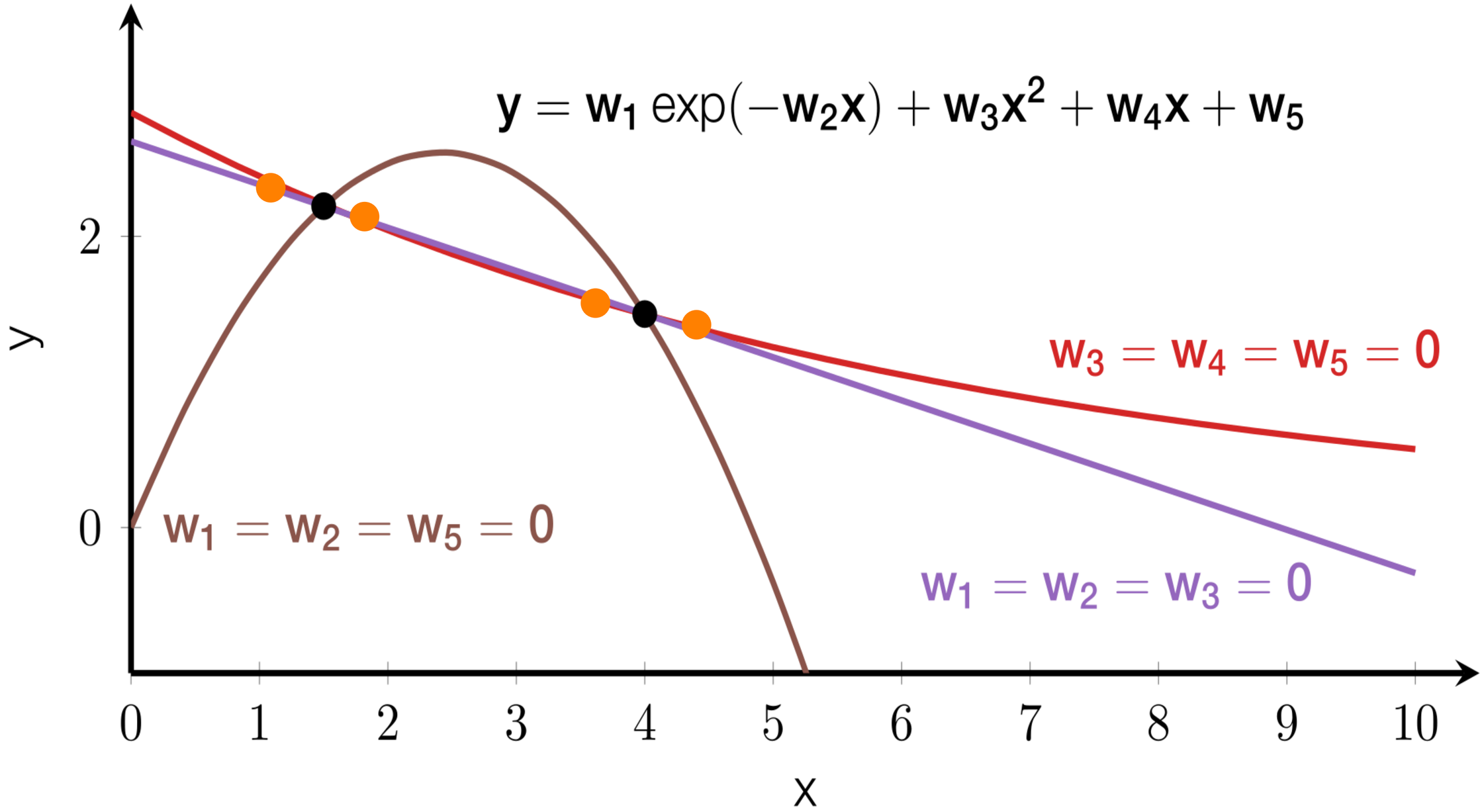
review: No Free-lunch Theorem (data augmentation)



Previous slide.

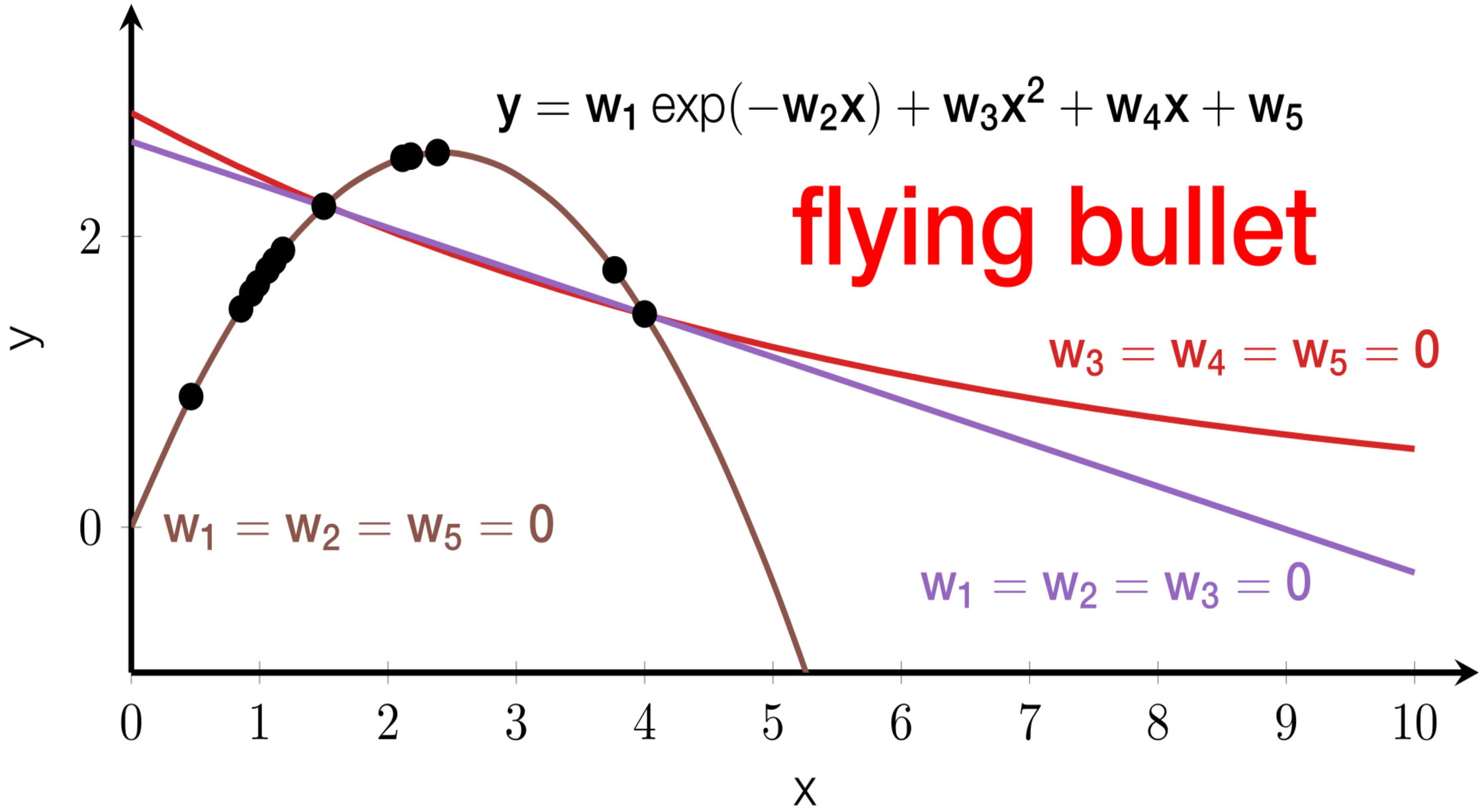
We may not know much about the data or find it difficult to define an explicit inductive bias but we have the intuition that the outcome should not change much if we transform the input in a certain way, e.g. if we would move the data points in the training set a bit to the left or to the right the outcome y should not be very different, as indicated with the orange data points.

With this augmented dataset we can fit all parameters w_1 to w_5.

Caveat: It may be difficult to find transformations that really leave the outcome invariant; getting more actual training data may be more worthwhile than data augmentation.

Thanks to data augmentation, or transfer from related problems we may be able to find a fit to only two data points that generalizes well if the data actually came from a measurement of radioactive decay.

review: No Free-lunch Theorem (the wrong inductive bias)



Previous slide.

Even though the training error is zero for all three strong inductive biases considered here and we have the same degrees of freedom in each case (namely two parameters to be fitted) the performance on the test set can be terrible, if we pick the wrong inductive bias for the data at hand. If the data came from the measurement of the trajectory of a flying bullet the fit with the exponential decay or the straight line would not generalize well.
In other words, we don’t get good generalization for free; there is no free lunch here. Only if we choose the inductive bias that matches the data we get good generalization.

Inductive bias

Induction = finding a rule (function) from specific examples
Inductive bias = prior preference for specific rules (functions)

1) Explicit inductive bias (transfer reasoning)

"For radioactive decay I know that $w_3 = w_4 = w_5 = 0$"

2) Inductive bias through transfer learning

"I train first different models on data from other radioactive elements and choose the best for my current case"

3) Inductive bias through data augmentation

"For radioactive decay neighboring points have similar values"

Quiz: Inductive bias

[ ] With a strong (and correct) inductive bias, I can reach a low test error with very little training data.

[ ] With a strong inductive bias the test error will always be low.

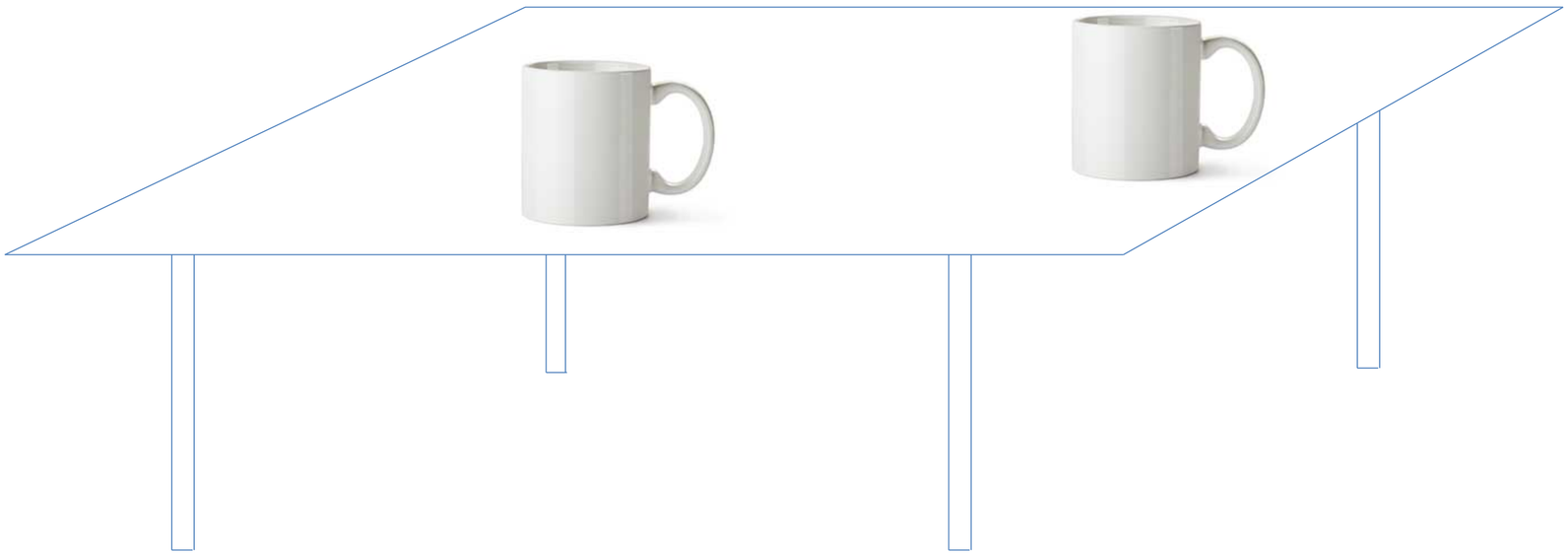
[ ] Data augmentation is a heuristic method to get more training data.

[ ] In data augmentation there is an inductive bias in the form of our assumptions about reasonable transformations to be applied to the data

[ ] Choosing a specific neural network architecture is equivalent to choosing an explicit inductive bias.

Aim of the lecture:

Convolutional networks provide an excellent inductive bias for image recognition: object invariance to (local) translation



Translation invariance: a mug is a mug, wherever it is placed on the table.

The aim of convolutional networks is to exploit this inductive bias.

→ **Inductive bias for images**

Reading for this lecture:

Goodfellow et al.,2016 *Deep Learning*

- Ch **9**

Further (optional) reading/watching for this lecture:

K. Fukushima, S. Miyake, **Neocognitron: a new algorithm for pattern recognition tolerant of deformations and shifts in position** Pattern Recognition, 15 (6) (1982), pp. 455-469

Geoff Hinton, Q&A https://www.reddit.com/r/MachineLearning/comments/2lmo0l/ama_geoffrey_hinton/clyj4jv/

Springenberg et al. , Striving for simplicity: the all convolutional net https://arxiv.org/abs/1412.6806

Patel, Nguyen, Baraniuk, A probabilistic framework for deep learning, 30th Conference on Neural Information Processing Systems (NIPS 2016), Barcelona, Spain. http://papers.nips.cc/paper/6231-a-probabilistic-framework-for-deep-learning

See also references in the slides as well as
Lectures by Andrew Ng

Reading: as usual the textbook on Deep Learning.

The viewpoint taken in this lecture puts more emphasis on the inductive bias.

Some of the points are controversial

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Part 2: Convolutional filters as inductive bias for images

1. Inductive bias in machine learning
2. Convolution filters as inductive bias for images

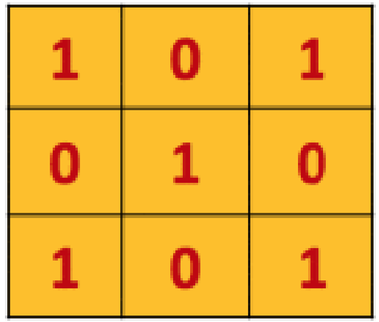
Previous slide.

For tasks involving natural images, like object recognition, we have now strong empirical evidence, that neural networks with convolutional layers work better than networks with only dense layers (all-to-all connectivity).

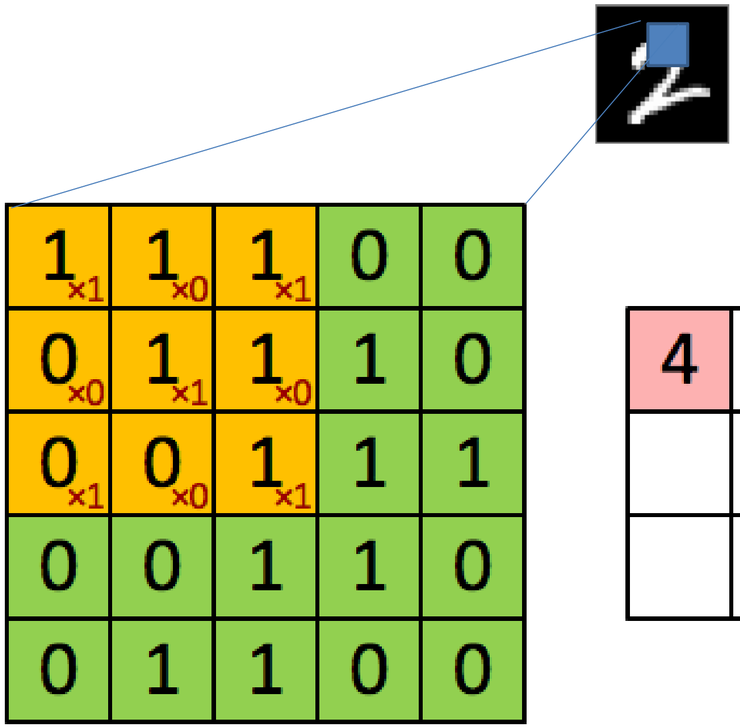
Thus we may conclude that convolutional layers provide a better **explicit inductive bias**

We will start now with a recap of convolution.

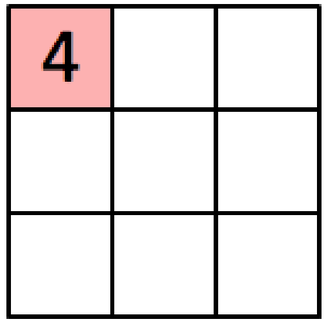
Convolution: one feature



Filter (or Feature)



Image



Convolved Feature

w_{xy}

I_{xy}

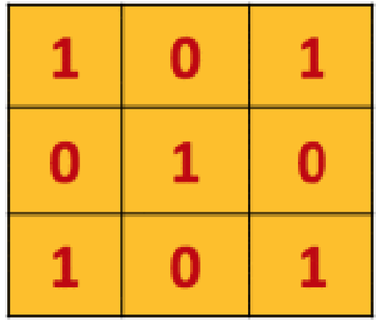
$$a_{ij} = b + \sum_{x=1}^{3} \sum_{y=1}^{3} I_{i+x-1,j+y-1} w_{xy}$$

http://deeplearning.stanford.edu/wiki/index.php/Feature_extraction_using_convolution

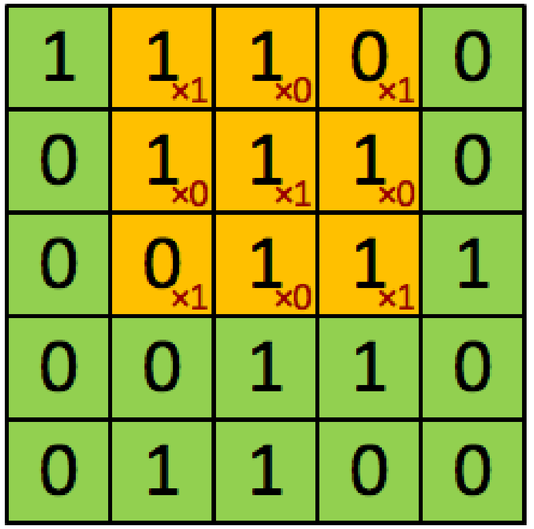
Previous slide.

On the left we have a 3x3 filter. Applying it to the top-left corner of the 5x5 image in the middle means multiplying each filter weight with the corresponding pixel value and summing the results to get the activation $a_{11} = 4$ for bias $b = 0$.

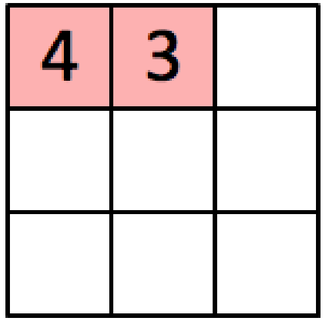
Convolution: one feature



Filter (or Feature)



Image



Convolved Feature

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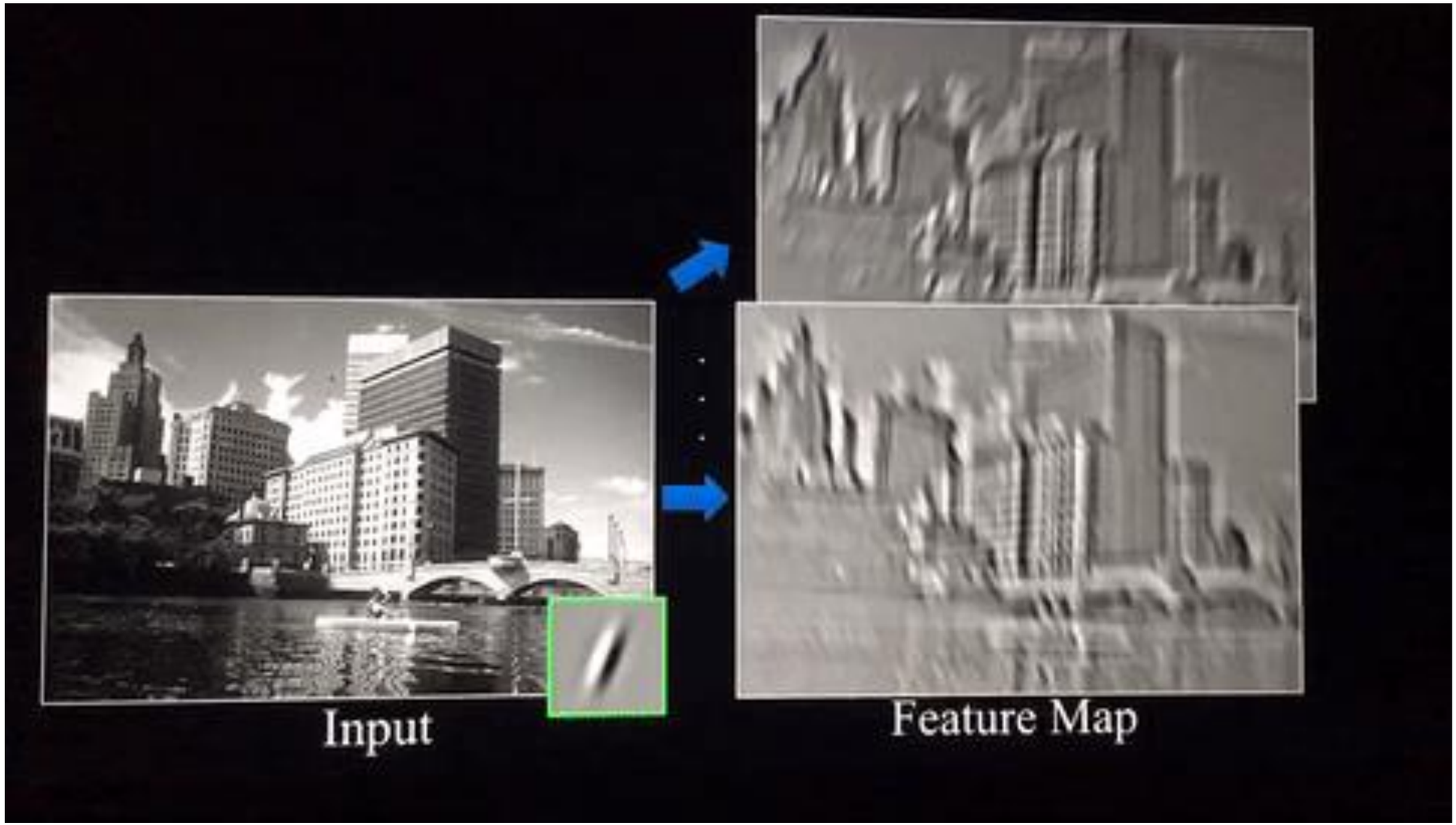
http://deeplearning.stanford.edu/wiki/index.php/Feature_extraction_using_convolution

Previous slide.

Next we move the filter over the image and repeat the multiplication and summation at each position to get all values of the convolved feature matrix on the right.

Note that in contrast to a “standard neuron” that would take the full image as input and compute one activation value, we get multiple activation values with a “convolutional neuron” (filter): there is one value for each position at which the filter (neuron) is applied.

Convolution: multiple features



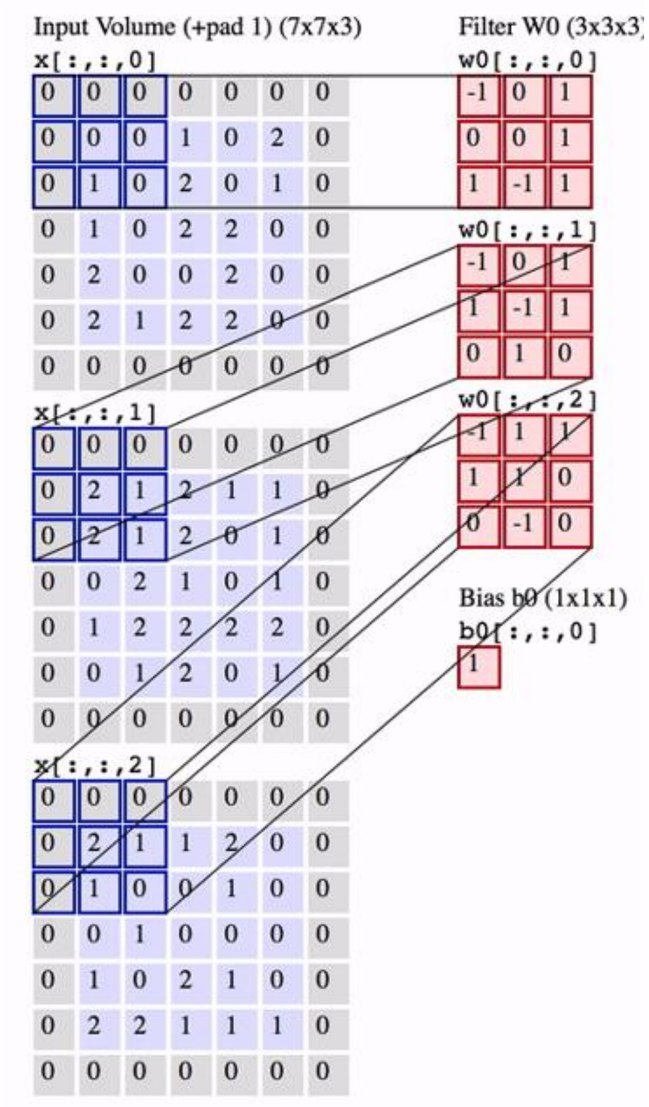
https://ujjwalkarn.me/2016/08/11/intuitive-explanation-convnets/

Previous slide.

For multiple filters we get multiple convolved features, also called feature maps. Note how the different simple filters in this example extract edges of different orientation.

One convolutional layer is usually composed of multiple filters. Since each feature map is two-dimensional we can think of the output of a convolutional layer as a three-dimensional object. We will discuss this in more details on the next-but-one slide.

Convolution: colors and padding



- 3 color channels

- Filter 3x3 (spatial) in each color dimension
  → filter tensor W0 denoted as (3x3x3)

- Apply several filters: W0, W1, W2, W3, …

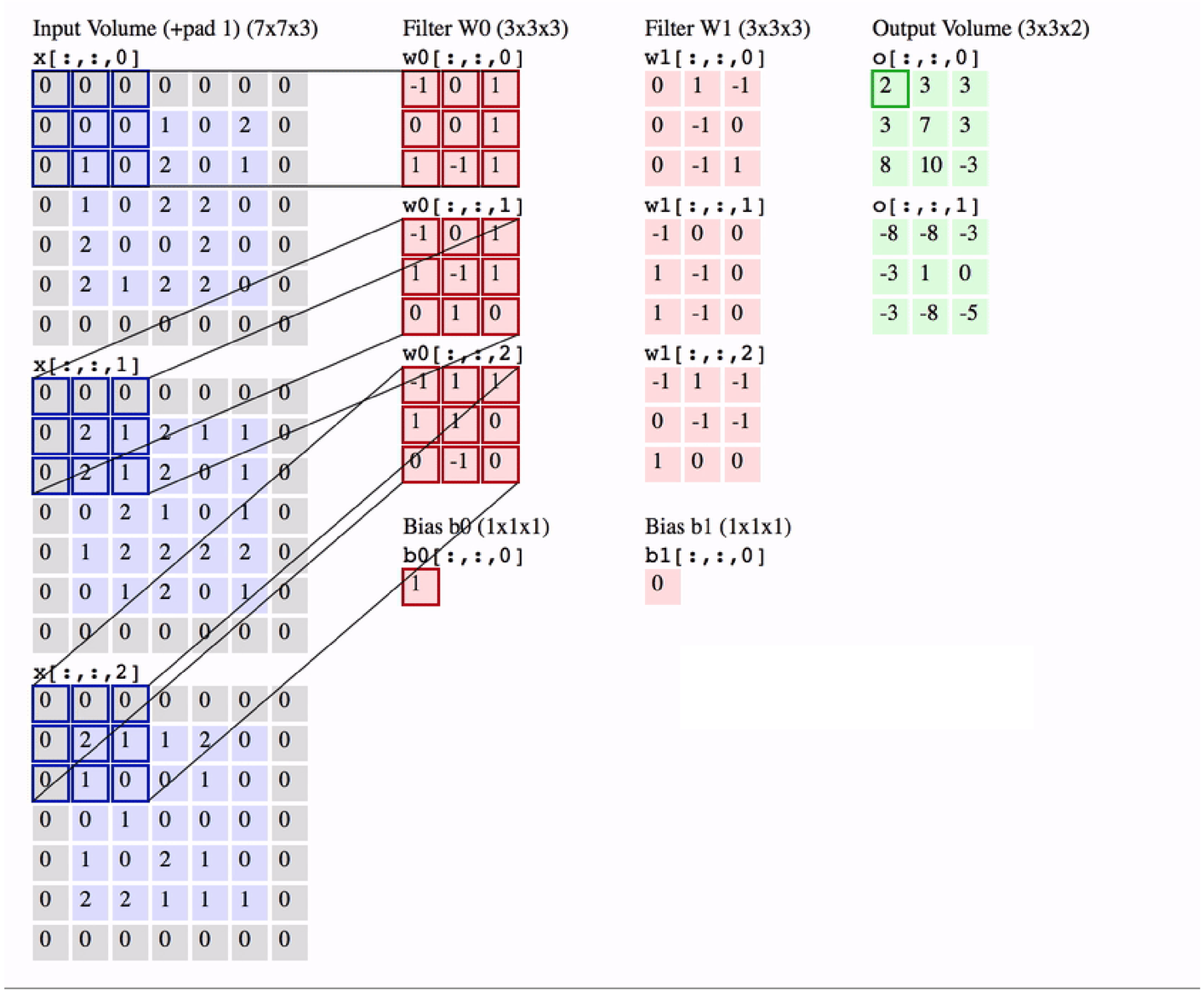
- Padding: extend images at borders with 0

Previous slide.

For color images, we have three color channels.
Therefore, the filter also works (potentially differently!) in each color domain.
A filter that has a spatial 3x3 layout becomes in total a filter with 3x3x3=27 parameters.

Usually we apply several filters of form (nxnx3).

Convolution: stride



- Stride 2

filters jump 2 pixels at a time as we convolve

http://cs231n.github.io/convolutional-networks/

Previous slide.
We can apply each filter starting at the top left-corner and moving it column by column, row by row over the image. This is known as convolution with padding 0 and stride 1.
Here we consider two generalizations:

First, if filters are applied only at every n’th position, we speak of stride n. The example was done with stride 2.

Second, if one wants to position the center of a filter to the top-left pixel of an image, this can easily be achieved with padding the original image, i.e. appending rows and columns of zeros to the original image.

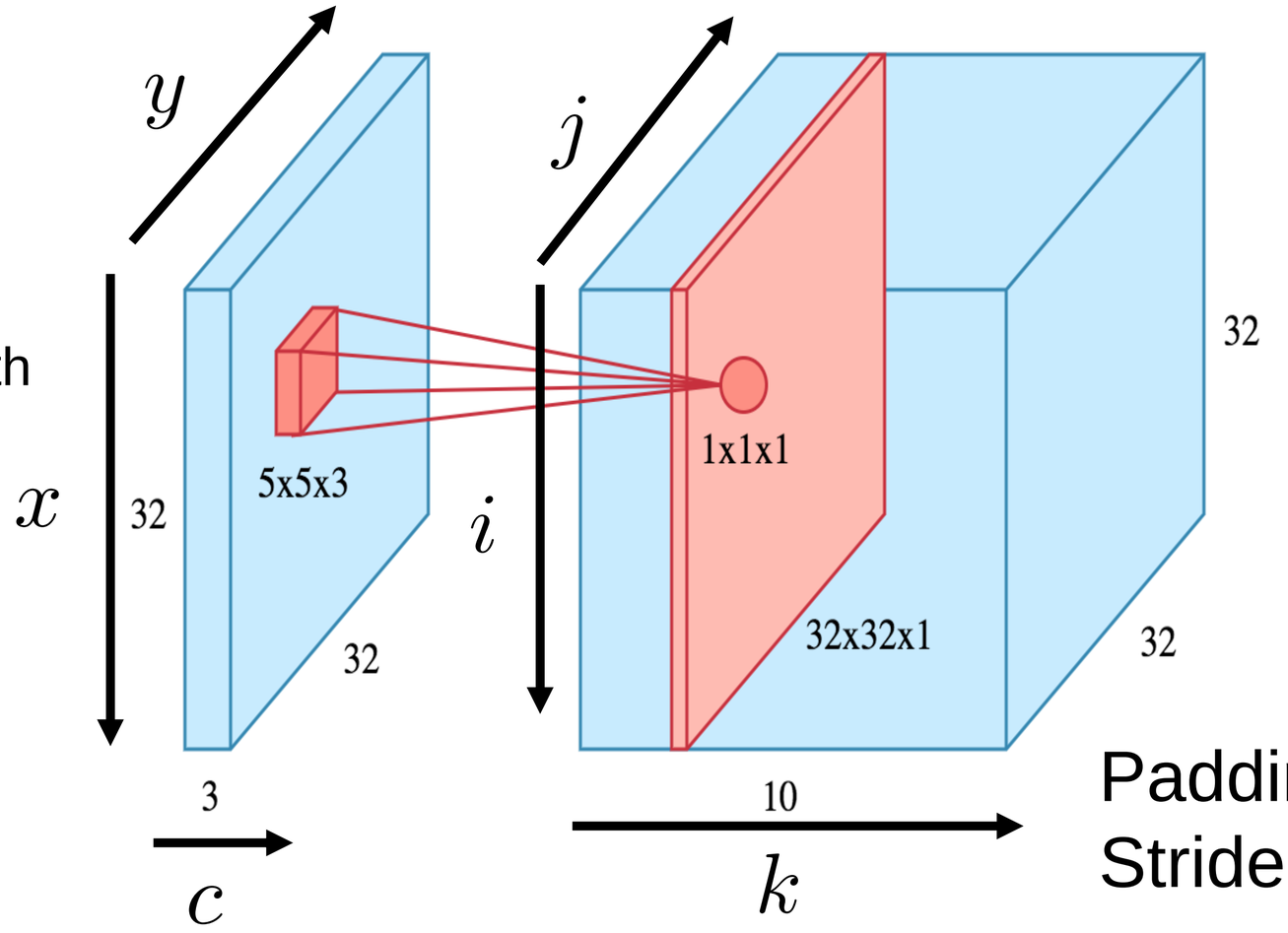
The green boxes show the result of the application of two filters (each one applied at 9 different positions, stride 2.).

Of course, stride and padding can in general be different in x and y direction.

Note, that using a stride > 1 results in a reduction of the x-y dimensions of the feature map, but if we would use stride 1 in this example, the input without padding and the output would have the same x-y dimensions (5x5) thanks to padding.

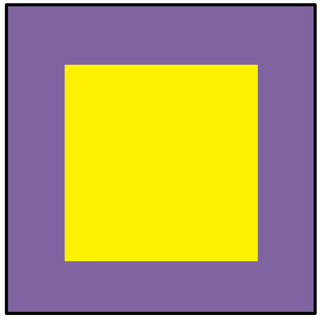
Convolution: one layer

color image
width x height x depth
32 x 32 x 3



filters, e.g

w_{xyc1}



w_{xyc2}

Padding 2
Stride 1

I_{xyc}

$$a_{ijk} = b_k + \sum_{x=1}^{5}\sum_{y=1}^{5}\sum_{c=1}^{3} I_{i+x-1,j+y-1,c} w_{xyck}$$

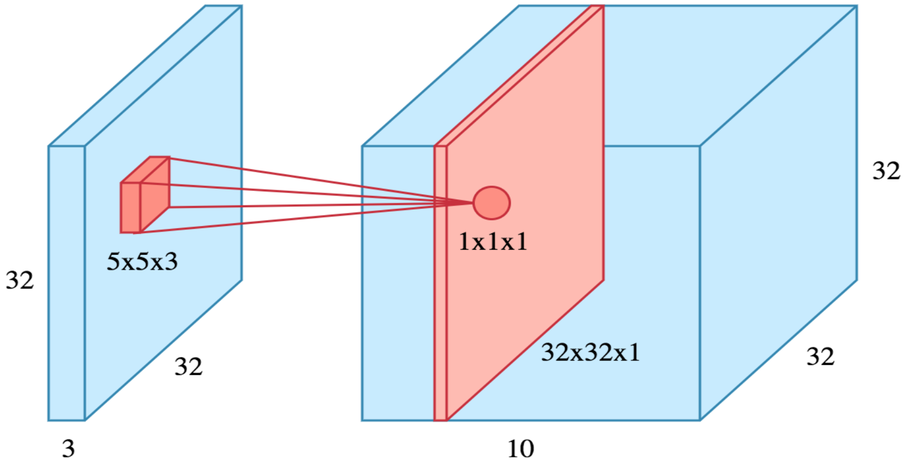
https://towardsdatascience.com/applied-deep-learning-part-4-convolutional-neural-networks-584bc134c1e2

Previous slide.

On the left we have a color image of 32x32 pixels with 3 color channels. We represent it as a volume of dimensions 32x32x3. If we convolve it with 10 filters of size 5x5x3 with padding 2 and stride 1 we get a volume of dimensions 32x32x10. One of those 10 filters could be a detector for blue crosses or for vertical yellow bars. (In case there is a single filter the volume would be 32x32x1, as indicated on the left).

What would be the output volume if we used padding 0 instead of padding 2? 28x28x10
What would it be if we used stride 2 instead of stride 1 for padding 2? 16x16x10
What would it be if we used 24 filters instead of 10? 32x32x24

Quiz: Convolutions



With padding 2, stride 1 and 10 filters we get in the above configuration a volume of 32x32x10.

- [ ] With padding 0, stride 1 and 10 filters we will then get a smaller volume of 28x28x10
- [ ] With padding 0, stride 2 and 10 filters we will then get a smaller volume of 14x14x10
- [ ] With padding 0, stride 2 and 10 filters, the filter will be placed in all borders
- [ ]The volume is proportional to the number of filters.

Previous slide.

Convolution: computing the volumes

n x n x c input volume (e.g. image) k filters of size f x f x c

stride s
padding p

$$\left(\frac{n+2p-f}{s}+1\right)\times\left(\frac{n+2p-f}{s}+1\right)\times k$$

Note: combination of stride and filter size where the above formula does not give a natural number should be avoided, because corners are not reached.

Previous slide.

In general the size of the output volume can be computed with the above formula. You have to make sure that you choose a stride that (when starting in the left corner) brings you with a number of steps to the right corner (no fractions!).

If fractions appear: solution is biggest integer small than the calculated fraction.

Inductive bias of a convolutional layer

1) Independence to translation

   “A feature detector (such as a vertical edge detector) that’s useful in one part of the image is probably useful in another part of the image.”

2) local feature detectors are useful

NOTE: A conv layer is a special case of a dense layer with

- many neurons having the same weights → 1)
- many weights zero, except in a small neighborhood, →2)

Previous slide.

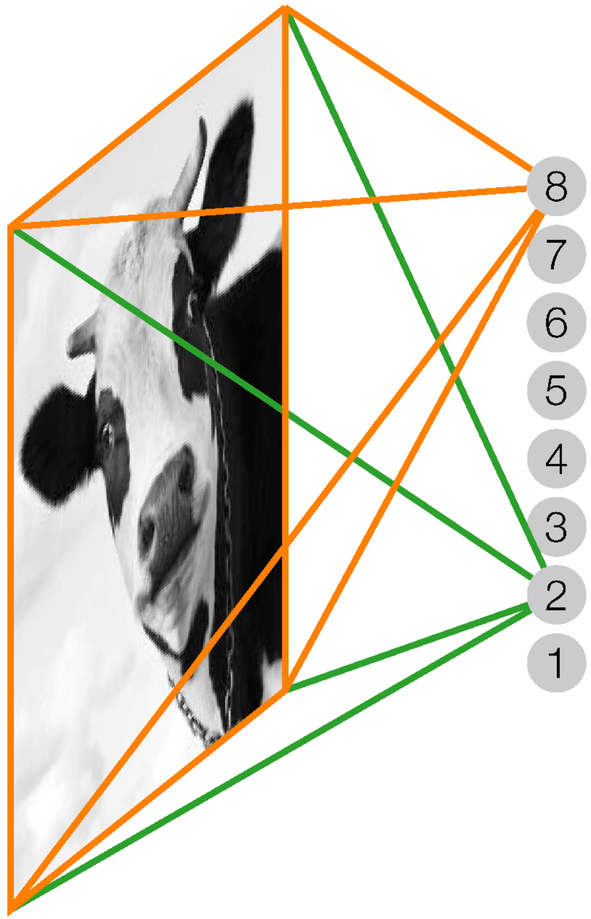
With a convolutional layer, even if the training set had a certain feature only in one part of the images – say, always in the upper part of the images – it will be detected in a test image also when it appears in another region.

Since the filters are typically much smaller than the full image, they are sensitive only to the configuration of pixels in a small neighborhood.

With dense layers one could achieve something similar to a convolutional layer with setting for each neuron all weights to zero except those in a small region of the input space and using data augmentation.

We will look at this in more details on the next slides.

From dense to convolutional layers

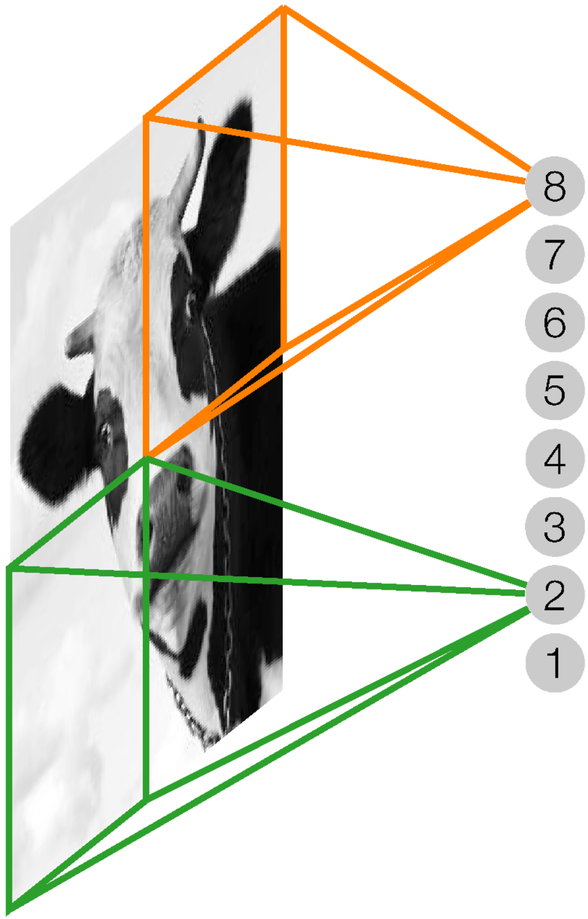


dense (all-to-all) connectivity

Previous slide.

Let us take the image on the left as input and a dense layer with 8 neurons that all connect to all pixels of the input image.

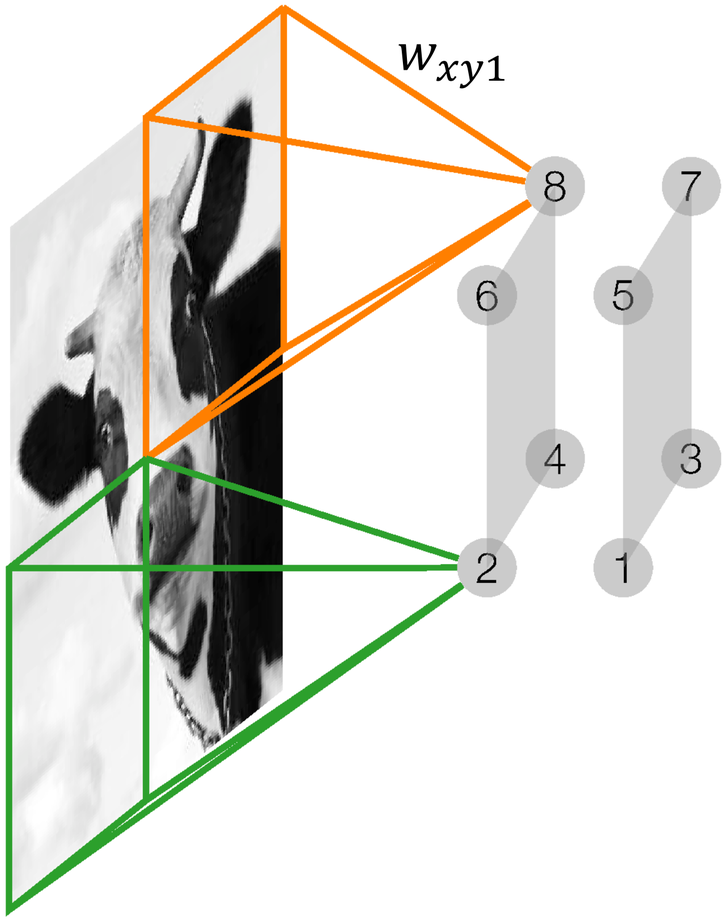
From dense to convolutional layers



Previous slide.

Now let us set most weights of these neurons to zero, such that neurons 1 and 2 only have non-zero weights to pixels in the bottom left quadrant of the image, neurons 7 and 8 only have non-zero weights to pixels in the top right quadrant etc.

From dense to convolutional layers



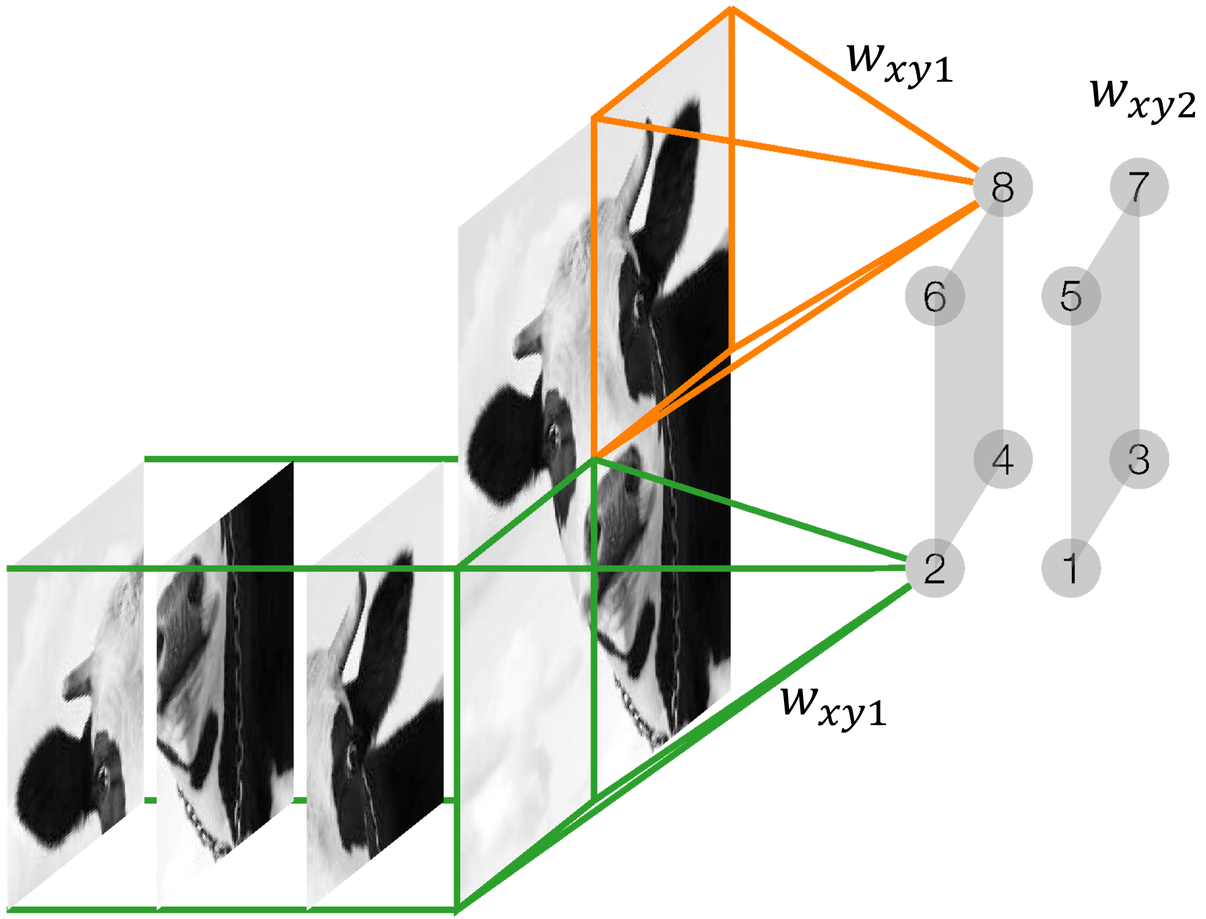
patchy connectivity

Previous slide.

Now we rearrange the neurons according to the region where they have non-zero input weights. This already resembles a lot the volumes we saw for convolutional networks.
We will call the left vertical plane the “even feature map” and the other one the “odd feature map”.

Let now all neurons in each feature map have exactly the same weight vector, e.g. the orange weight vector and the green weight vector are the same. Think of the green weight vector as the concatenation of the weights from each row of the patch.
But neurons in different feature maps shall have different weight vectors.

From dense to convolutional layers



patchy connectivity & data augmentation

Previous slide.

During training we now do data augmentation, such that each neuron sees each patch of the image equally often. Since the weight vectors of the neurons in each feature map were initialized to the same values and all neurons see the same training data, the weight vectors of all neurons in the same feature map will always remain the same.

You should realize now that we have total equivalence to a convolutional layer. But instead of keeping different neurons with the same weight vectors we replace all the neurons in one feature map with a single filter and instead of data augmentation, we move this filter over all patches of the image.

In the introductory example we discussed, how setting some weights to zero and doing data augmentation installs an inductive bias. Here we see how such an inductive bias can be absorbed in the architecture of the network.

Summary: Inductive bias of a convolutional layer

A convolutional layer is a special case of a dense layer with

- many neurons having the same weights
- many weights zero, except in a small neighborhood

→A normal deep network with dense connections should be able to learn the same tasks!

A convolutional layer brings two intuitions in the form of an inductive bias:

1) Independence to translation of filter

   "A feature detector that's useful in one part of the image is probably also useful in another part of the image."

2) Local features are useful to understand images

Previous slide.
Your notes.

Artificial Neural Networks

Convolutional Neural Networks

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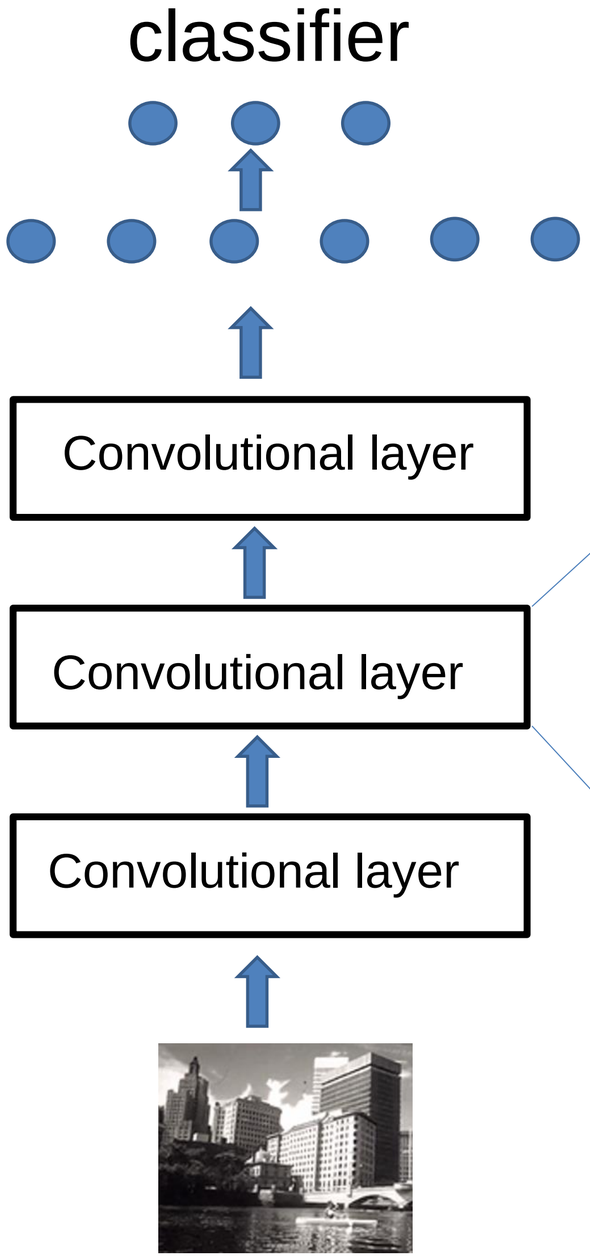
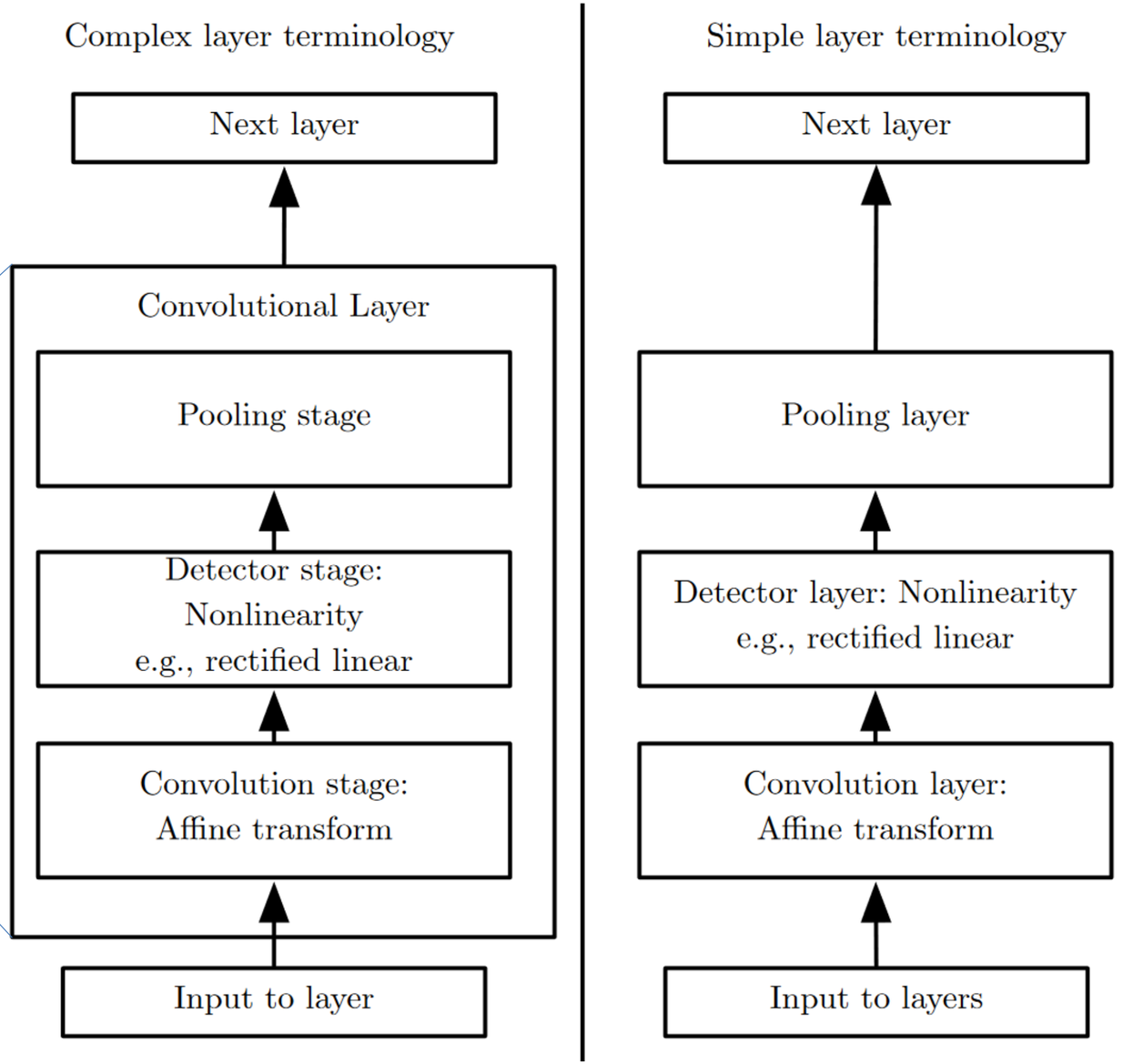
Part 3: MaxPooling as inductive bias for images

1. Inductive bias in machine learning
2. Convolution filters as inductive bias for images
3. MaxPooling as inductive bias for images

Previous slide.
After the convolutional layer follows a max-pooling layer.

Components of a typical conv-net layer



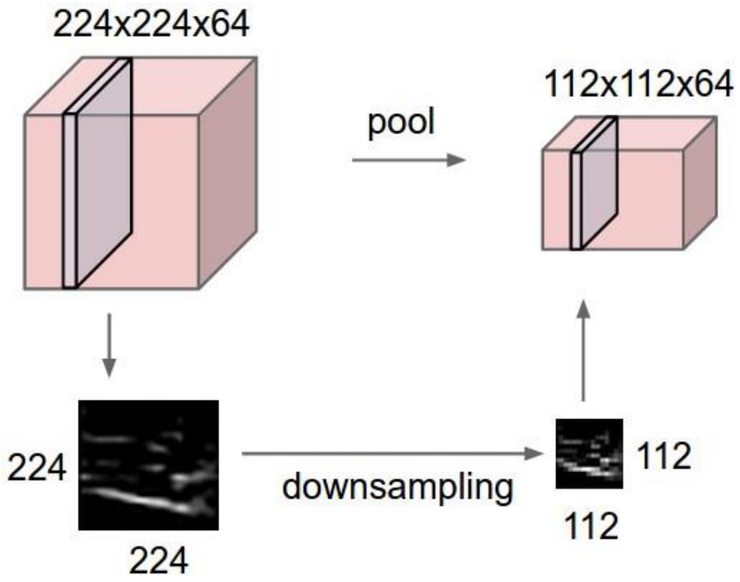


Goodfellow et al. Chapter 9.3

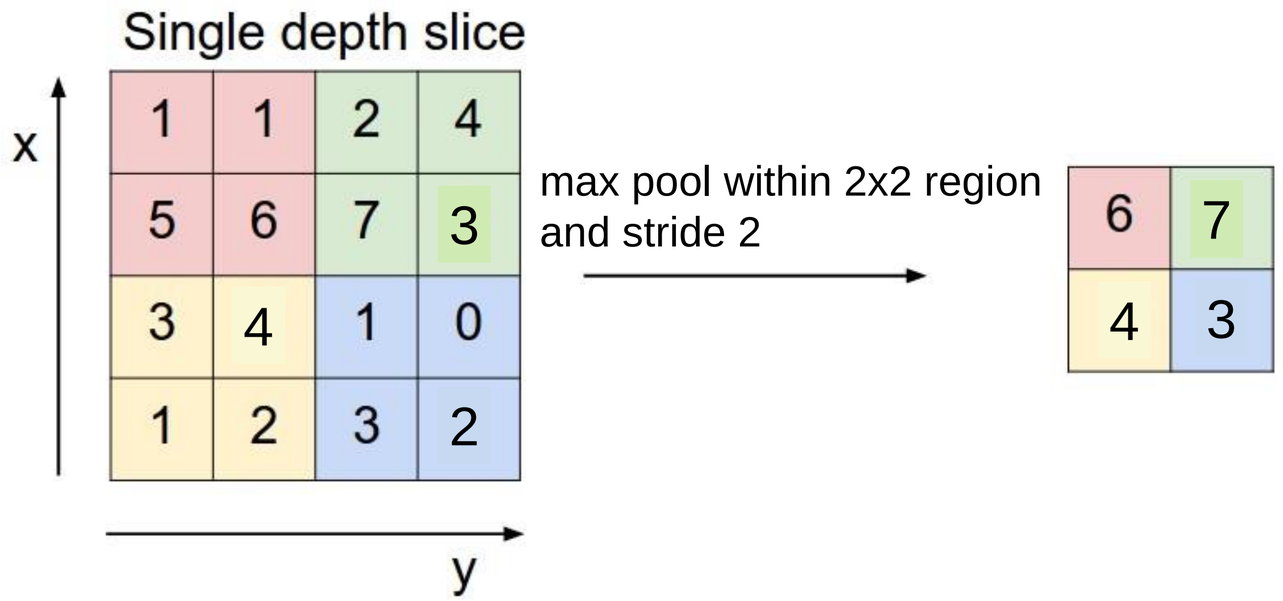
Previous slide.

Nevertheless, pooling was so popular that even in textbooks the definition of convolutional layers sometimes includes a pooling stage, as in the complex layer terminology on the left, where one layer consists of a convolution, nonlinearity and a pooling stage.

Pooling stage: Max Pooling



Response of filter No 17:



Previous slide.

Let us move on to a second component that is often used in convolutional networks: max pooling. Any kind of pooling reduces the x-y dimension of a volume; by a factor two in each dimension in the example on the left.

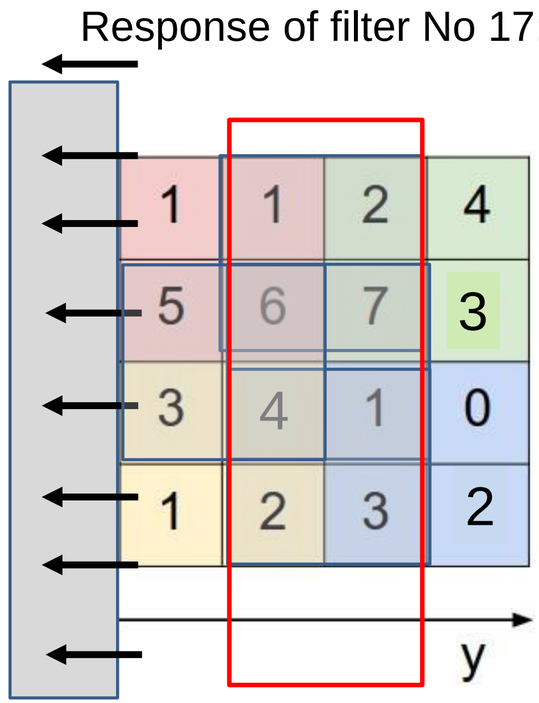
A max-pooling layer with a filter of size 2x2 and stride 2 picks the maximal number in groups of 4 neighboring pixels, as illustrated on the right.

If we move the input image on the right one pixel to the left and pad with zeros, the red, green and blue output would remain the same and the yellow output would switch from 3 to 2. A max-pooling layer thus implements an inductive bias that small translations should not have a large effect on the output of the neural network. Note, however, that the outcome changes quite a bit, if instead one moves the input one pixel to the right.

Instead of using the maximum, people sometimes use also the median or the mean.

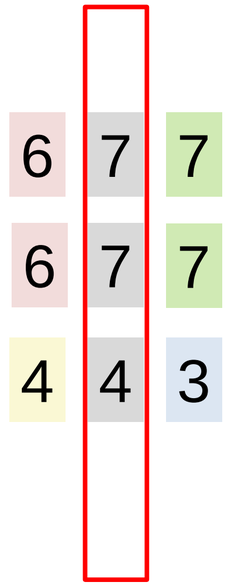
Reducing the size of the output volume may be desirable from the perspective of computational resources. Pooling is however only one way to achieve this: increasing the stride in a convolutional layer achieves the same.

Max Pooling: stride one

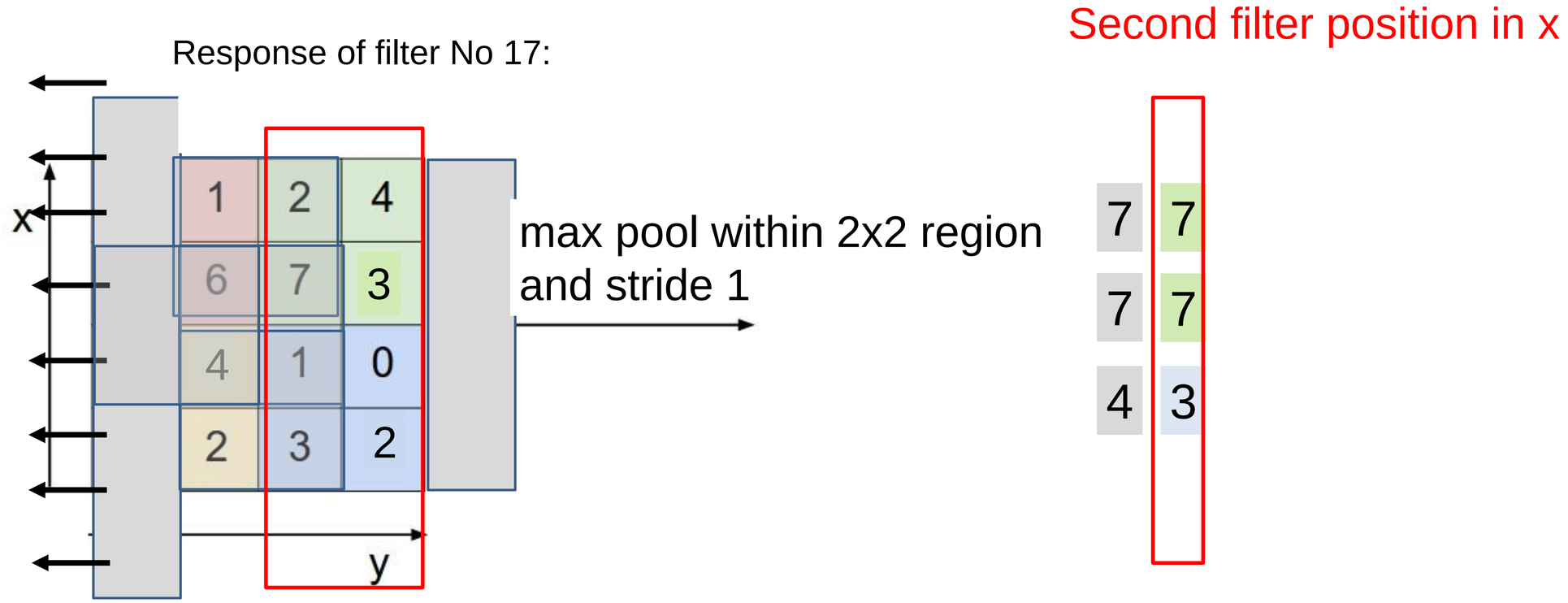


max pool within 2x2
and stride 1

Second filter position in x



Max Pooling: stride one, small shift of image



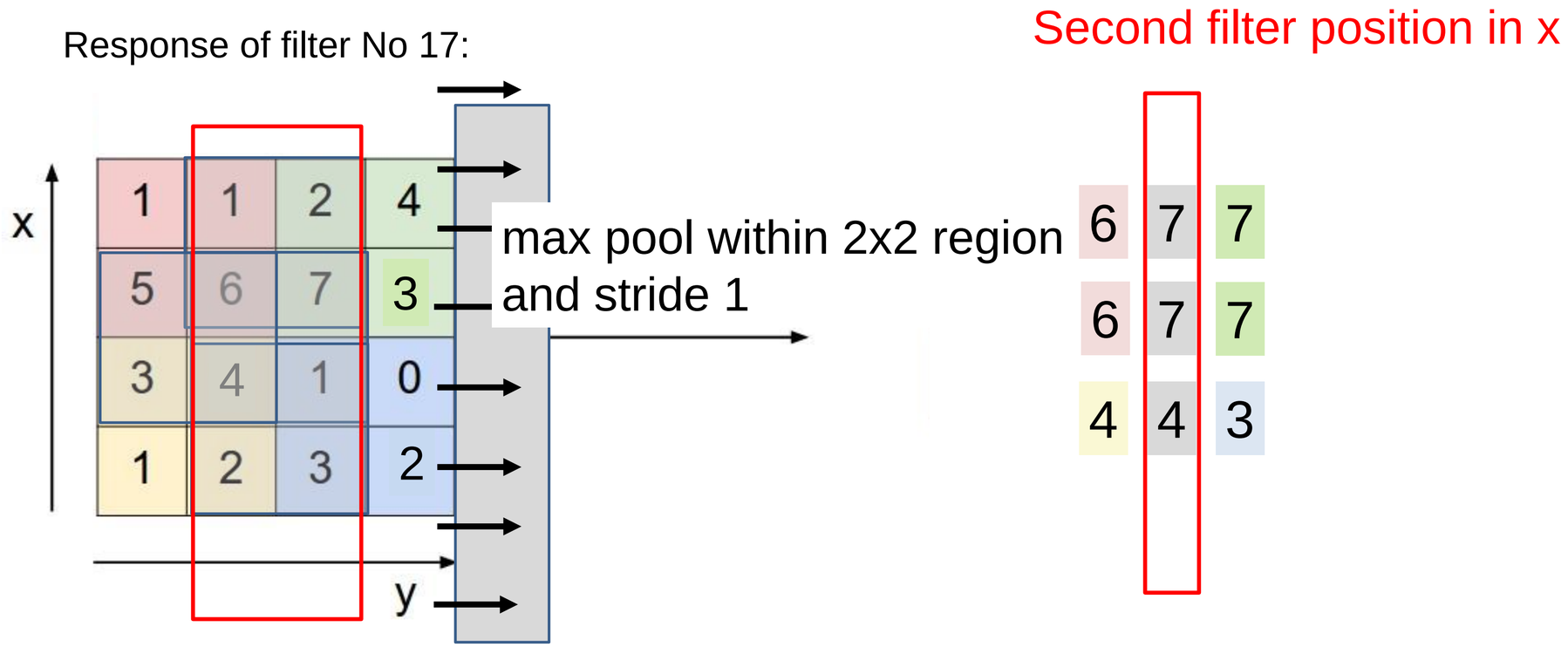
Inductive bias of max pooling

Invariance to small translations with some probability

f(x) is invariant to local translation T if $f(T(x)) = f(x)$

“quite a few activations remain the same when wiggling the image”

Max Pooling: stride one, small shift of image



Inductive bias of max pooling

Invariance to small translations with some probability

f(x) is invariant to local translation T if $f(T(x)) = f(x)$

“quite a few activations remain the same when wiggling the image”

Previous slide.

The invariance to small translations is best implemented with a stride of one in the pooling layer

If we move the input image one step to the left and pad with zeros, quite a few responses remain identical (see the responses in the second column, highlighted by red rectangle).

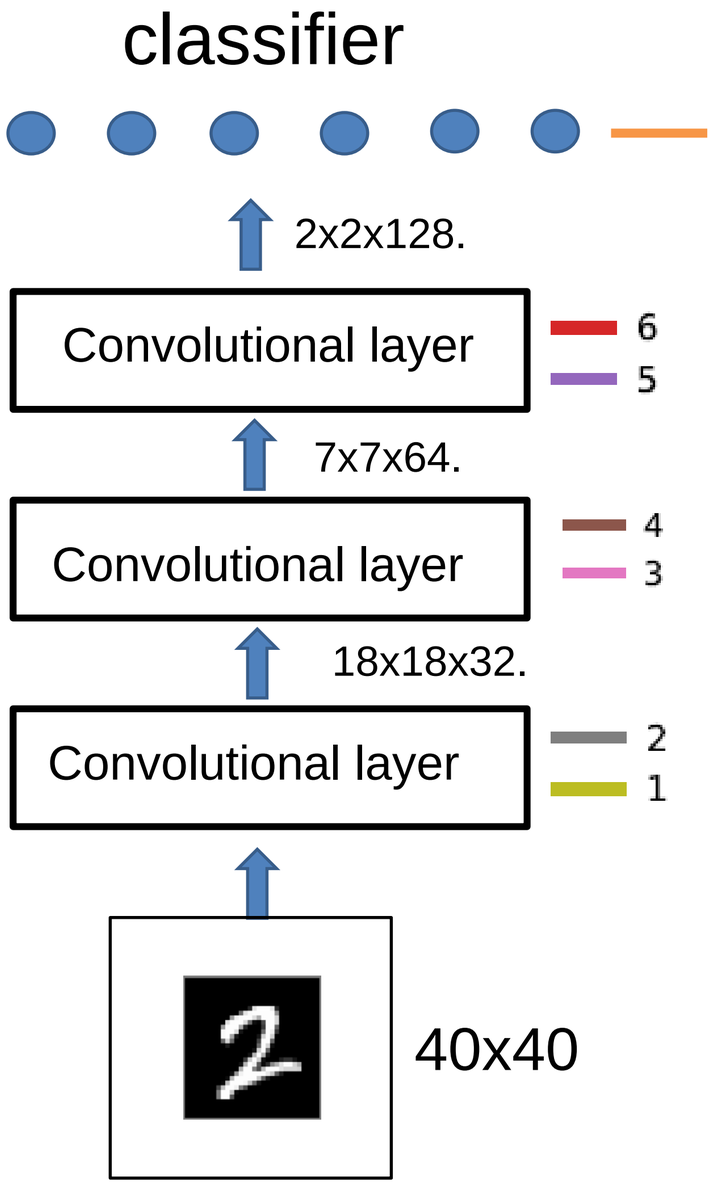
The one that changes (at the bottom of the second column) is in turn stable if we shift the image one step to the right.

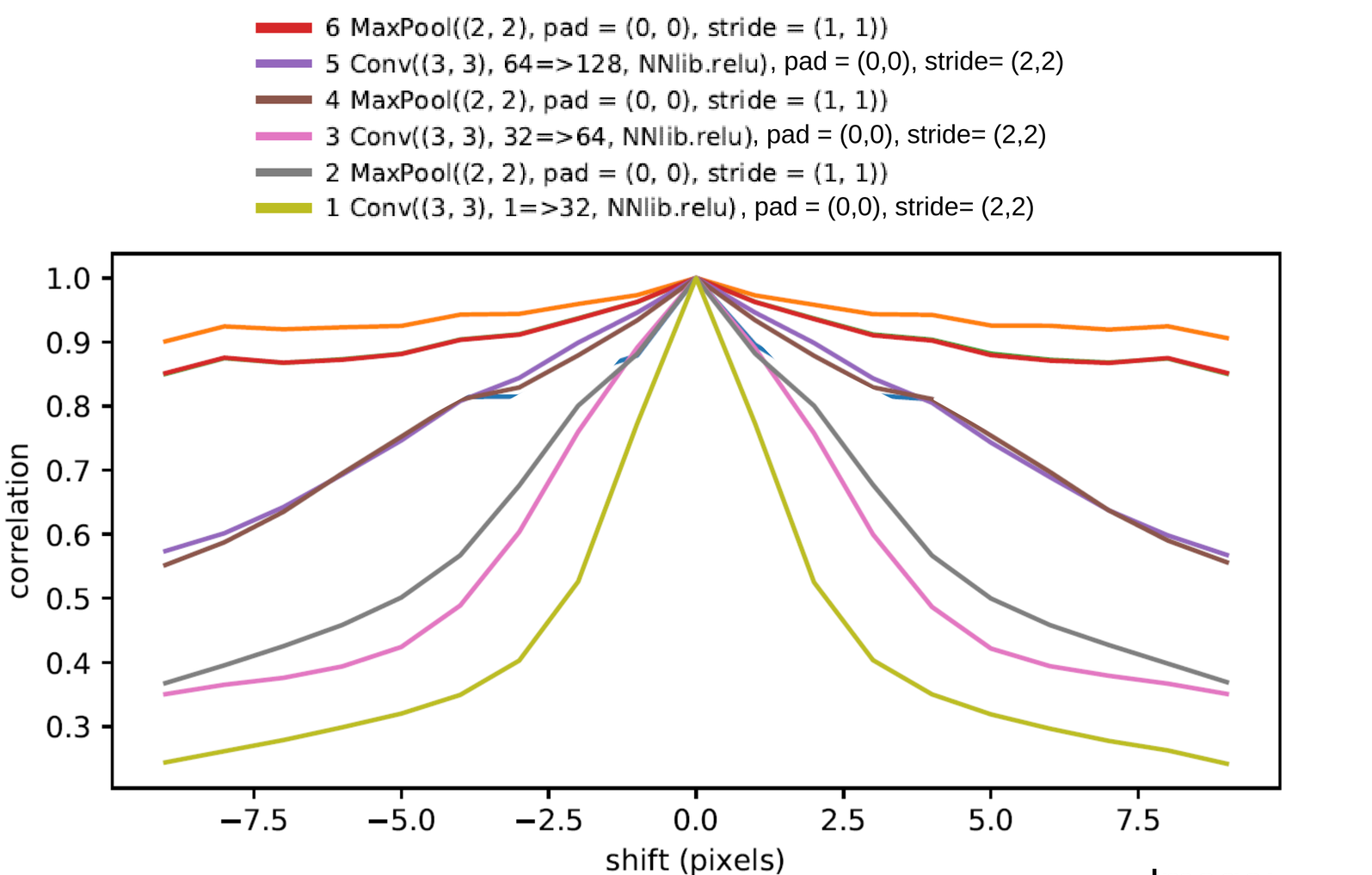
Note that when I say ‘shift by one step’ it really means ‘shift the original image at the entrance to the convolutional layer by an amount that corresponds to the stride in the convolutional layer’. So if the **first** convolutional layer has a stride of 2 then this means we shift the original image by 2 pixels. Thus a sequence of convolutional filter followed by nonlinearity followed by maxpooling implements an (approximate) local invariance to a translation by two pixels.

If in addition you also want to ensure translation invariance to a shift by a single pixel, you should use data augmentation where each image is jittered by one pixel to the left and to the right.

In the end we will stack many convolutional layers. If each time we get invariance by two pixels, then we have after 10 layers invariance across 1000 pixels!

ConvNet with Max Pooling stride one: global shift of image





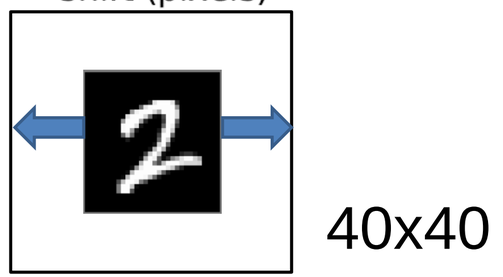


Image:
Bernd Illing

Previous slide.

In the end we will stack many convolutional layers. If each time we get invariance by two pixels, then we have after 10 layers invariance across 1000 pixels!

In the example here, Bernd Illing used 28x28 MNIST images embedded (by zero padding) into a 40x40 input field. He used 3 convolutional layers, each with a filtering stage, a ReLu nonlinarity, and a pooling stage. Filtering had a stride of 2, whereas pooling had stride 1. No padding.

The first convolutional stage used 32 different filters, applied with stride 2, makes 19x19x32. The pooling stage with stride 1 (region 2x2) reduces this to 18x18x32.

The second convolutional stage used 64 filters, applied with stride 2, makes 8x8x64. The pooling stage with stride 1 (but region 2x2) reduces this to 7x7x64

The red line, labeled 6, is the output of the third convolutional layer, after the pooling stage (2x2x128=512). If the image is now shifted to the left or to the right, the activity pattern changes only very little.

Critic of max-pooling in Deep Convolutional Networks

"The pooling operation used in convolutional neural networks is a big mistake and the fact that it works so well is a disaster." *Geoff Hinton*

https://www.reddit.com/r/MachineLearning/comments/2lmo0l/ama_geoffrey_hinton/clyj4jv/

https://openreview.net/pdf?id=HJWLfGWRb https://www.youtube.com/watch?v=pPN8d0E3900

"We find that max-pooling can simply be replaced by a convolutional layer with increased stride without loss in accuracy on several image recognition benchmarks." *Springenberg et al.* https://arxiv.org/abs/1412.6806 See also https://openreview.net/forum?id=HJeuOiRqKQ

(In addition to max-pooling …) "Other popular pooling functions include the average of a rectangular neighborhood, the L2 norm of a rectangular neighborhood, or a weighted average based on the distance from the central pixel." *Goodfellow et al., Deep Learning*

Support of max-pooling in Deep Convolutional Networks (DCN)

"we see that architectures and layer types commonly used in today's DCNs can be derived from precise probabilistic assumptions that entirely determine their structure"

Patel, Nguyen, Baraniuk http://papers.nips.cc/paper/6231-a-probabilistic-framework-for-deep-learning

Previous slide.

The popularity of pooling layers in practical applications has decreased in recent years.

However, theoretical studies have shown that Deep Convolutional Networks (DCN) arise naturally in the framework of probabilistic generative models (called DRMM) if local translation variance is part of the generative process:

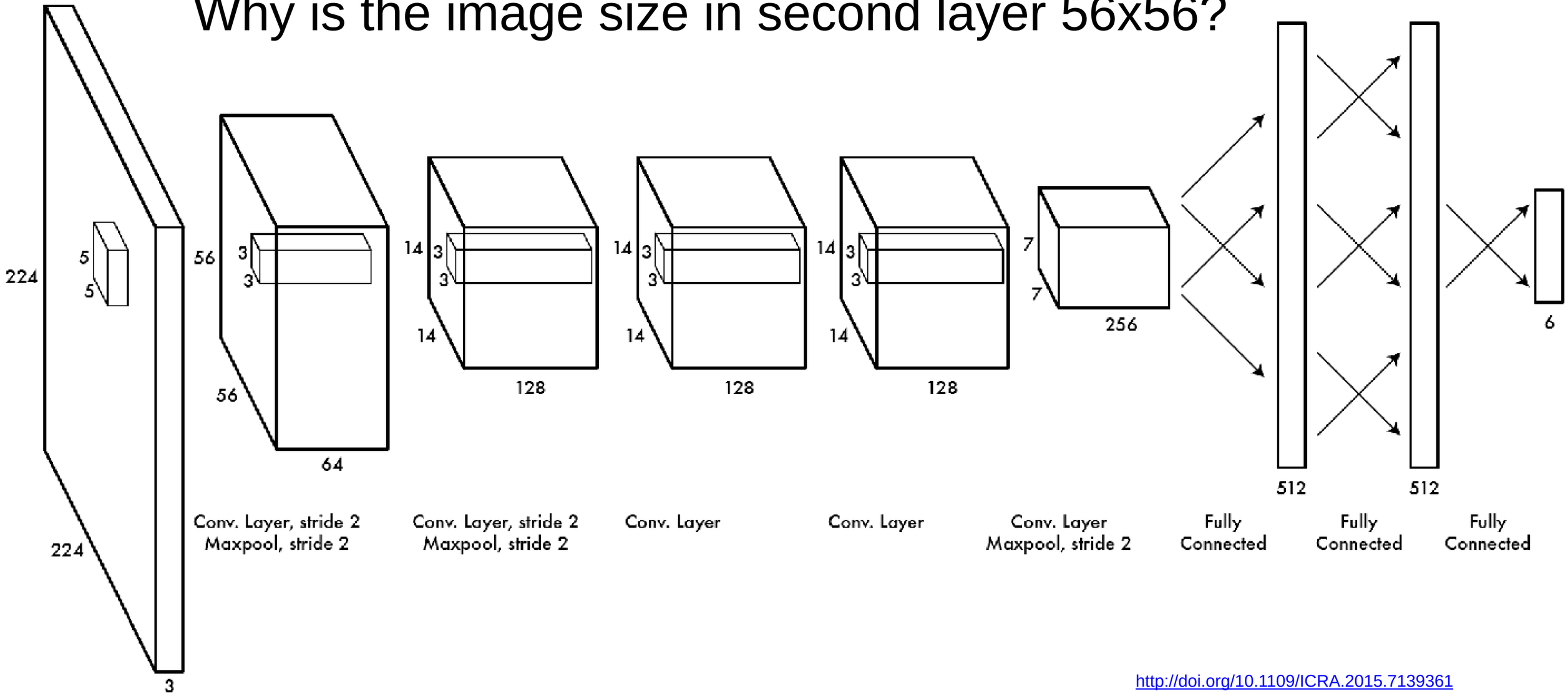
“The convolution, Max-Pooling and ReLu operations in a DCN correspond to max-sum/product inference in a DRMM. … Thus, we see that architectures and layer types commonly used in today’s DCNs can be derived from precise probabilistic assumptions that entirely determine their structure. The DRMM therefore unifies two perspectives—neural network and probabilistic inference”

Patel, Nguyen, Baraniuk, A probabilistic framework for deep learning,
30th Conference on Neural Information Processing Systems (NIPS 2016), Barcelona, Spain.
http://papers.nips.cc/paper/6231-a-probabilistic-framework-for-deep-learning

Convolutional network

How many filters are used in first layer?

Why is the image size in second layer 56x56?



http://doi.org/10.1109/ICRA.2015.7139361

Previous slide.

Putting everything together we can draw nice images like these to describe the network architecture. You should now be able to understand such graphs.

E.g.
How many filters are there in the first convolutional layer? The answer is 64

What padding was used in the first convolutional layer? 2. Two times stride of 2 reduces each dimension by a factor 4. Since 224/56 = 4, we conclude that the center of the filters was placed also at all edges of the image, which for a filter of size 5x5 means that a padding of 2 is needed.

Summary: Convolutional network

1) Convolutional networks exploit (partial) translation invariance of objects in images.
2) The inductive bias (“local translation invariance”) is encoded in the network architecture (several “convolutional layers”)
3) Pooling with stride 2 is often used to reduce dimensionality. However, pooling with stride 1 is more compatible with the idea of local translation invariance and hence recommended.
4) Dimensionality reduction can be achieved by strides >1 in the convolutional filtering stage.
5) Maxpooling can be replaced in practice by mean-pooling.
6) People claim that in practice a pooling stage is not even necessary.

Your notes.

Quiz: Convolutional Networks

[ ] The number of adjustable weights in a convolutional layer with 10 filters of size 5x5x3 followed by max-pooling in a neighborhood of 2x2 is 750 (excluding biases).

[ ] The number of weights in a convolutional layer does not depend on the size of the x-y dimension of input layer.

[ ] The number of outputs of a convolutional layer with 10 filters of size 5x5x3 followed by max-pooling in a neighborhood of 2x2 cannot be larger than 750.

[ ] A deep convolutional network with a sufficient number of filtering and max-pooling layers is also invariant under rotations of the image.

[ ] You have a dataset of centered portraits (passport photos) on white background. Global translation invariance is a good inductive bias for this dataset.

[ ] Pooling by averaging over a rectangular neighborhood is a special case of standard deep multilayer networks with feedforward architecture, whereas max-pooling is not.

Your notes.

Artificial Neural Networks

Convolutional Neural Networks

Johanni Brea & W. Gerstner

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Part 4: The gradient of a convolutional layer

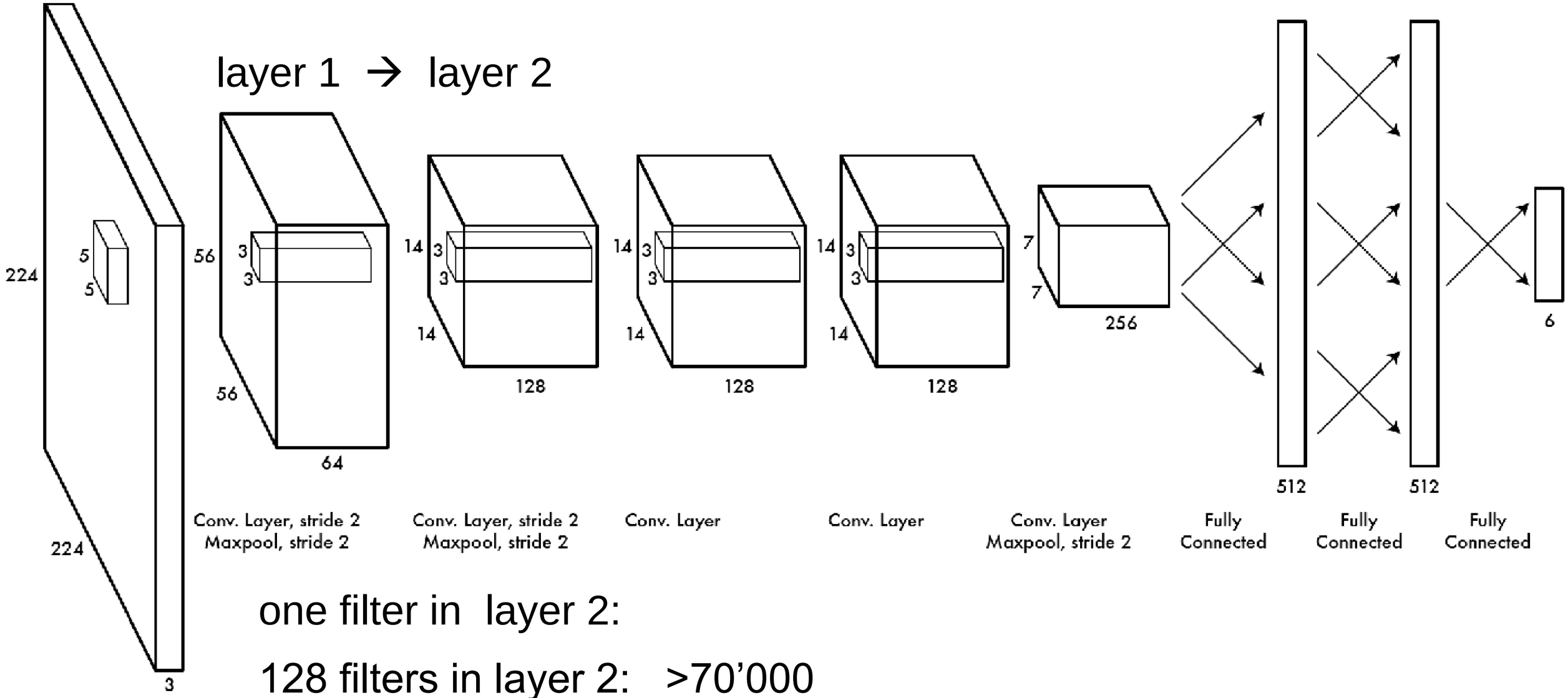
1. Inductive bias in machine learning
2. Convolution filters as inductive bias for images
3. MaxPooling as inductive bias for images
4. **The gradient of a convolutional layer**

Previous slide.
As in standard deep networks parameters are adjusted by gradient descent using Backprop. But BackProp is just a special case of automatic generalization.
We now sketch the framework of automatic differentiation as a generalization of Backprop.

Convolutional network: Filters are optimized by learning

How many weight parameters from layer 1 to layer 2?

layer 1 → layer 2



one filter in layer 2:

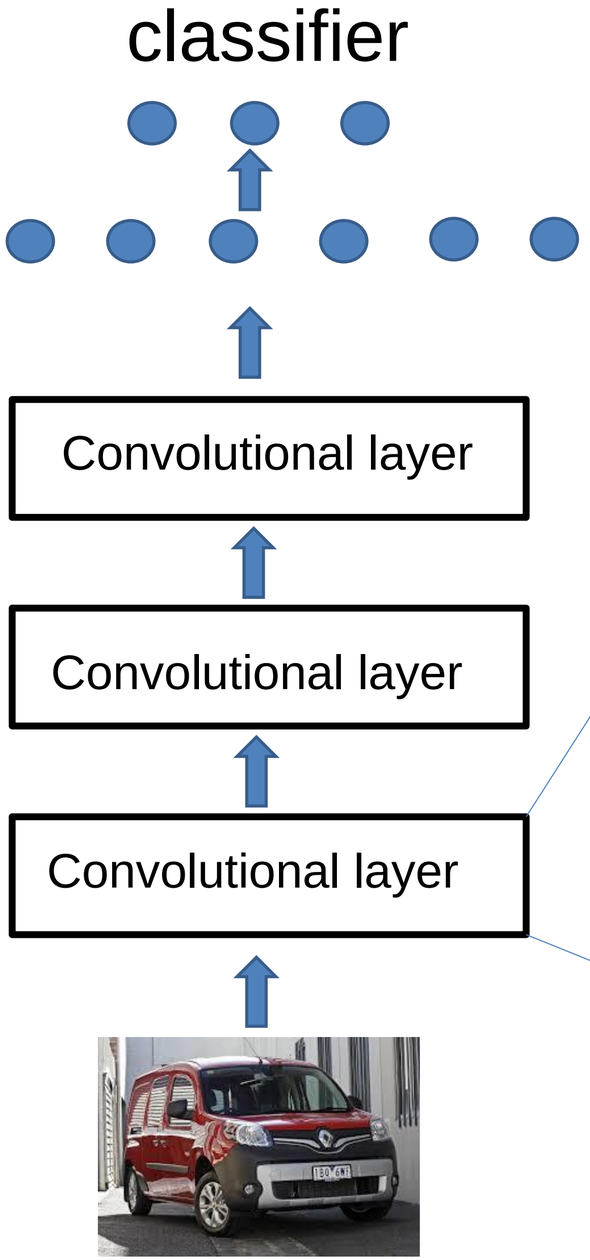
128 filters in layer 2: >70’000

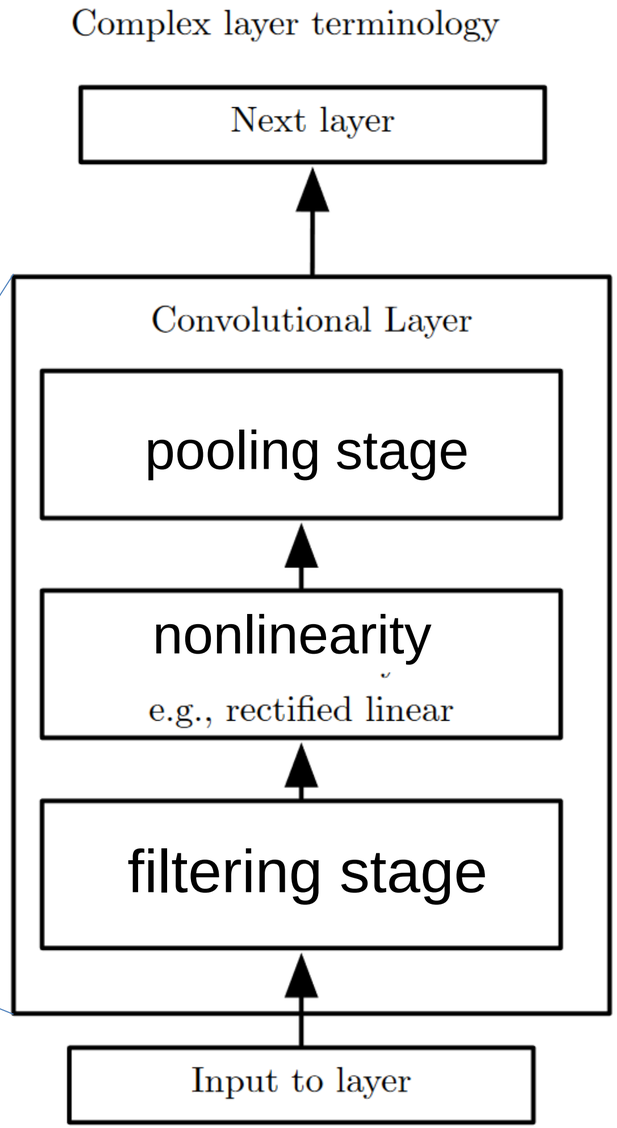
http://doi.org/10.1109/ICRA.2015.7139361

There are many parameters.
The only novel aspects are that
 (i) we have some weight sharing (easy to deal with) and
(ii) that the max-operation comes into play.

The following examples shows what this implies (see also Exercise 1)

Components of a typical conv-net layer





pooling in 2x2 region with stride 1

$$x_{ijk}^{(1,out)} = \max\{x_{i,j,k}^{(1)}, x_{i+1,j,k}^{(1)}, x_{i,j+1,k}^{(1)}, x_{i+1,j+1,k}^{(1)}\}$$

$$x_{ijk}^{(1)} = \sigma(a_{ijk})$$

$$a_{ijk} = b_k + \sum_{x=1}^{5}\sum_{y=1}^{5}\sum_{c=1}^{3} I_{i+x-1,j+y-1,c} w_{xyck}^{(1)}$$

Previous:
Just a reminder of the steps within one convolutional layer.

Filters are optimized by learning

Blackboard/ Exercise 1

Novel aspects: - weight sharing
- max-pooling

Example: $1 \leq k \leq K$ convolutional filters 5x5x3 in layer 1 followed by nonlinearity σ (and max pooling) transmitted to a single unit (index *o*) in layer 2:

$$\hat{y}_o = \sum_{ijk} w^{(2)}_{ijko} x^{(1)}_{ijk}$$

$$x^{(1,out)}_{ijk} = \max\{x^{(1)}_{i,j,k}, x^{(1)}_{i+1,j,k}, x^{(1)}_{i,j+1,k}, x^{(1)}_{i+1,j+1,k}\}$$

$$x^{(1)}_{ijk} = \sigma(a_{ijk})$$

$$a_{ijk} = b_k + \sum_{x=1}^{5}\sum_{y=1}^{5}\sum_{c=1}^{3} I_{i+x-1,j+y-1,c} w^{(1)}_{xyck}$$

with max pooling over region 2x2, stride 1

Optimize weight w_{1135} of filter with filter index $k = 5$, input channel *c=3* and location (1,1). Quadratic loss.

→selects those indices (i*,j*) that gave the argmax for some location.

Consider a very simple convolutional neural network with 3 dimensional input (e.g. RGB image), one convolution layer with 5x5x3 filters, stride 1, non-linearity σ and one linear layer, i.e.

$$a_{ijk} = b_k + \sum_{x=1}^{5}\sum_{y=1}^{5}\sum_{c=1}^{3} I_{i+x-1,j+y-1,c} w^{(1)}_{xyck} \tag{1}$$

$$x^{(1)}_{ijk} = \sigma(a_{ijk}) \tag{2}$$

$$\hat{y}_o = \sum_{ijk} w^{(2)}_{ijko} x^{(1)}_{ijk} \tag{3}$$

a) With loss $L = \frac{1}{2}\sum_o (t_o - \hat{y}_o)^2$, compute $\partial L / \partial w_{1111}$.

b) Add a max pooling layer of 2x2 region 4 that takes the outputs of equation (2) and transforms it into

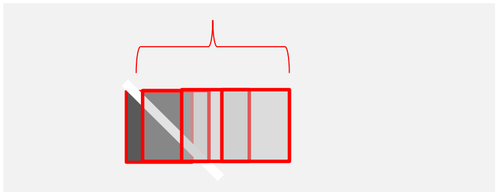
$$x^{(1,out)}_{ijk} = \max\{x^{(1)}_{i,j,k}, x^{(1)}_{i+1,j,k}, x^{(1)}_{i,j+1,k}, x^{(1)}_{i+1,j+1,k}\} \qquad (2b)$$

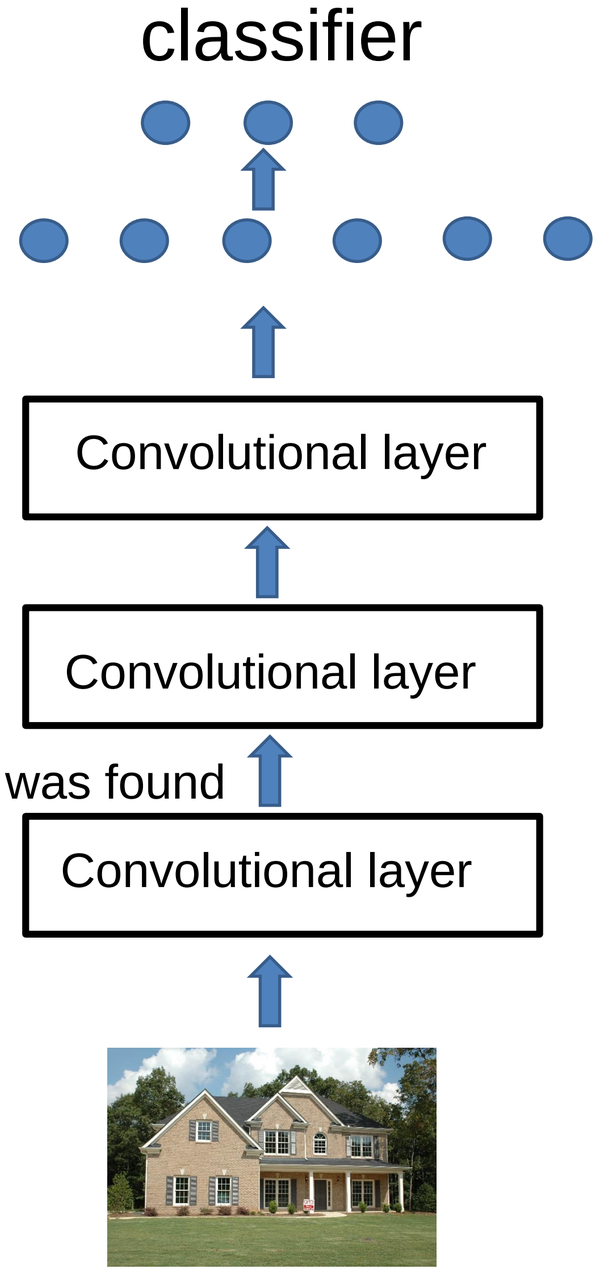
before sending it to the output, In other words, use Eq. (3) but with the upper index (1,out) instead of (1) and calculate again the gradient. What changes?

BackProp for Max-pooling: Attentional focus

with max pooling over region 5x5, stride 1

→selects those indices (i*,j*) that gave the argmax for some location.





Gradient finds location
where a specific feature was found

Your notes.

Summary: Gradient across MaxPooling layer

Gradient is taken across max-pooling layer.

Backward path during backpropagation leads to a focusing on the location i*,j* that was the winner.

→ Selection of region/attentional effect

Previous slide.

The gradient across the max-pooling stage automatically focuses on the ‘relevant’ location, i.e., the location that was the winner (within the small region of max-pooling).

Over several layers this leads to a region-finding effect, similar to an attentional focus and can be used to find the relevant regions that decided in favor of a certain output (such as diagonal stripes for the decision of a house with a classic roof).

Artificial Neural Networks

Convolutional Neural Networks

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Part 5: Automatic Differentiation: BackProp revisited

1. Inductive bias in machine learning
2. Convolution filters as inductive bias for images
3. MaxPooling as inductive bias for images
4. The gradient of a convolutional layer
5. **Automatic Differentiation: BackProp revisted**

Previous slide.
As in standard deep networks parameters are adjusted by gradient descent using Backprop. But BackProp is just a special case of automatic generalization.
We now sketch the framework of automatic differentiation as a generalization of Backprop.

Shall we manually compute the gradients?

No. Use automatic (reverse mode) differentiation.

- Record computation tree on forward path to
- apply chain rule in reverse order.
  (Same idea as backprop in simple feedforward net)

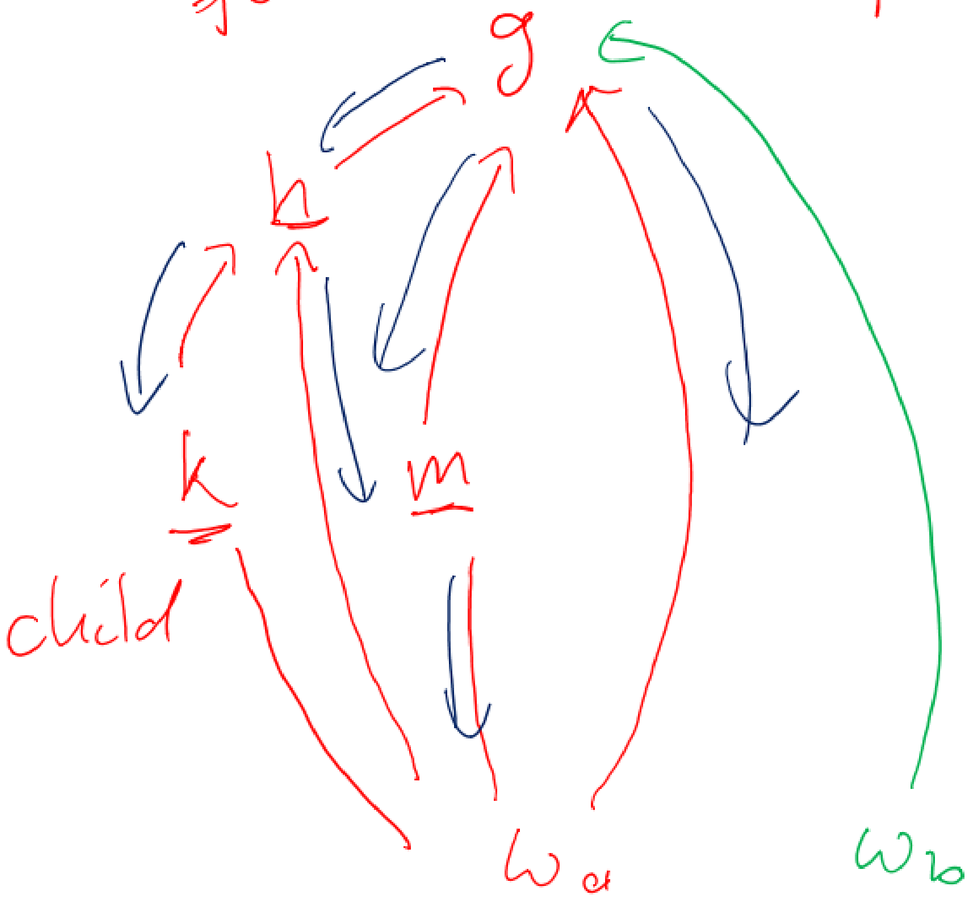
- An implementation requires just a Tracker (to record the computation tree) and analytical expressions of primitive operations

Your notes.

Example: Automatic differentiation

$$f(w_a^{\square}, w_b^{\square}) = g[h(k(w_a^{\square}), w_a^{\square}), m(w_a^{\square}), w_a^{\square}, w_b^{\square}]$$

forward calcul. path



$$\frac{\partial f}{\partial w_a} = \frac{\partial g}{\partial h}\left[\frac{\partial h}{\partial k}\frac{\partial k}{\partial w_a} + \frac{\partial h}{\partial w_a}\right]$$

$$+ \frac{\partial g}{\partial m}\cdot\frac{\partial m}{\partial w_a}$$

$$+ \frac{\partial g}{\partial w_a}$$

w_a has 4 children

Previous slide.

In the simple example, the parameter wa has 4 children.
This gives rise to 4 terms in the summation induced by the chain rule.

None of the intermediate functions like m or h has more than one child. Therefore no additional summations are generated.

Summary: Automatic Differentiation (generalized BackProp)

1. Determine children nodes of weight variable.

2. Find a backward (revers ancestral) schedule:
   all children scheduled before node itself.

3. Start with top node and run through reverse schedule

4. Look up primitive operations. Define intermediate variables.

5. Sum over children and multiply (→chain rule)

→Takes care of max pooling, weight sharing
→Takes care recurrent connections

Your notes.

Artificial Neural Networks

Convolutional Neural Networks

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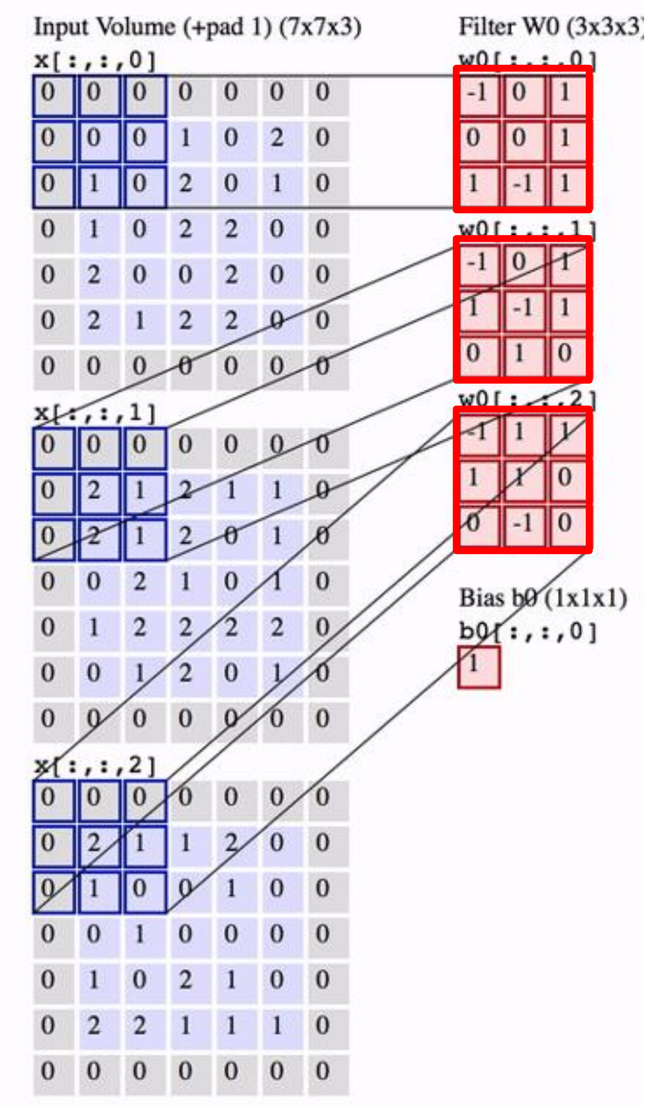
Part 6: Reducing the number of parameters: space and depth

1. Inductive bias in machine learning
2. Convolution filters as inductive bias for images
3. MaxPooling as inductive bias for images
4. The gradient of a convolutional layer
5. Automatic differentiation: BackProp revisited
6. **Reducing the number of parameters: space and depth**

Previous slide.

We have a large number of parameters. With some additional assumptions (inductive bias!) we can reduce the number significantly.

Convolution: colors and padding

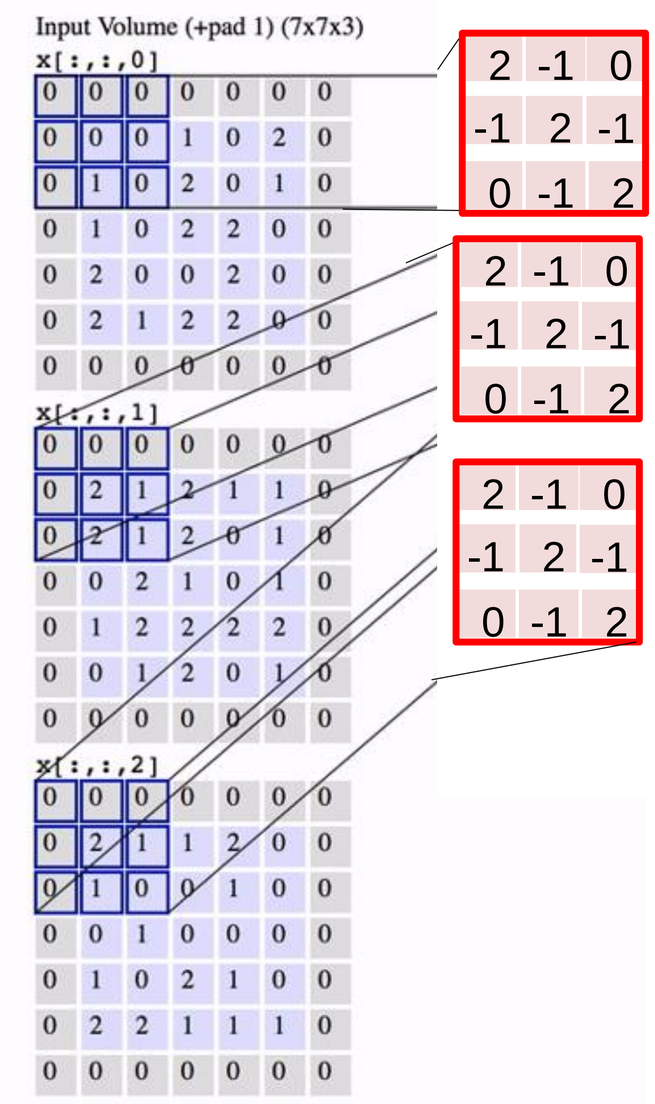


- 3 color channels

- Filter 3x3 (spatial) in each color dimension
  → filter tensor W0 denoted as (3x3x3)

- everal filters: W0, W1, W2, W3, … index k

Convolution: colors and padding

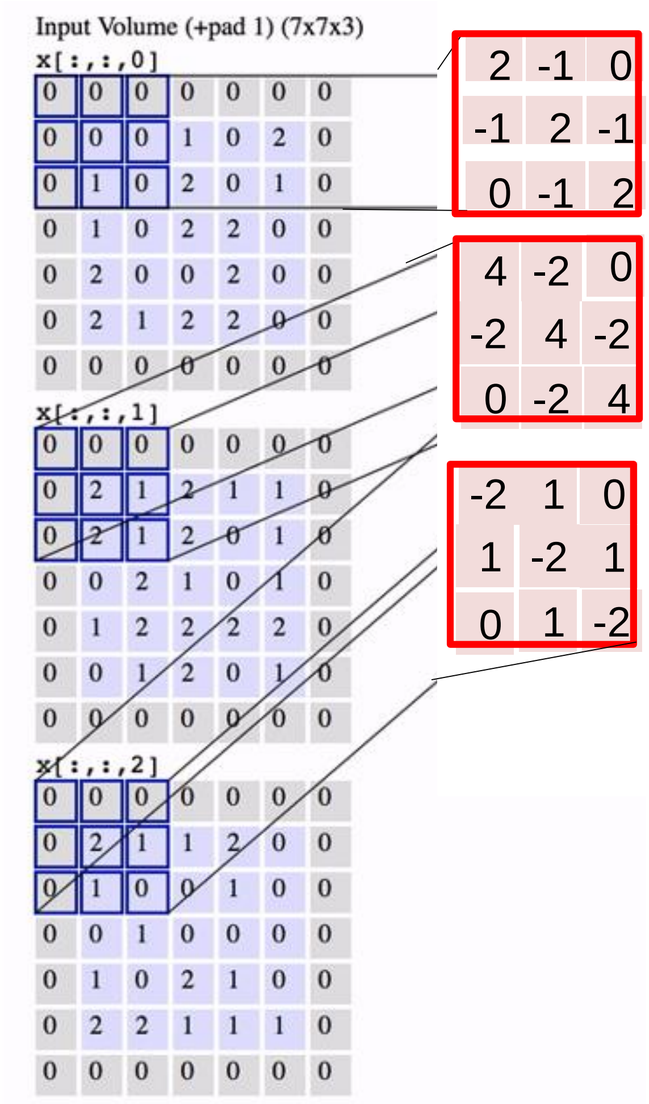


- 3 color channels

- Filter 3x3 (spatial) in each color dimension
  → filter tensor W0 denoted as (3x3x3)

But why not use the SAME spatial filter for 3 channels?

Convolution: colors and padding



- 3 color channels

- Filter 3x3 (spatial) in each color dimension
  → filter tensor W0 denoted as (3x3x3)

But why not use the SAME spatial filter for 3 channels?

And may be with different scaling?

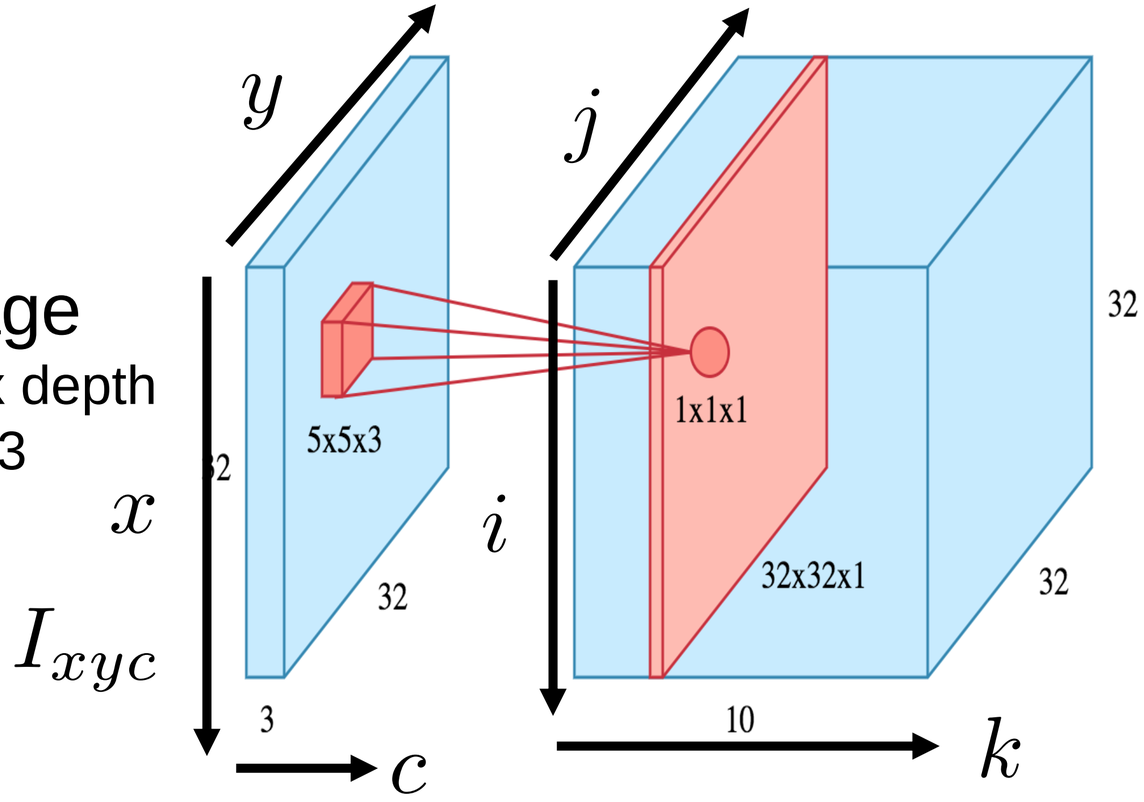
Previous slide.

Basic idea:
Let us separate the spatial dimension from the color dimension.

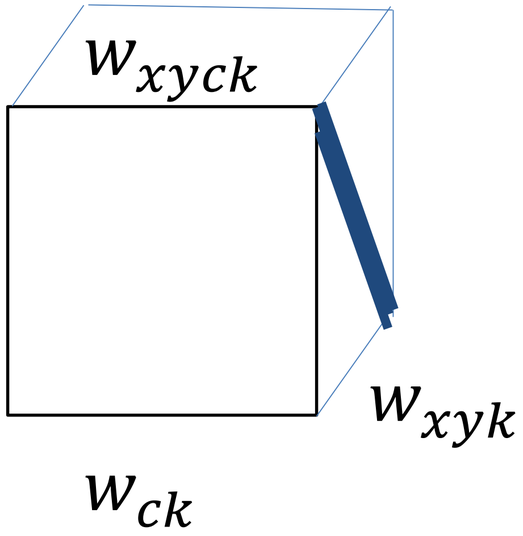
More generally:
Let us separate the spatial dimension from the color dimension.

Convolution: outer product

color image
width x height x depth
32 x 32 x 3



Filter k

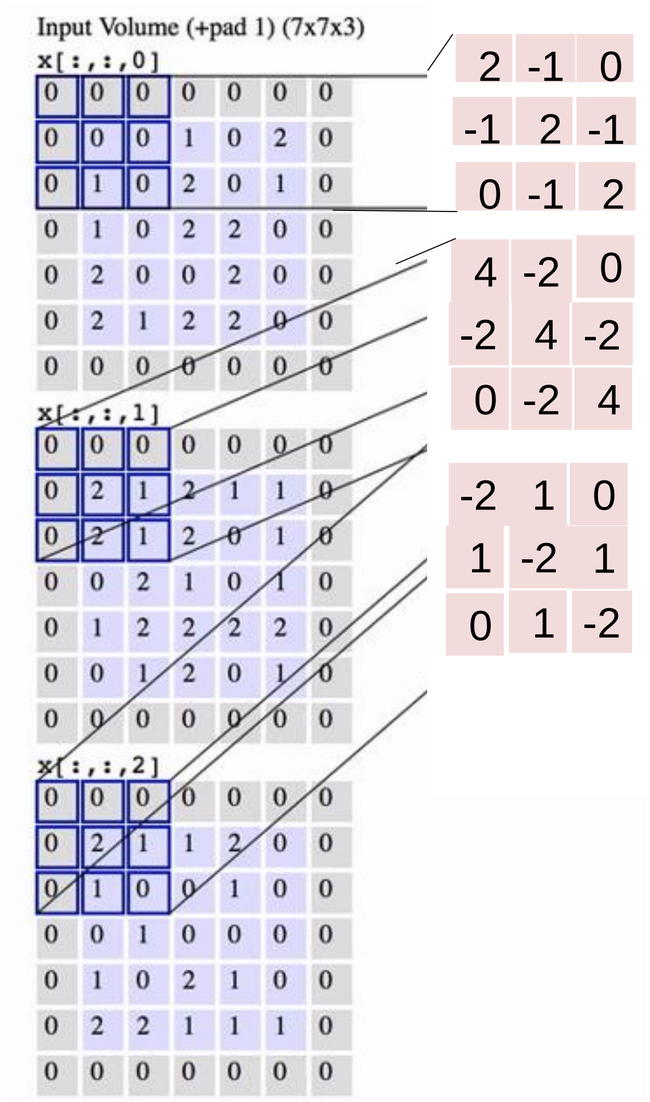


$$w_{xyck} = w_{xyk} w_{ck}$$

$$a_{ijk} = b_k + \sum_{x=1}^{5} \sum_{y=1}^{5} \sum_{c=1}^{3} I_{i+x-1,j+y-1,c} w_{xyck}$$

Your notes (calculation).

Convolution: outer product representation



Filter k

$$w_{xyck} = w_{xyk} w_{ck}$$

- 3 color channels

- Filter 3x3 (spatial) in each color dimension
  → filter tensor W0 denoted as (3x3x3)

SAME spatial filter for all color channels
= first calculating a (shaded) grey value

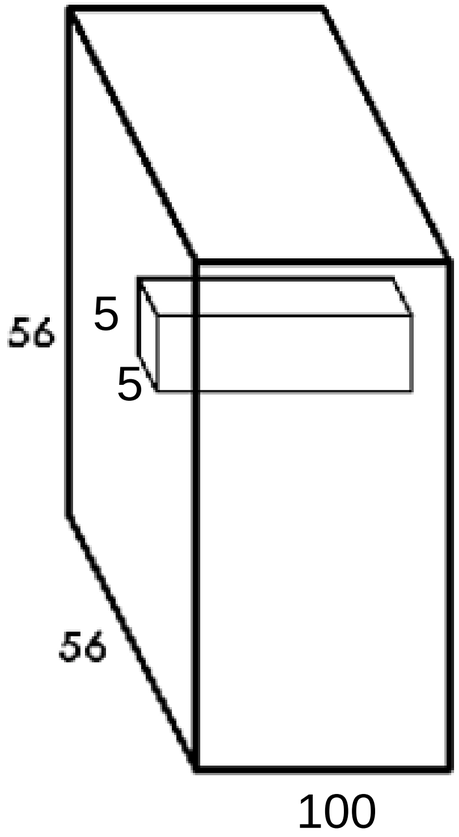
Previous slide.
The calculation shows that the separation of the filter into a spatial component and a depth component can be used to FIRST integrate out the depth: this amounts to a weighted average of all the values in a given xy position.
Then the spatial filtering can be performed later with this ‘average’.

In the specific case of three color channels, the averaging with equal weights would amount to first calculating the grey-scale image (getting rid of the color information). Since it is a weighted average it amounts to a grey with, e.g., a redish or greenisch shading.

Note that instead of the interpretation of the depth filter (a vector of length d) as a weighted averaging filter, we can also say that we apply a tensor of 1x1xd.

Then the full filter of size 5x5xd in outer product representation, can be calculated as the sequence of two filter applications:
First the filter 1x1xd (the depth filter), followed by a filter 5x5x1.
This idea is used later in the inception module.

Parameter reduction with outer product representation



Filter k

- 100 depths channels: 1 ≤ c ≤ 100

- Filter 5x5 (spatial)
  → filter tensor W denoted as 5x5x100

Parameters:

SAME spatial filter for all color channels

$$w_{xyck} = w_{xyk} w_{ck}$$

Parameters: (5x5)+100

Previous slide.

Reduction of parameters by more than a factor of 10!

The outer-product representation is used in many fields of science as a means to reduce the number of parameters in a large filter tensor. It is not restricted to ANN.

Artificial Neural Networks

Convolutional Neural Networks

Johanni Brea & W. Gerstner

EPFL, Lausanne, Switzerland

Part 7: Modern Convolutional Networks and Image Tasks

1. Inductive bias in machine learning
2. Convolution filters as inductive bias for images
3. MaxPooling as inductive bias for images
4. The gradient of a convolutional layer
5. Automatic differentiation: BackProp revisited
6. Reducing the number of parameters: outer-product
7. **Modern ConvNets and Image Recognition Tasks**

Previous slide.

We now look at recent developments around Convolutional Networks.

Recent Imagenet challenges: object detection task

http://www.image-net.org/
https://www.kaggle.com/c/imagenet-object-detection-challenge

Previous slide.

The task here is to find several objects in the image, and tell where these are.

Imagenet challenge 2012

example images of australian terriers




- 1.2 million images training set
- 1000 categories
- 50 thousand validation set
- 100 thousand test set

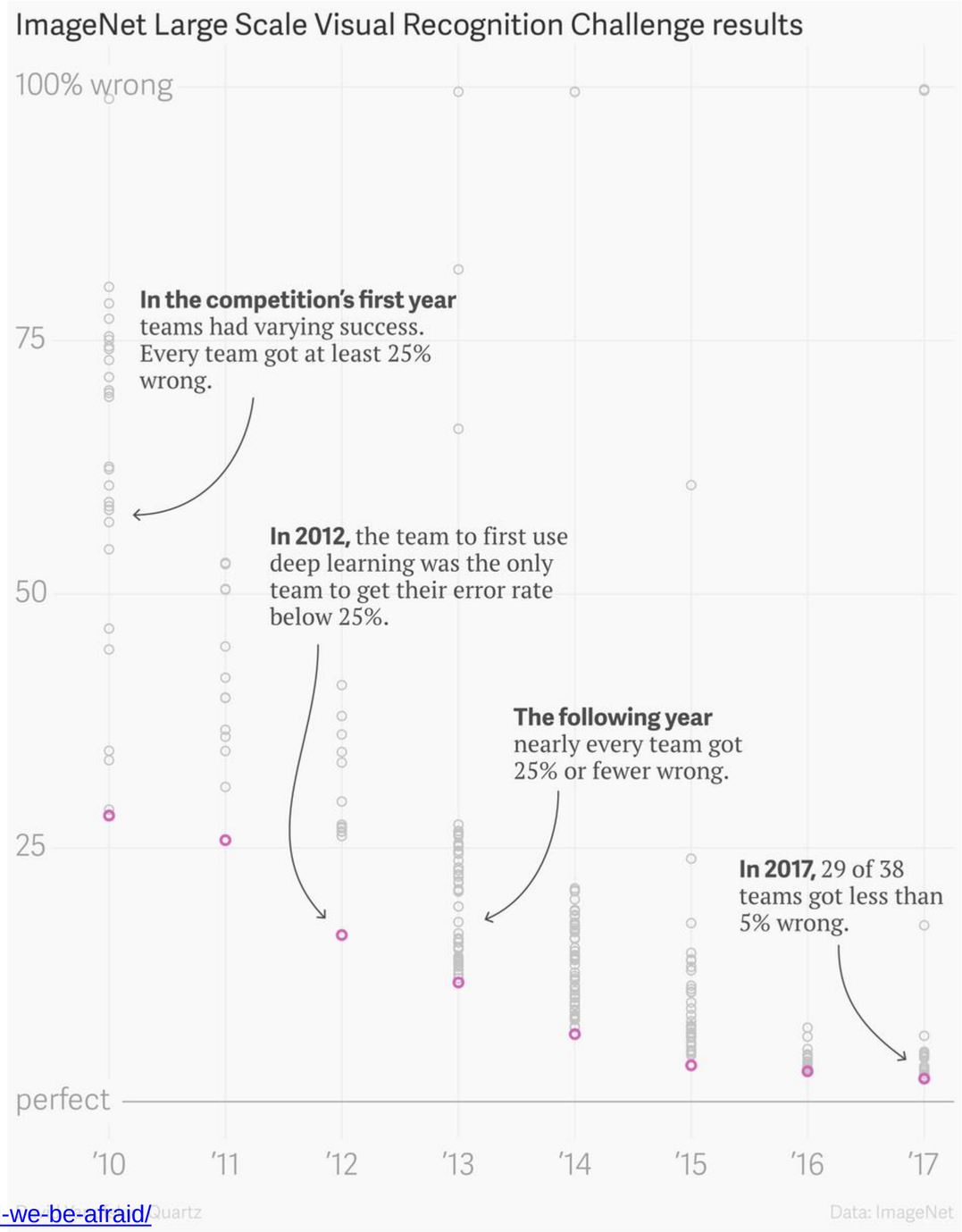
http://www.image-net.org/

Previous slide.

In the classic ImageNet challenges, the task was to find a single object. Note that the data base had a sizable fraction of different dog breeds.

The imagenet competitions have extensively been used to demonstrate progress in deep learning. The competition in 2012 contained 1.2 Million hand-labeled images containing objects in 1000 classes; these numbers have increased in the meantime by at least an order of magnitude.

Imagenet challenge progess

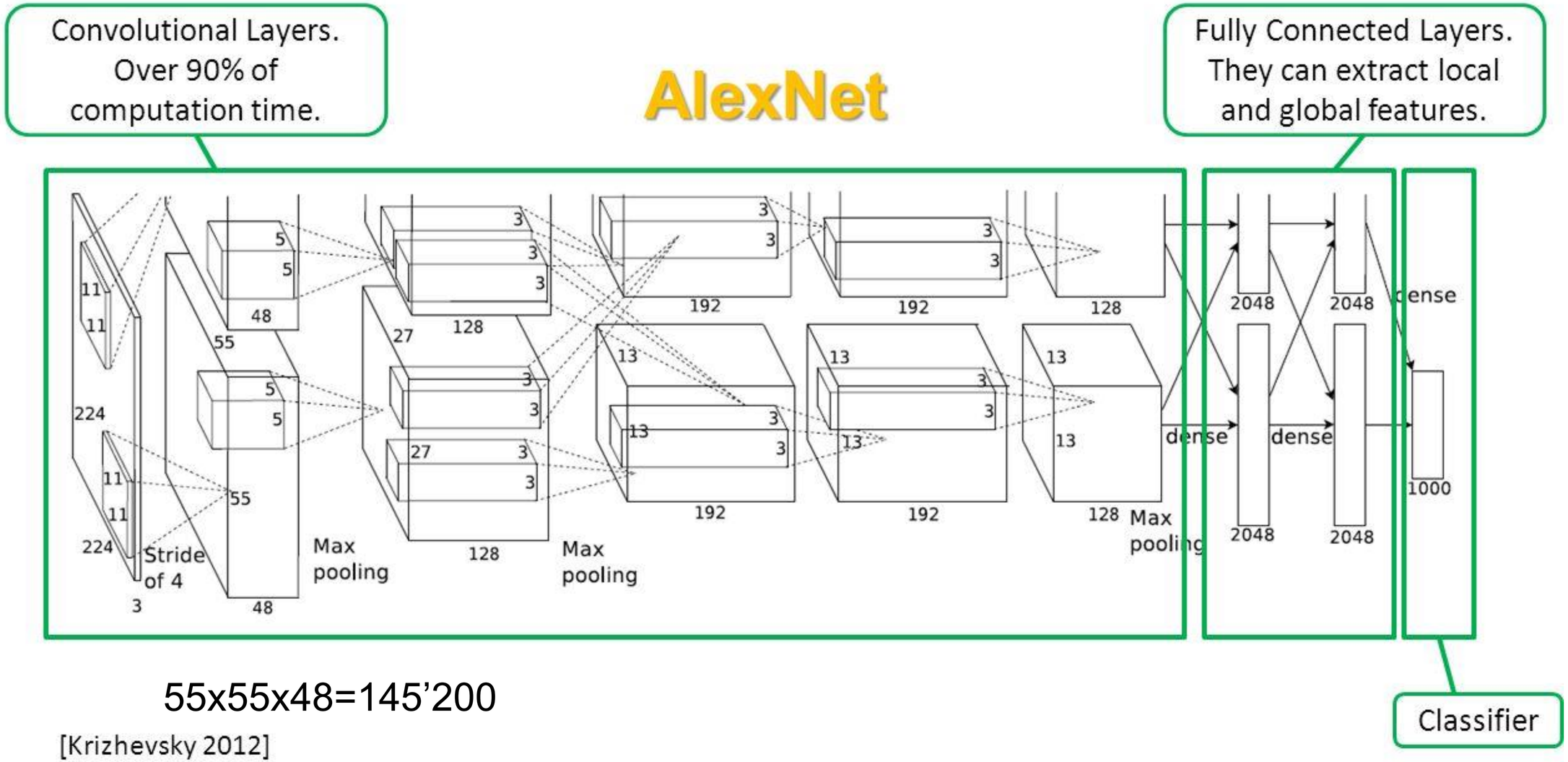


https://qz.com/1046350/the-quartz-guide-to-artificial-intelligence-what-is-it-why-is-it-important-and-should-we-be-afraid/

Previous slide.

Right: Within only a few years the error rate decreased from 25 to 5 percent thanks to advances in the design and training of convolutional networks.

AlexNet: winner of Imagenet challenge 2012



55x55x48=145’200

[Krizhevsky 2012]

http://slideplayer.com/slide/10259573/34/images/5/AlexNet+Convolutional+Layers.+Over+90%25+of+computation+time..jpg

Previous slide.

The best method of the 2012 competition, now called AlexNet, still had a pretty conventional architecture with elements that you have seen up to now in this lecture.

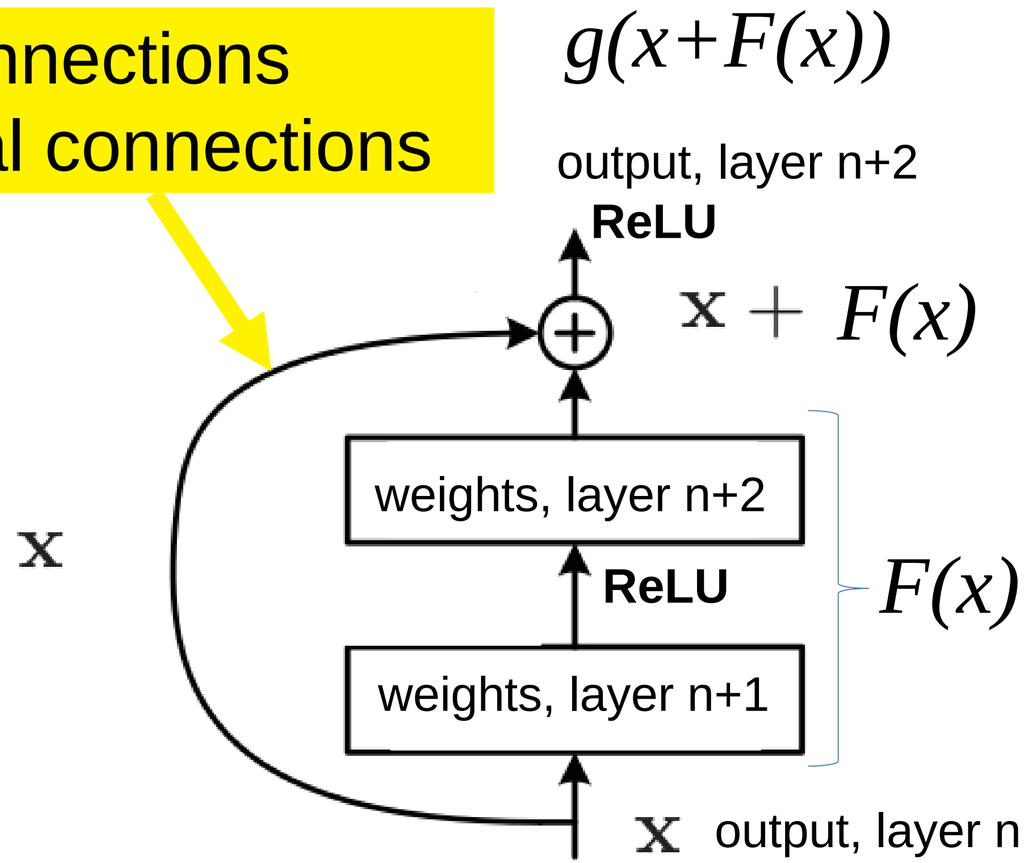
In the first layer, there are about 250’000 neurons. The two processing streams at top and bottom correspond to the implementation on two GPUs.

We will now start to look at newer features that helped to further improve convolutional networks.

ResNet

Better training in very deep (>100 layers) networks.

Skip connections
Residual connections



How many layers?
→ Let the data decide!

https://arxiv.org/pdf/1512.03385.pdf

Previous slide.

In theory, the training loss should decrease with deeper and deeper networks, simply because they have more parameters. In practice, however, it was observed that successfully training very deep convolutional networks is difficult.

A possibly reason are vanishing gradients, but it is not clear whether this is the true reason.

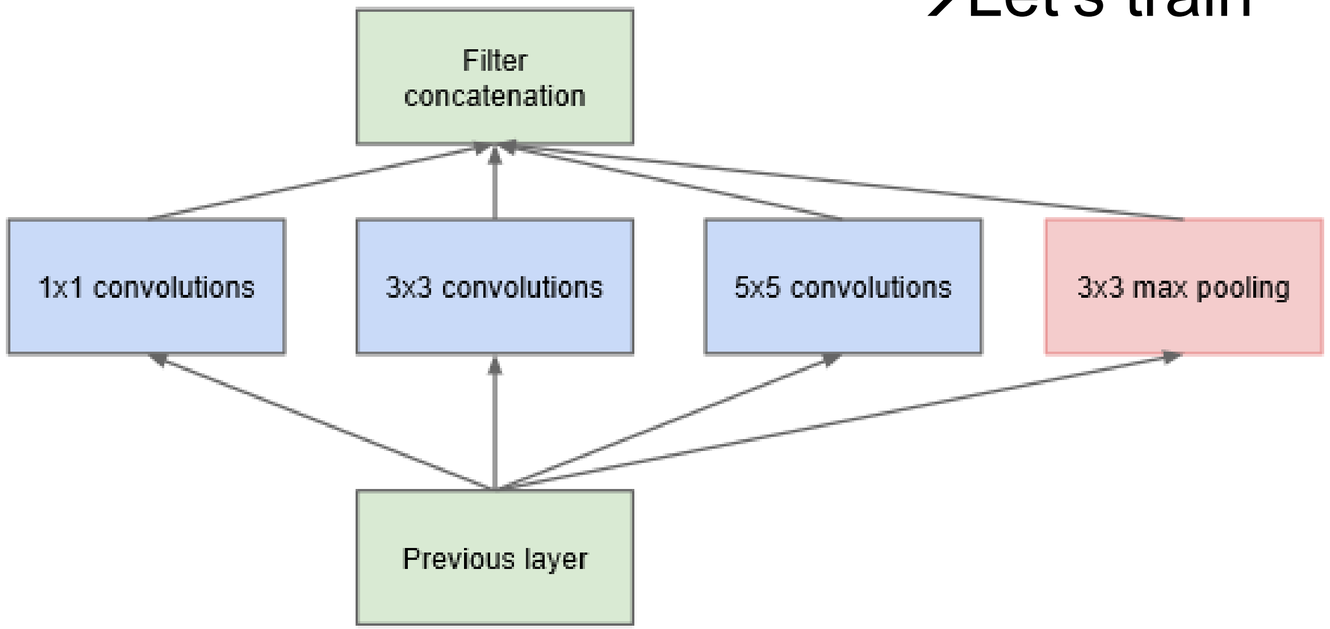
The idea of skip connections is to allow the network to “dynamically choose the number of layers” (see blackboard). The layers within the skip-connections learn the residual F(x), the part that is not yet learned by the network up to layer n (which has the output x). Thus, the function F(x) can be used to adapt to special cases, exceptions, fine-tuning etc – without affecting the main network function (which is just copied to layer n+2 via the skip connections).

Note that, when calculating the gradients, the skip connection contributes no derivative g’, so that multiplication of small gradients is avoided along the skip connections: the skip connection = identity/copy has always a gradient of one. This also means that the combined vanishing gradient problem/linearity problem/bias problem is avoided. The skip connection act linear (and avoids the vanishing gradient and bias problem) and the F(x0) avoids the linearity problem because for some data the nonlinearity F might come into play (if that extra layer is needed).

Inception module (naïve version)

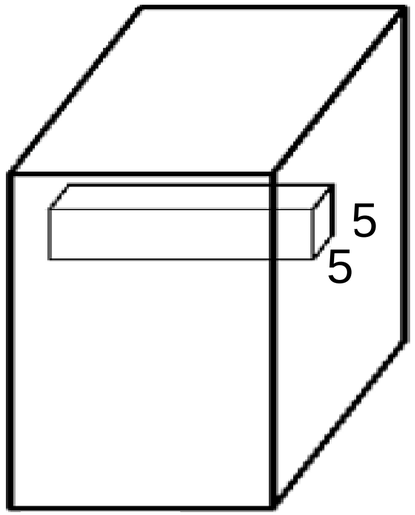
Are 3x3 convolutions better or 5x5 convolutions?
→Let the data decide
→Let's train

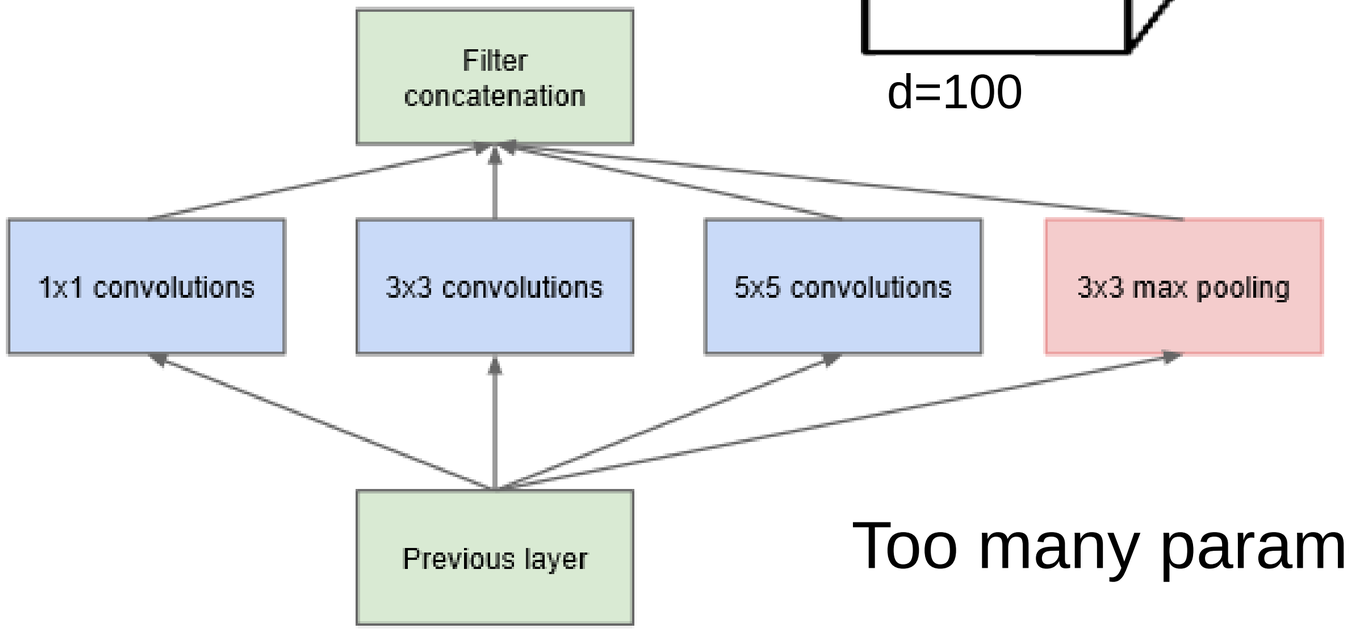
Do we need max-pooling? Let the data decide
→Let's train



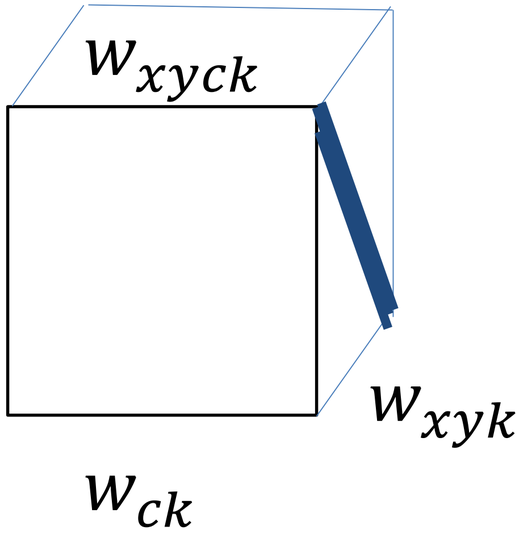
https://arxiv.org/pdf/1409.4842.pdf

Inception module (naïve version)





Filter k

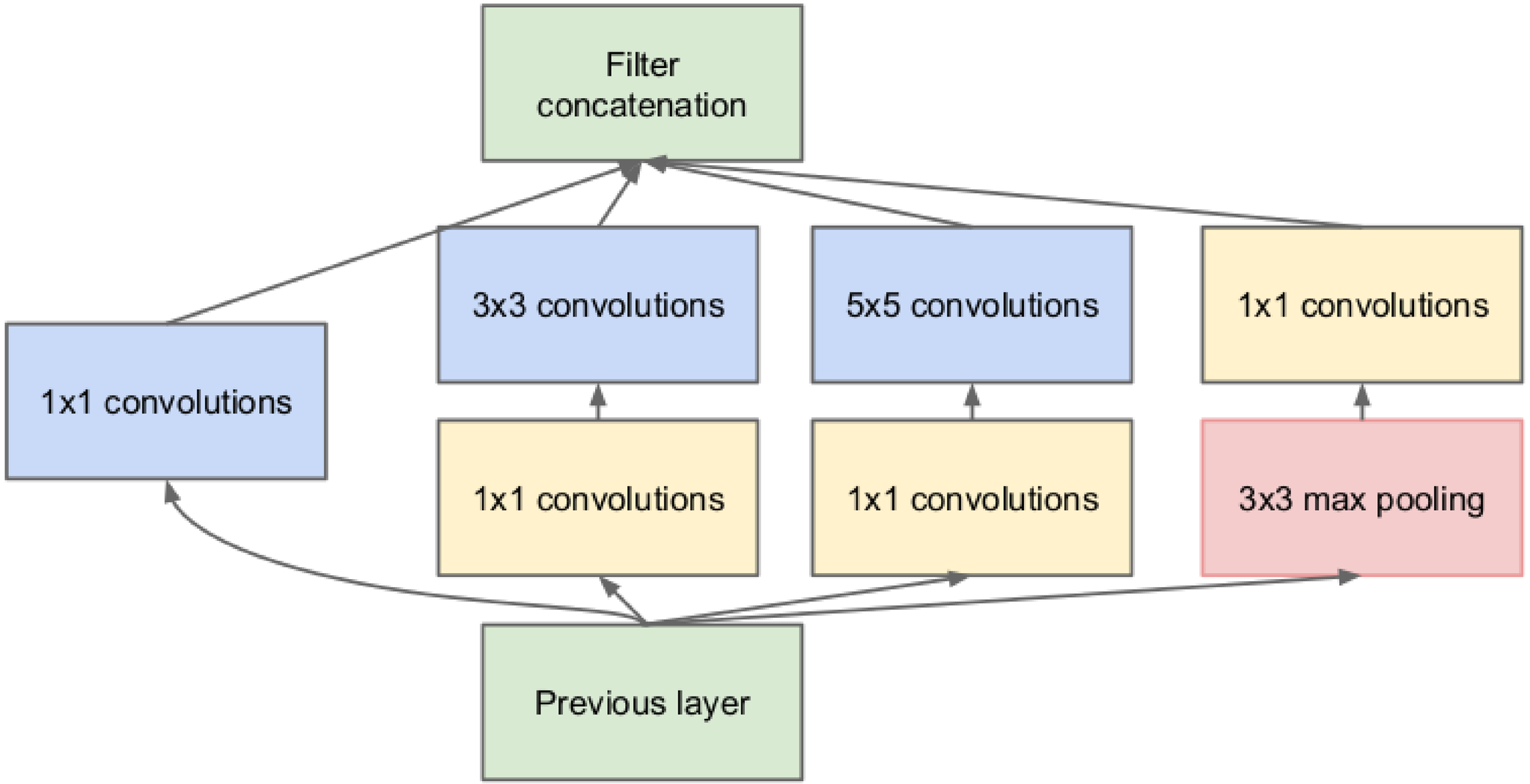


$$w_{xyck} = w_{xyk} w_{ck}$$

Too many parameters! Let's use outer product

https://arxiv.org/pdf/1409.4842.pdf

Inception module



https://arxiv.org/pdf/1409.4842.pdf

Previous slide.

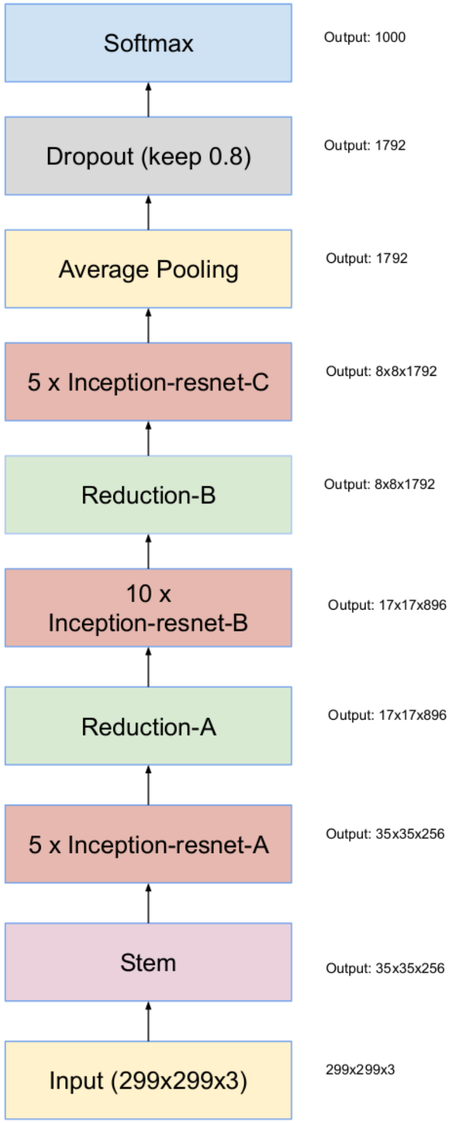
For AlexNet the designers decided where to place convolutional layers with filters of certain sizes and where to place pooling layers. But how should we know what the best architecture is? The inception module leaves this choice to the network itself:

Each layer contains convolutions with different filter sizes and pooling operations in parallel. Depending on the relative weight of each block, the layer can be used as max-pooling layer, or as convolution layer, or as combination of both.

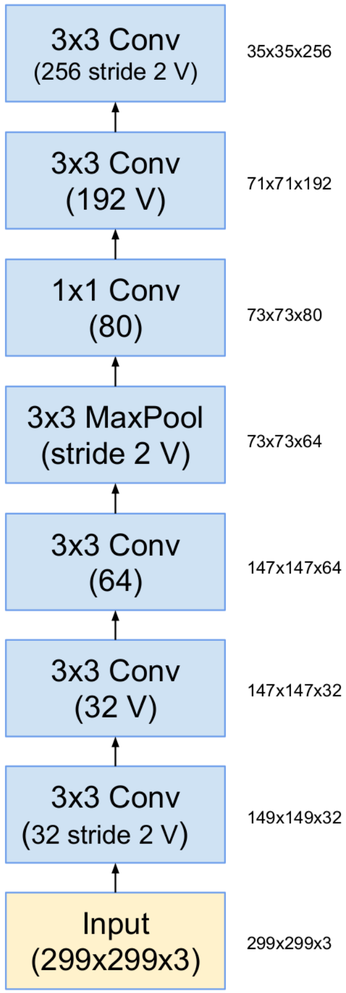
The 1x1xd filters learn to select the relevant features or mixture of features from the concatenation of the outputs. The 1x1xd filters also help to reduce the computational load and the number of parameters (outer-product representation).

Note that the convolution and max-pooling layers have to use the same stride, i.e. if stride 1 is used for the 3x3 and 5x5 filters, than also stride 1 should be used for the max-pooling layer.

Inception-ResNet-v2



Full model



Stem

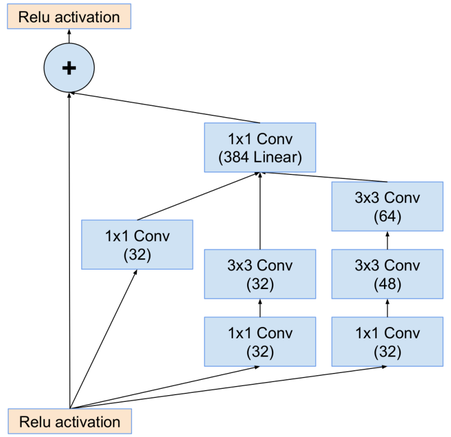
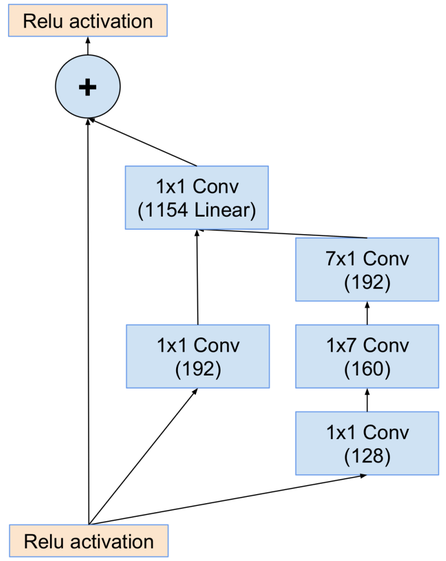


Figure 16. The schema for 35 × 35 grid (Inception-ResNet-A) module of the Inception-ResNet-v2 network.



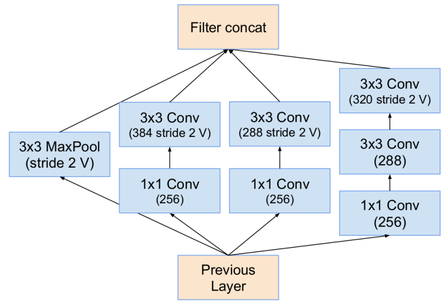


Figure 18. The schema for 17 × 17 to 8 × 8 grid-reduction module. Reduction-B module used by the wider Inception-ResNet-v1 network in Figure 15

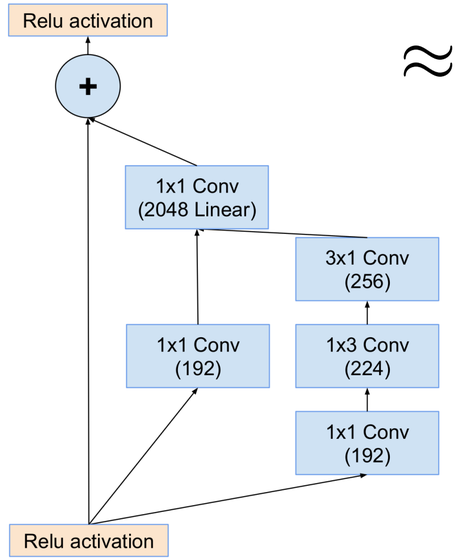


Figure 19. The schema for 8×8 grid (Inception-ResNet-C) module of the Inception-ResNet-v2 network.

Network	k	l	m	n
Inception-v4	192	224	256	384
Inception-ResNet-v1	192	192	256	384
Inception-ResNet-v2	256	256	384	384

Special Layers

≈ 60 million parameters

https://arxiv.org/pdf/1602.07261.pdf

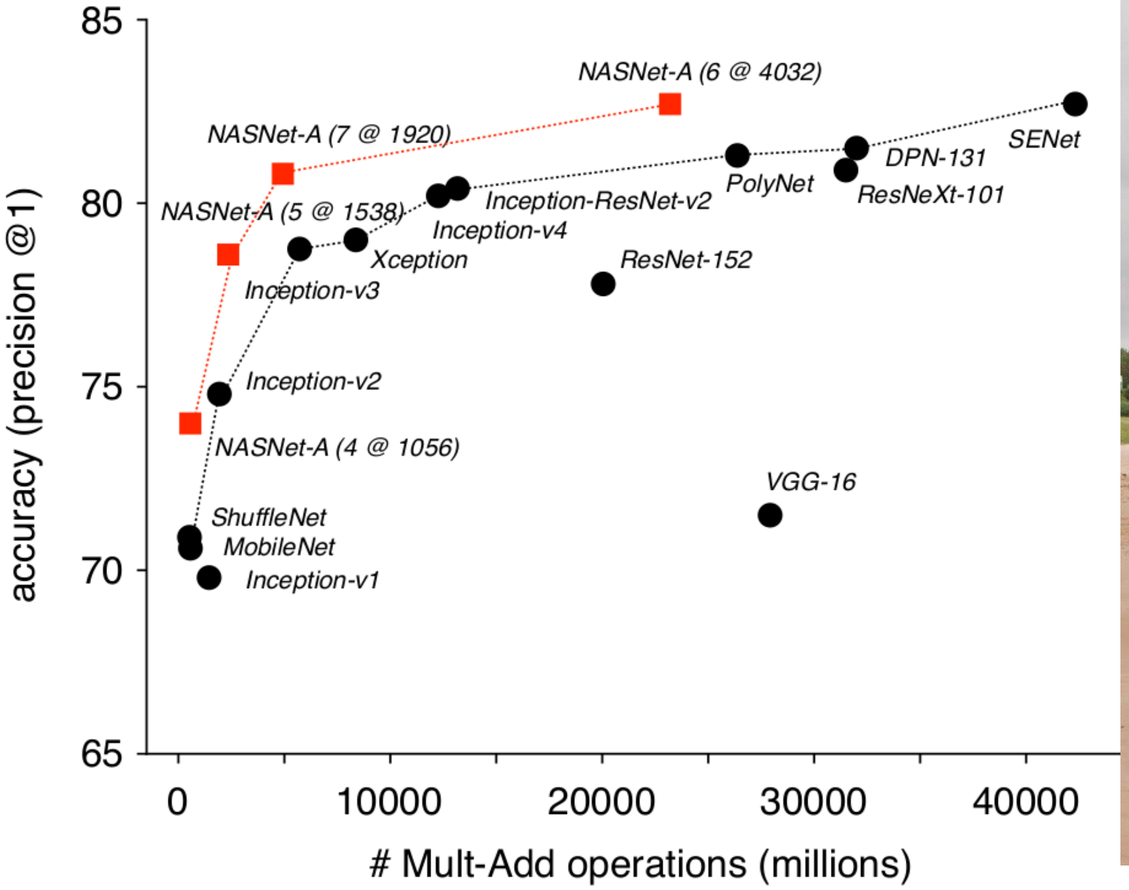
Previous slide.

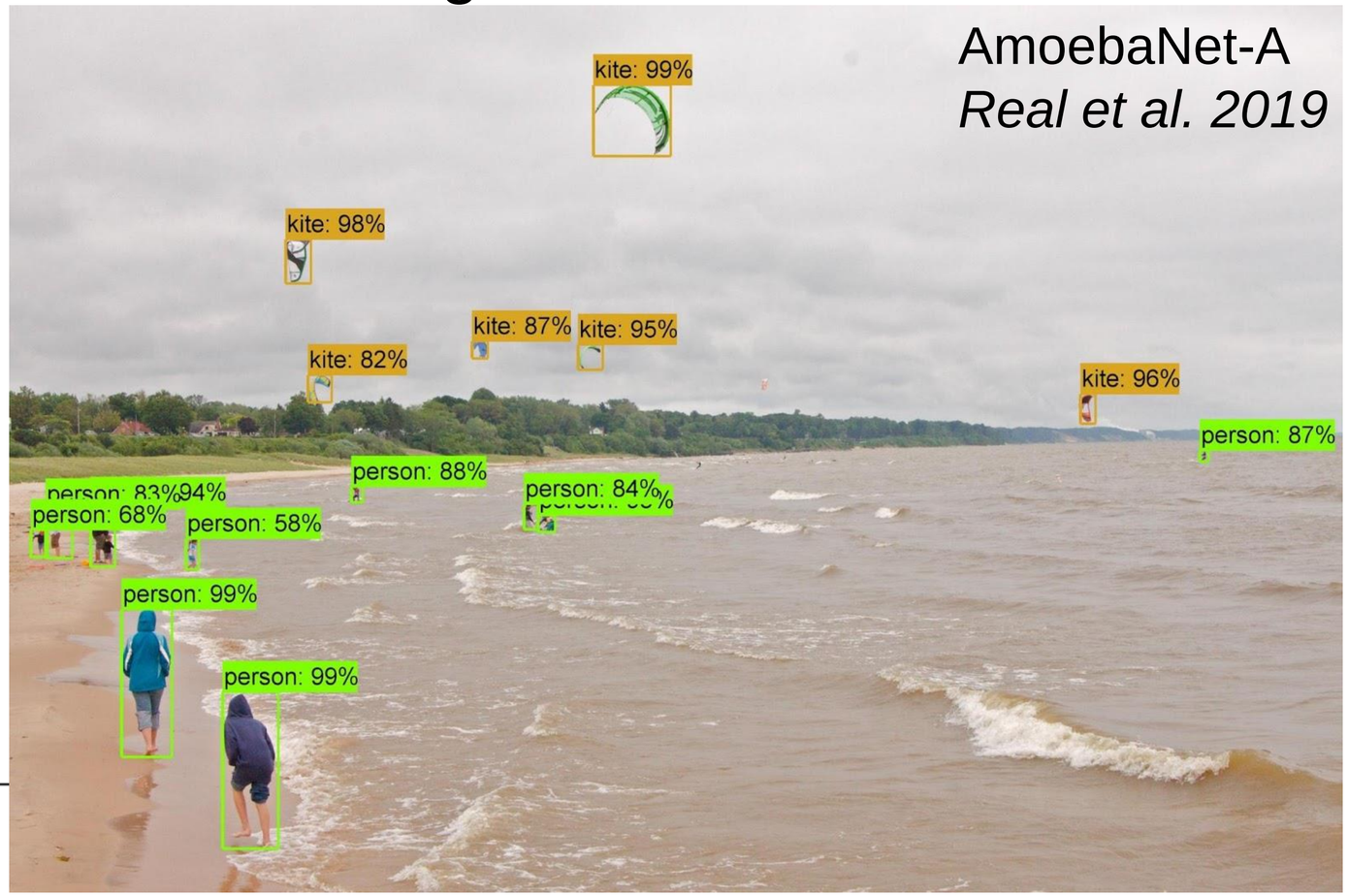
Inception modules have been the basis for GoogLeNet.

A full network may look quite scary and use these inception modules several times.

Transfer Learning for Image Recognition

- Find network architecture with reinforcement learning or evolutionary programming on a small dataset
- Use the same architectural elements on Imagenet





https://arxiv.org/pdf/1707.07012.pdf

https://arxiv.org/abs/1802.01548

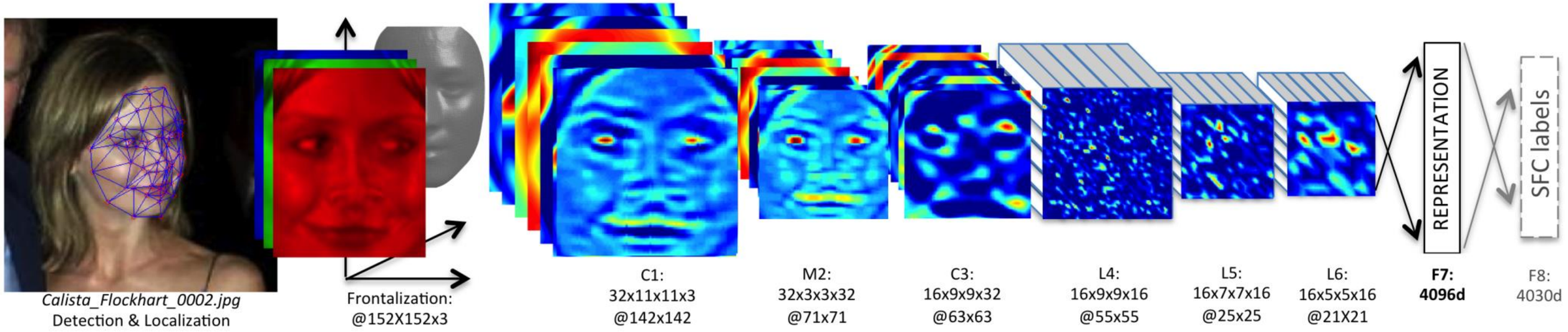
Previous slide.

With skip connections and Inception modules, allowed significant improvements over AlexNet. Yet, the search for the optimal network architecture, i.e. the best inductive bias, continues. Can we automatize this process? Yes, with sufficient computing power one may search for architectures, e.g. with reinforcement learning methods or evolutionary programming. State-of-the-art architectures, like NASNet and AmoebaNet were found like this.

On the left we see that NASNet reaches higher performance on imagenet than hand-crafted variants. In particular the NASNet variants reach consistently higher performance than alternatives with the same number of basic operations. VGG-16 is a famous hand-crafted network with a simple stucture of 13 convolutional layers with 3x3 filters and 3 fully connected layers.

On the right we see an impressive example of what can nowadays be achieved with these networks.

DeepFace and Transfer Learning



Closing the Gap to Human-Level Performance in Face Verification (2014)

- 4 million user - labeled faces on FaceBook images (for free!)
- 4000 individuals
- Retrain fully-connected layers at the top on Labeled Faces in the Wild (LFW) dataset reaching (human level) accuracy of 97.35%

Previous slide.

Even though good inductive biases help us reduce the amount of training data needed, it is still crucial to have a lot of labeled data. I find the example of Facebooks face recognition network interesting, because some years ago, millions of users worked for Facebook by labeling their images, without ever getting paid for it and probably without knowing that they helped Facebook to create a state-of-the-art face recognition tool.

Also interesting about this work is the demonstration of transfer: although the network was trained on the faces of Facebook users, it was sufficient to just retrain the fully connected layers at the top to beat all benchmarks on a dataset of images of celebrities.

Summary

- Inductive bias by ConvNet structure
- Architectural choices reduced by more flexibility in architecture
- Needs a lot of data
- Energy hungry
- Transfer learning:
    - basic representation is stable across tasks
    - train a few output layers for specific task

Previous slide.

Training deep networks consumes a huge amount of energy. Think about this before your train your own deep network from scratch.

Transfer learning also helps to reduce energy consumption.

Artificial Neural Networks

Convolutional Neural Networks

Part 8: Beyond object recognition

Johanni Brea & W. Gerstner

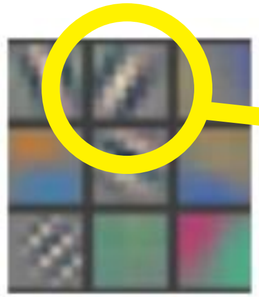
EPFL, Lausanne, Switzerland

1. Inductive bias in machine learning
2. Convolution filters as inductive bias for images
3. MaxPooling as inductive bias for images
4. Modern ConvNets and ImageNet competitions
5. Gradient of a max-pooling layer
6. Automatic Differentiation
7. Modern Deep Networks and image recognition
8. **Applications beyond object recognition**

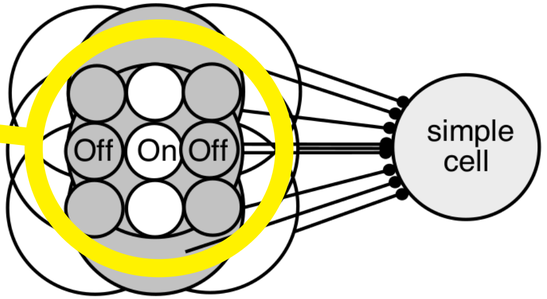
Next 4 slides

Convolutional neural networks share some features with biological neural networks.
In particular neurons in higher layers tend to respond to more abstract features than neurons in lower layers. In layer 1 we find features like those of complex cells, described in the famous work of Hubbel and Wiesel that studied experimentally the visual system of cats. For layers 2 to 5 on the following slides, we find the top 9 image patches that maximally activated a trained neural network together with a reconstruction of the feature within each patch that was responsible for the high activation. We see that neurons in the second layer respond to basic features like colors or edges of different orientation. In layer 5 for example, we see neurons that respond to the face of a dog with a high degree of invariance, i.e. the feature is not “distracted” by different backgrounds of kinds of dogs.

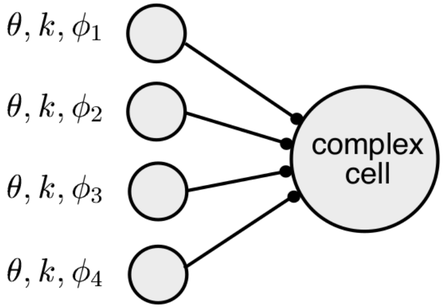
Neuroscience



Layer 1



LGN receptive fields



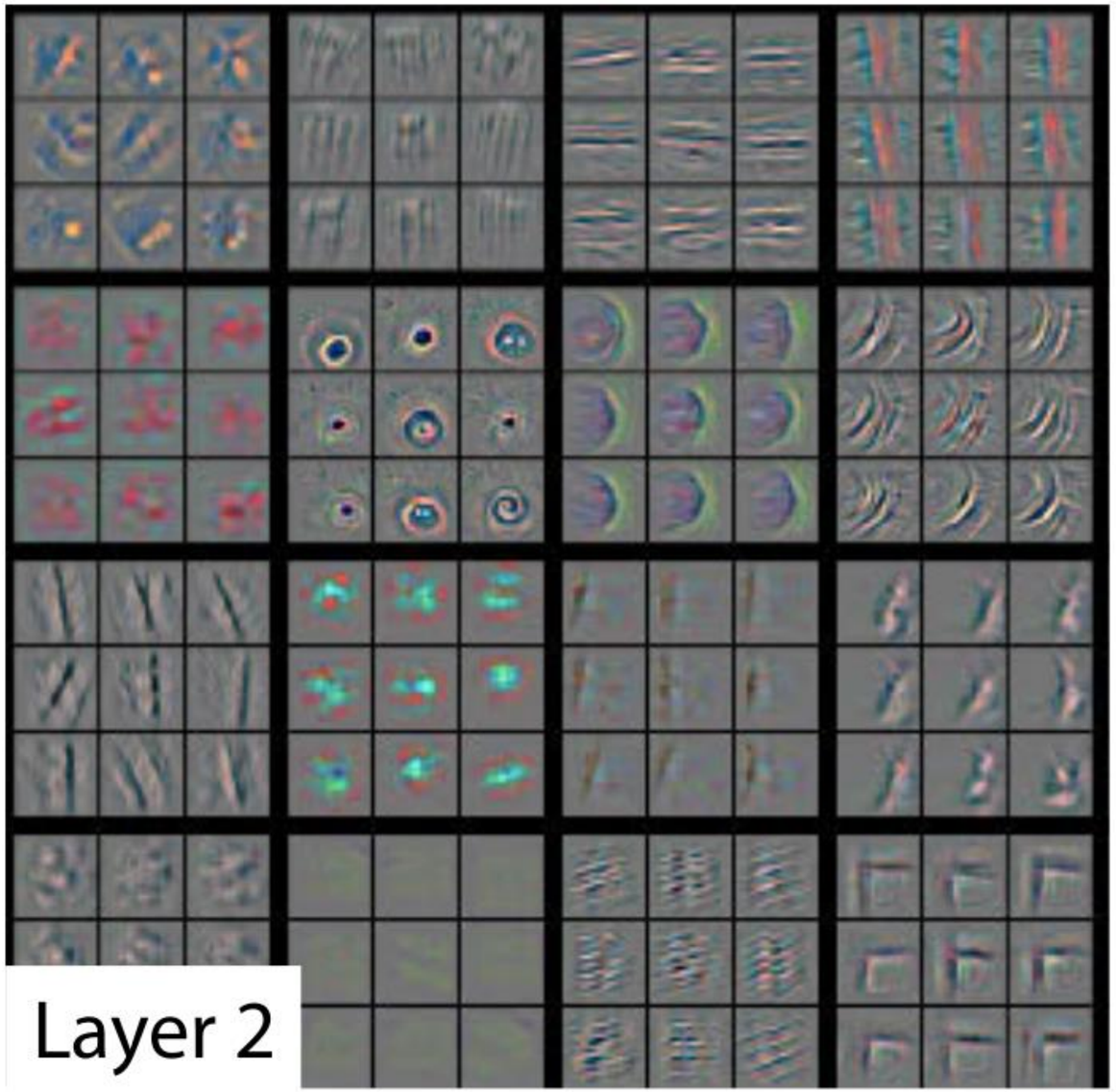
simple cells

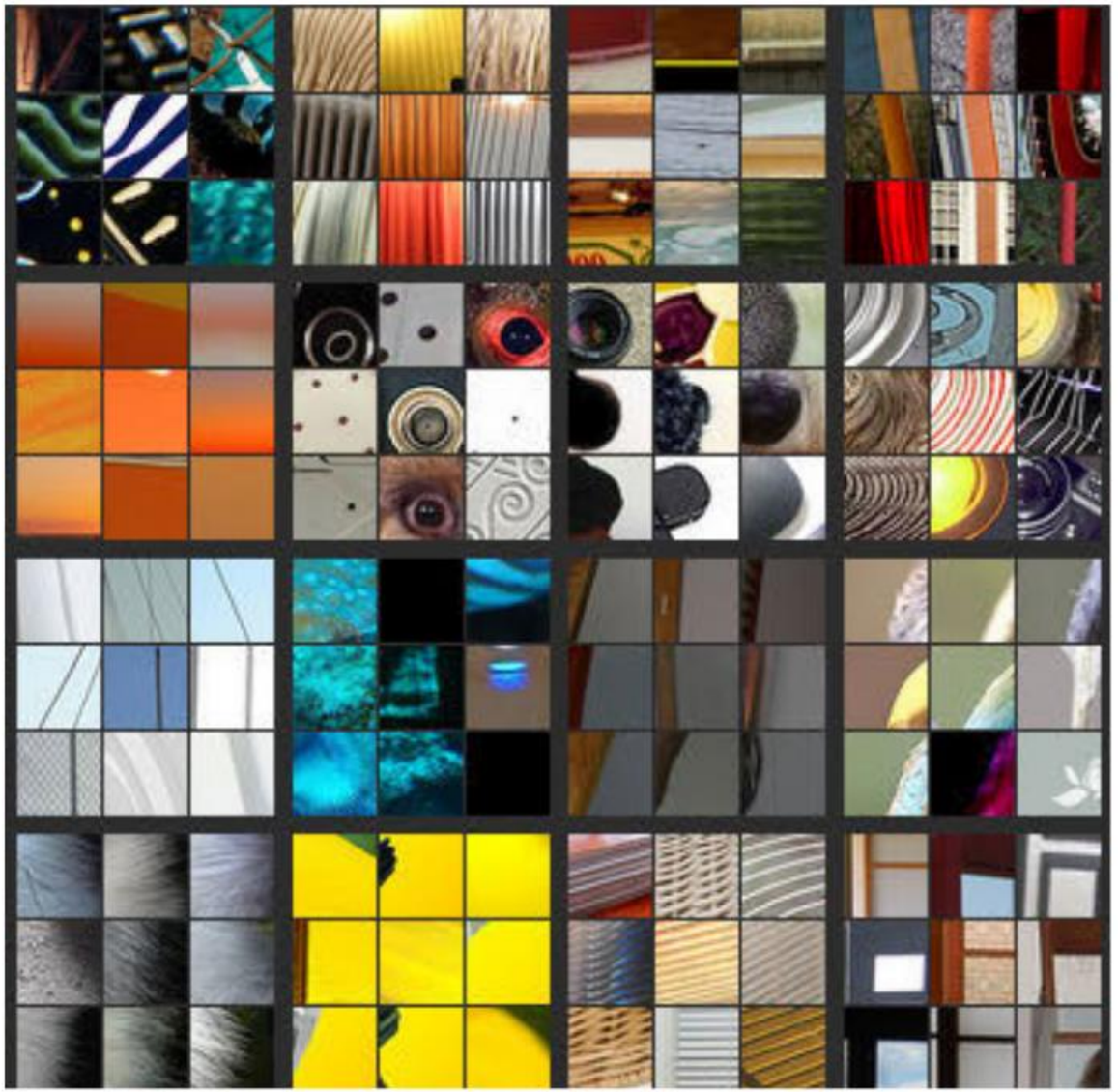
- ConvNets have similar architecture and similar features as the visual system of animals and humans

Dayan and Abbott, Theoretical Neuroscience, 2001

https://arxiv.org/pdf/1311.2901.pdf

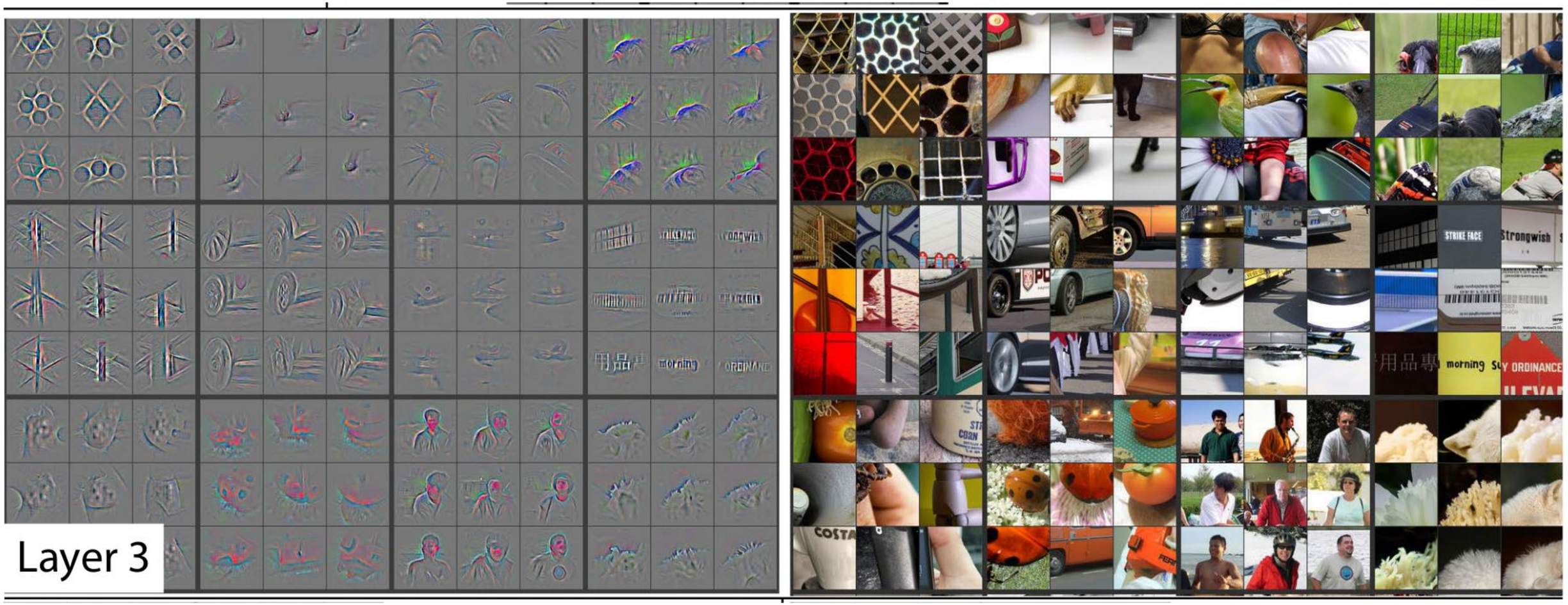
Visualizing convolutional networks





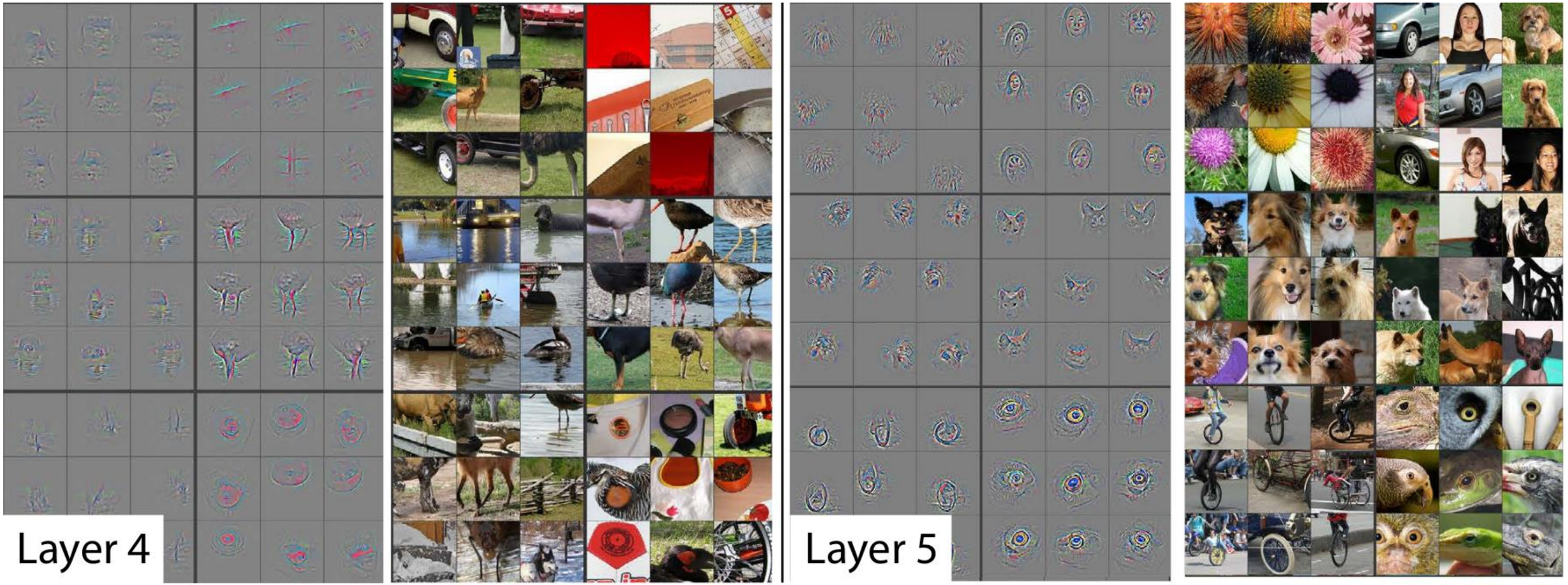
https://arxiv.org/pdf/1311.2901.pdf

Visualizing convolutional networks



https://arxiv.org/pdf/1311.2901.pdf

Visualizing convolutional networks



https://arxiv.org/pdf/1311.2901.pdf

Neural style

https://arxiv.org/pdf/1508.06576.pdf

Previous slide.

Neural style transfer is a now famous application of convolutional neural networks. Shown is a photo of Tübingen, regenerated by the network in 3 different styles, where the style was extracted from the small images in the bottom left corner of each panel.

Neural style

$$E_L = \sum (G^L - A^L)^2$$

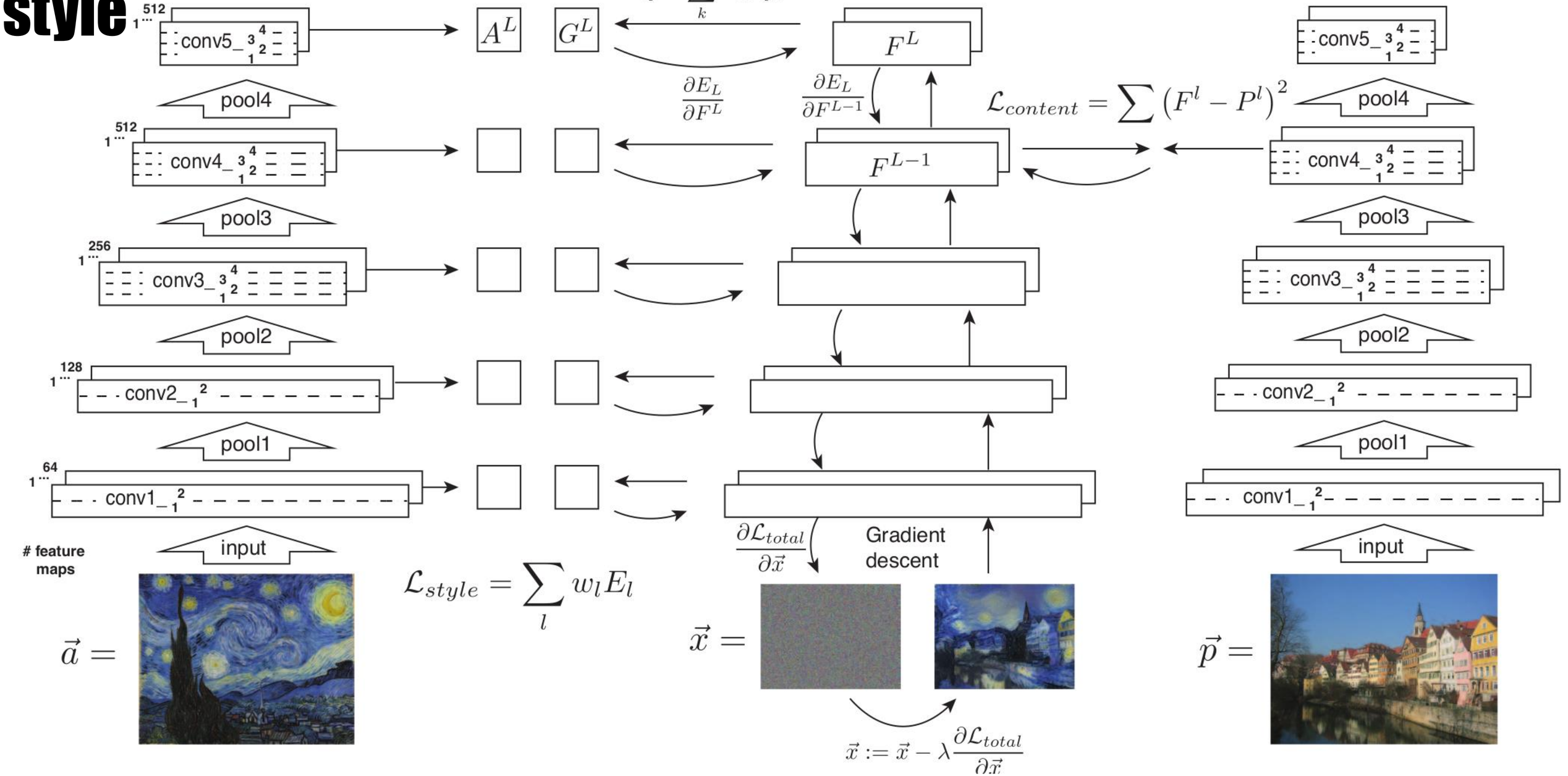
$$\mathcal{L}_{total} = \alpha \mathcal{L}_{content} + \beta \mathcal{L}_{style}$$

$$G^L_{ij} = \sum_k F^L_{ik} F^L_{jk}.$$

$$\mathcal{L}_{content} = \sum \left(F^l - P^l\right)^2$$

$$\mathcal{L}_{style} = \sum_l w_l E_l$$

$$\vec{x} := \vec{x} - \lambda \frac{\partial \mathcal{L}_{total}}{\partial \vec{x}}$$



Previous slide.

This works the following way:
A trained convolutional neural network is activated with an input image p. The activity P^l in each feature layer l in response to this input can than be used to reconstruct the image.

This is done in the following way: start with a new input of random pixel values x and compare the feature response F^l to P^l. Now we minimize the difference between F^l and P^l with gradient descent to find back a reconstruction of p. When using the features in the lower layers the reconstruction is almost perfect.

The style of an image can be reconstructed by following a very similar procedure but instead of comparing the responses, the difference between feature correlations G^l and A^l are minimized.

If the losses of style reconstruction and content reconstructions are mixed, the resulting image will have the content of input image p with the style of input image a.

Caption generation and generative models of images

a cow is standing in the middle of a street

https://cs.stanford.edu/people/karpathy/deepimagesent/

ConvNet + bidirectional RNN

https://thispersondoesnotexist.com/

ConvNets in generative adversarial networks (GANs)

Previous slide.

Convolutional networks together with recurrent neural networks have also enabled better automatic caption generators. On the left the image is given as input and the caption is generated as the output from recurrent neural network that received as input abstract features extracted with a convolutional neural network.

You have seen already in the previous lecture the images of persons that don’t exist that where generated with generative adversarial networks that use convolutional neural networks.

Summary

1) Use good explicit inductive biases together with transfer learning and data augmentation.

2) Convolutions (and max-pooling) are reasonable inductive biases for natural images.

3) Residual connections help to learn in very deep networks.

4) Modern architectures like Inception-Resnet-v2, DeepFace and NASNet reach human level performance on recognition tasks.

5) AutoDiff is a flexible tool to train different architectures.

6) ConvNets and neurons in the visual systems have similar receptive fields.